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Original article

Numerical simulations of seepage flow in rough single rock fractures

Qingang Zhang [a], Yang Ju [b, c, *], Wenbo Gong [a], Liang Zhang [a], Huafei Sun [a]

[a] School of Mechanics and Civil Engineering, China University of Mining and Technology, Beijing 10083, China
[b] State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou 221116, China
[c] State Key Laboratory of Coal Resources and Safe Mining, China University of Mining and Technology, Beijing 100083, China

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ABSTRACT

To investigate the relationship between the structural characteristics and seepage flow behavior of rough single rock fractures, a set of single fracture physical models were produced using the Weierstrass–Mandelbrot functions to test the seepage flow performance. Six single fractures, with various surface roughnesses characterized by fractal dimensions, were built using COMSOL multiphysics software. The fluid flow behavior through the rough fractures and the influences of the rough surfaces on the fluid flow behavior was then monitored. The numerical simulation indicates that there is a linear relationship between the average flow velocity over the entire flow path and the fractal dimension of the rough surface. It is shown that there is good agreement between the numerical results and the experimental data in terms of the properties a of the fluid flowing through the rough single rock fractures.

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1. Introduction

The permeability of natural rock is composed of two parts: the permeability of intact blocks and the permeability of macro-rock fractures (including single fractures and fracture networks) [1].

Actually, at present, the seepage coupling rock fracture models, including theoretical and numerical models, have been well discussed by the aforementioned pioneer researchers. Due to page limitation, we did not put detailed discussion about these methods in the original manuscript. Ref. [2] provided a detail discussion about the previous studies. Actually, the proposed methods or models, to just mention a few, include Representative Elementary Volume (REV), Discrete Fracture Network (DFN), Monte-Carlo Technique, Hydrological-Mechanical-Chemical (HMC), Parallel Plate Model, Channel Model, etc. For instance, Ref. [3] used REV and DFN models to study the scale dependency of the permeability of fractured rock, indicating significant scale-dependence. The combined Monte-Carlo technique and HRFRGM model based on field geological investigations and tests can perfectly simulate the heterogeneity and the random fracture distribution in rock [4]. The hydrological–mechanical–chemical (HMC) method has been used to explain the enigmatic spontaneous changes in permeability that develop within a fracture in limestone under simulated in situ conditions and has successfully replicated experimental measurements in limestone [5]. The parallel-plate model, which includes contact areas and artificial fractures, has been proposed to evaluate the effects of contact area and surface roughness on the behavior of fluid flow through rock fractures [6], and because the model is based on cubic laws of smooth single fractures, it has been widely used in seepage analyses of

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* Corresponding author. State Key Laboratory of Geomechanics and Underground Engineering, China University of Mining and Technology, Xuzhou 221116, China.

E-mail addresses: zqg_hank@163.com (Q. Zhang), juy@cumtb.edu.cn (Y. Ju), 11106300406@student.cumtb.edu.cn (W. Gong), zhangliang_619@126.com (L. Zhang), shf@cumtb.edu.cn (H. Sun).

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rock fractures. However, because the surface structure of natural rock is typically rough and irregular, satisfying the smooth fracture assumption of a parallel plate model is often difficult. Groove models include the fluid flow and solute transport through channels of variable aperture in a tight fractured medium. Tsang and others [7,8] have proposed using groove models, which are based on high stress rock seepage. They also incorporated the fracture model from the parallel plate model into the groove model. However, the groove distribution of fracture surfaces under stress is very complex, resulting in numerous accuracy problems when using the groove model. Therefore, research and application of the groove model has been very limited [2].

Nevertheless, challenges still remain in quantitatively characterizing the flow mechanisms of complex structures. Few studies regarding the influence of irregular rough fractures on fluid flow behavior and velocity distribution are available. Especially, there is a lack of physical experiments to uncover the complex seepage mechanisms. Indeed, the authors [1] have conducted a series of laboratory tests to investigate the behavior of fluid flow through a set of rough single fractures. In these experiments, the Weierstrass–Mandelbrot fractal function and PMMA material were employed to generate fractures with various fractal roughnesses. As a supplement to our experimental investigations, this study reports a numerical approach to determining the velocity distribution of fluid flowing through the complex rough structures. In addition, it may provide an effective way to characterize the stress distribution in the walls of flow paths during fluid flow which is considerably difficult to identify through experiments.

The purpose of this paper is to numerically investigate the effects of surface roughness on the seepage properties of single rough fractures, and the accuracy of the simulations is validated through comparison with experimental data.

2. Theory

The rough single fractures models with various fractal roughness were produced by the Weierstrass–Mandelbrot function with MATLAB and CAD programming code. The Weierstrass–Mandelbrot function [9,10] is formulated as follows:

$$W(t) = \sum_{n=-\infty}^{\infty} \left(1 - e^{ib^n t}\right) e^{i\phi_n} \Big/ b^{(2-D)n} \tag{1}$$

where b refers to a real number that is greater than1, φ_n is any angle and $D \in (1, 2)$ is the fractal dimension. Taking the real part of $W(t)$ as the fractal governing function, $C(t)$ yields:

$$C(t) = \mathrm{Re}W(t) = \sum_{n=-\infty}^{\infty} (1 - \cos b^n t) \Big/ b^{(2-D)n} \tag{2}$$

where C(t) to a continuous, non-differentiable function, with the fractal dimension D complying with [11]:

$$D_{HB} - (B/b) \le D \le D_{HB} \tag{3}$$

where B is a real number, and D_{HB} refers to the Hausdorff–Besicovitch dimension.

We define the permeability coefficient of a single rough fracture as follows, to quantify the influence of a rough structure on water flow through the single fracture:

$$K^d = \frac{\mu \cdot Q \cdot L^d}{\Delta P \cdot A} \tag{4}$$

whereK^d represents the fractal permeability coefficient. Q is the average flow flux per unit time. L^d refers to the total length of the rough fracture. A is the cross-sectional area. ΔP is the pressure difference [1].

3. Numerical models of rough single fracture

The assumption for fracture permeability was made with the following considerations: First, compared to intact blocks, macroscale fractures have much bigger water flow capacity and higher water permeability. The contribution of intact blocks to the water permeability of entire rock masses is negligible. Second, in order to focus on the effect of complexity of rough structures on fluid flow in macroscale fractures, we ignored the seepage flow from intact blocks to macroscale fractures. In the other words, in this study, we merely pay our attention to the fluid flow behavior within the macroscale fractures. Nevertheless, we are fully aware of that those intact blocks that comprise microscale (or even nanoscale) fractures could make great contribution to fluid (gas) flow capacity of entire rock masses under certain circumstances. Thus, we will present our simulation results of gas flow with taking account of contributions both from macroscale fractures and intact blocks containing micro fractures in the future.

Indeed, taking the real part of equation (1) as the fractal governing function, $C(t)$ yields equation (2) [9,10]; In terms of generation of the fractal curves, we took MATLAB code to generate the fractal curves following equation (2) where b equals to a constant of 1.4, t ($t \in (0 : 0001 : 1)$), n ($n \in (-100 : 100)$) and D equal to 1.0, 1.1, 1.2, 1.3, 1.4, and 1.5 respectively.

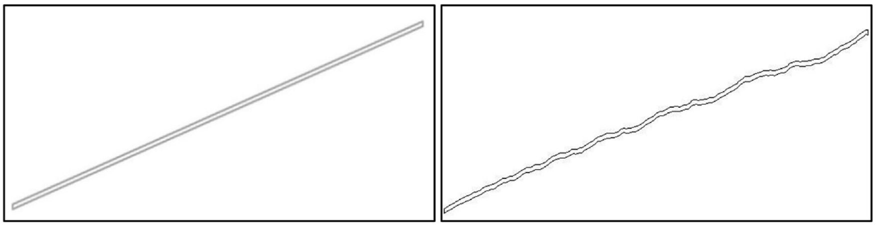
Researchers used to define fracture roughness through the following methods: Bump Height [12], JRC [13,14], and Fractal Dimension [12,15,16]. In this study, we adopted fractal dimensions to depict the roughness of various fractures. Fig. 1 shows a group of single rough fracture models with various surface roughness values, in which the fractal dimensions are equal to 1.0, 1.1, 1.2, 1.3, 1.4 and 1.5 respectively, and the model with $D = 1.0$ represents a smooth, flat fracture.

During the physical experiments, the physical models, which were made of transparent and homogeneous PMMA material, were used to observe the fluid seepage processes. Six fractal models, which were characterized by the fractal dimension, were constructed and used to investigate the fluid flow behavior through rough fractures, as well as the influences of rough surfaces on this behavior. A high-speed video camera was used to record the fluid flow process through the entire single rough fracture with a constant hydraulic pressure [1].

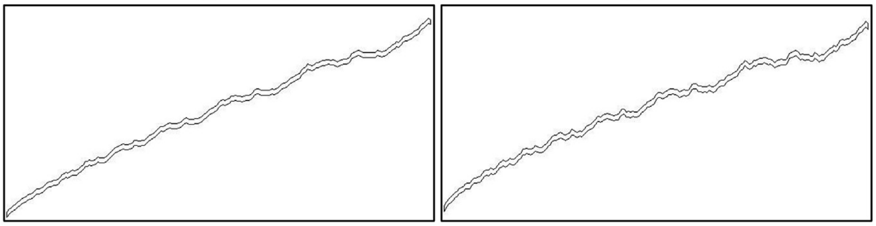
In equation (4), Ld refers to the total trajectory length of the fractal fracture, which is calculated by AUTOCAD based on equation (2). The Ld values for the six models are listed in Table 1. Index A refers to the cross section area of the fracture. The fractal depth and width are 5.0 mm and 2.0 mm, respectively, and the total area of the cross-section (A) is 10 mm^2.

4. Numerical simulation of rough single fracture

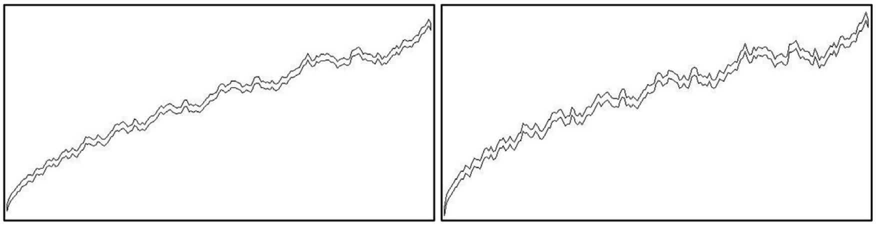
COMSOL multiphysics software was adopted to simulate the influences of the rough surfaces characteristic on the fluid flow behavior, particular the analysis of the mean fluid velocity and permeability of the entire flow path. The fluid properties, boundary conditions and the convergence are essential to the



(a) D=1.0 (b) D=1.1



(c) D=1.2 (d) D=1.3



(e) D=1.4 (f) D=1.5

Fig. 1. Numerical models of a rough single fracture with various surface roughnesses. From (a)–(f), the fractal dimension D is equal to 1.0, 1.1, 1.2, 1.3, 1.4, and 1.5, respectively, where D = 1.0 represents a smooth, flat fracture.

accuracy of the numerical simulation. Thus, relevant descriptions of the conditions and parameters used in the numerical simulation are as follows:

(1) The fluid properties: the chosen fluid in this paper is water, which has a density and viscosity of 1000 kg/m^3 and 0.001 $Pa \bullet s$, respectively.

(2) The boundary conditions: During the computational fluid dynamics (CFD) simulation, the basic boundary parameter conditions include the flow inlet boundary P1, the flow outlet boundary P2, open boundary conditions, symmetry boundary and the wall boundary V0. In our simulation, the boundary conditions are set as constant in the experimental setup. The far-right end of the model is the inlet, which is set as a constant pressure boundary at a pressure of 490 Pa. The far-left end of model is the outlet, which is also set as a constant pressure boundary, at a pressure of 0 Pa. The other parts of the model are set as no-slip boundaries, with fluid velocity set to zero in all directions.

(3) The convergence: The simulation convergence is controlled by mesh generation, fluid properties and number of iterations. As the scale of the numerical model is fairly small, “free triangular meshes” and the program code are adopted to ensure convergence in the numerical simulation.

Table 1
Total length of the profile trajectories of the fractal fractures with various fractal dimensions.

Fractal dimension D_i	0	1.10	1.20	1.30	1.40	1.50
Total length $L(D_i)$/mm	217.70	225.50	237.30	264.40	321.30	400.40

In this study, the complexity of flow paths is described by the fractal dimensions of the path profile trajectories. The projection lengths in the vertical direction are kept the same in the six fractal fracture models. In order to distinguish the effect of path roughness from other effects on seepage flow behavior in the single fractures, we set the fracture width and pressure difference to be constant for all tested models.

A laminar flow pattern was adopted in the numerical calculation of fluid flow behavior in the numerical models. During the physical flow experiments, the water pressure (hydraulic head) was held constant at 490 Pa. The water pressure was also set to 490 Pa in all numerical simulations. Due to the fine scale of the numerical models, the method of free triangular meshes was adopted in the numerical simulations to better calculate and analyze the effects of fracture structure characteristics on fluid flow behavior. In the numerical simulation, the fluid flow velocity (Fig. 2) over the entire flow path gradually decreased as the fractal dimension value increased.

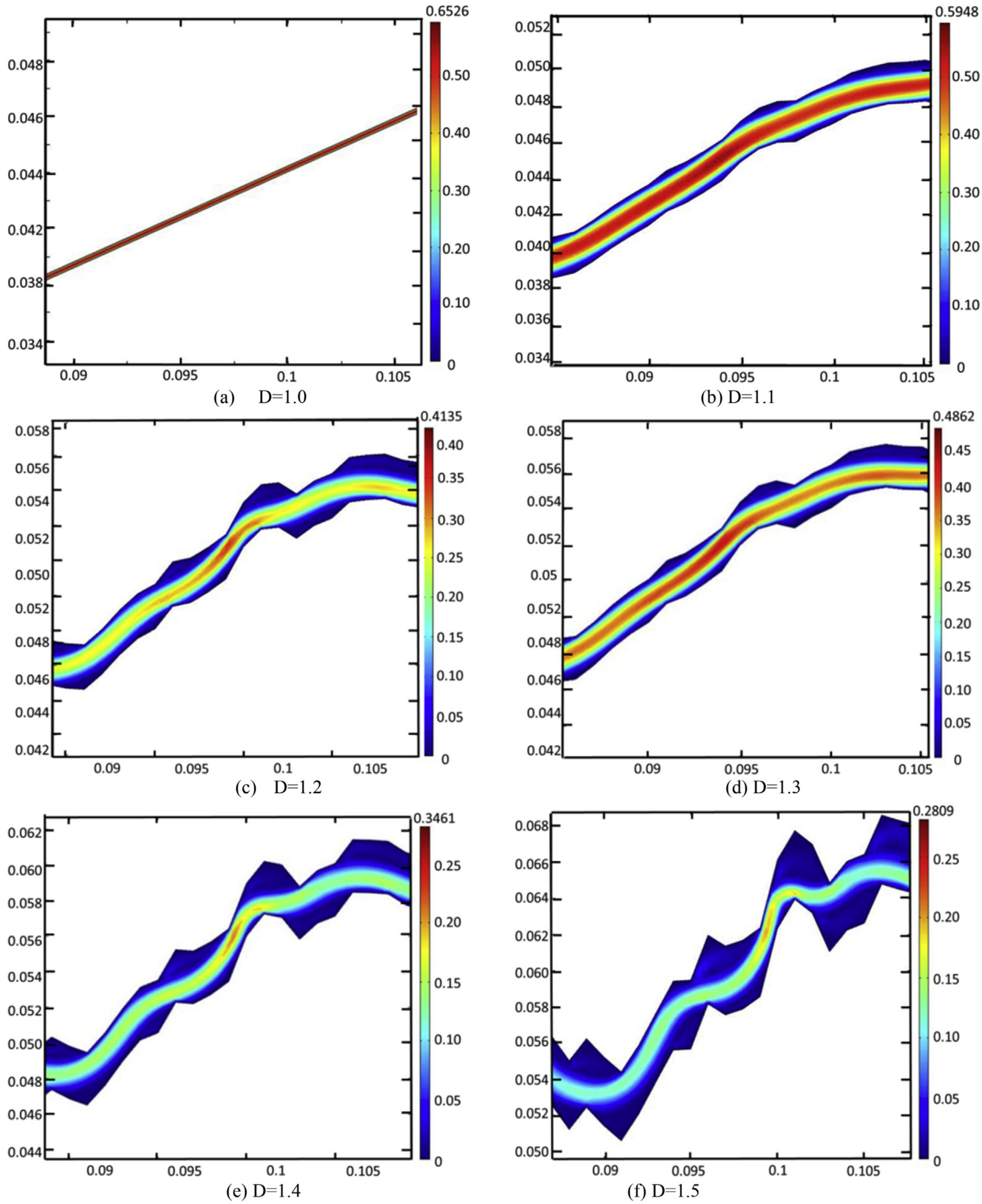
5. Results and discussion

During the fluid flow process, the fractal dimension increased as the mean fluid velocity over the entire flow path linearly decreased (Fig. 3). The relationship between the mean flow velocity over the entire flow path and the fractal dimension, D, can be approximated as follows:

$$V = -39.4D + 72.7 \quad R^2 = 0.9399 \tag{5}$$

$$V = -42.3D + 76.7 \quad R^2 = 0.9038 \tag{6}$$

The functions in equations (5) and (6) are the fitted curves based on the experimental data and the numerical results,



(a) D=1.0 (b) D=1.1

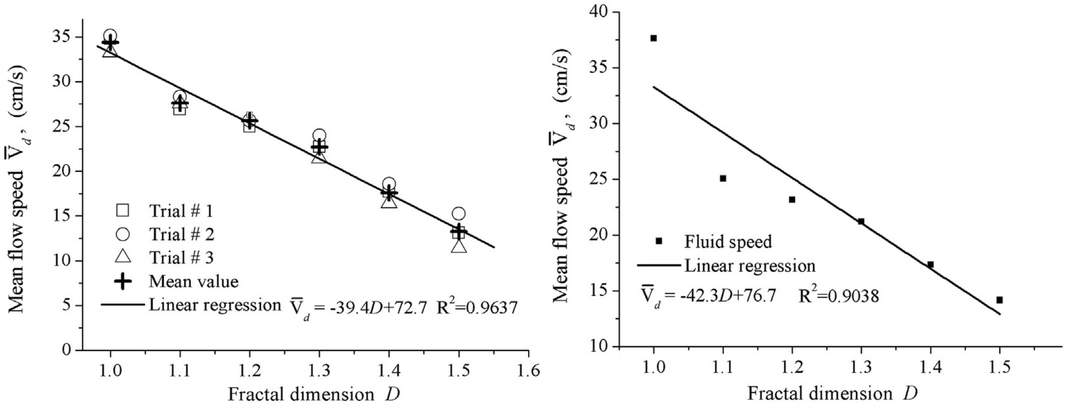
(c) D=1.2 (d) D=1.3

(e) D=1.4 (f) D=1.5

Fig. 2. Fluid velocity distribution of the rough fracture models. From (a)–(f), the fractal dimension D is equal to 1.0, 1.1, 1.2, 1.3, 1.4 and 1.5, respectively.

respectively. In statistics, the coefficient, denoted as R^2, is a number indicating how well the data fit a statistical model. It provides a measure of how well observed outcomes are described by the model. Under the same experimental conditions, the numerical results produced equivalent water flow velocities in the rough single fractures as the experimental data.

Fig. 4 illustrates that there is a good agreement between the numerical results and the experimental data in terms of the properties of the fluid flowing through the rough single rock fracture. The functional relationship between the permeability and fractal dimension, D, from the physical experimental data and numerical simulation results can be approximated by:



(a) Experimental data (Ju et al., 2013) (b) Numerical results

Fig. 3. Linear regression of the average water velocity and various values of fractal dimension based on the numerical results and experimental data.

$$k = e^{11.5-6D} + 16.7 \quad R^2 = 0.9992 \tag{7}$$

$$k = e^{12.3-7D} + 17.3 \quad R^2 = 0.9954 \tag{8}$$

6. Conclusions

Based on various rock fracture surface roughness values, a set of single fracture numerical models were produced using the Weierstrass–Mandelbrot functions to test the seepage flow performance. Six single fractures, with various surface roughnesses characterized by the fractal dimension, were built using COMSOL multiphysics software. The fluid flow behavior through the rough fractures and the influences of the rough surfaces on the fluid flow behavior of the fractures were then monitored. Fractal dimension (characterization of a rough fracture) was found to be linked to fracture roughness and fluid flow velocity. In the numerical simulations, the larger the fractal value of a rough fracture, the more rough and irregular the roughness of the fracture. In addition, the numerical simulation indicated that there is a linear relationship between the average flow speed over the entire flow path and the fractal dimension of the rough surface. Numerical results were found to be in good agreement with experimental data, in terms of the properties of the fluid flowing through the single rough rock fracture.

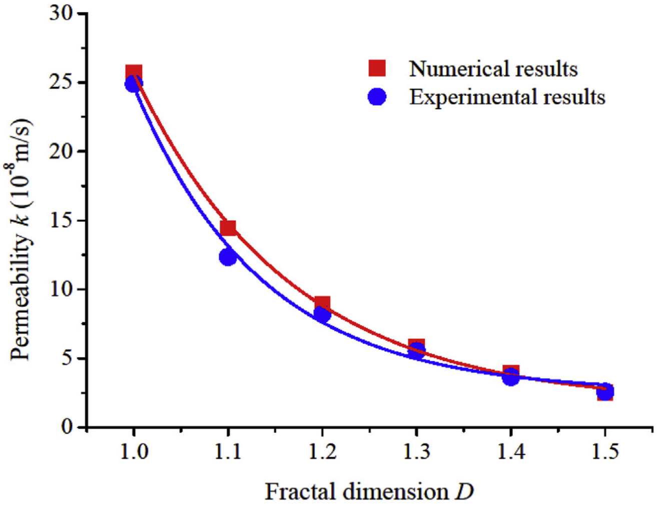


Fig. 4. Functional relationship between the permeability and the fractal dimension from experimental data and numerical results.

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