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Are deviation from bi-maximal mixing and non-zero U_{e3} related to non-degeneracy of heavy Majorana neutrinos?

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Abstract

We propose a scenario that the mass splitting between the first generation of the heavy Majorana neutrino and the other two generations of degenerate heavy neutrinos in the seesaw framework is responsible for the deviation of the solar mixing angle from the maximal mixing, while keeping the maximal mixing between the tau and muon neutrinos as it is. On top of the scenario, we show that the tiny breaking of the degeneracy of the two heavy Majorana neutrinos leads to the non-zero small mixing angle U_{e3} in the PMNS matrix and the little deviation of the atmospheric neutrino mixing angle from the maximal mixing.

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Thanks to enormous progress in solar, atmospheric and terrestrial neutrino experiments, we have now the robust evidence for the existence of neutrino oscillation which provides a window to physics beyond the standard model (SM). Until now, while the atmospheric neutrino deficit still points toward a maximal mixing between the tau and muon neutrinos, however, the solar neutrino problem favors a not-so-maximal mixing between the electron and muon neutrinos. There have been many attempts to explain the origin of the deviation of the solar mixing angle from the maximal mixing. Surprisingly, it has recently been noted that the solar neutrino mixing angle $\theta_{\rm sol}$ required for a solution of the solar neutrino problem and the Cabibbo angle θ_C reveal a striking relation [1], $\theta_{\rm sol} + \theta_C \simeq \frac{\pi}{4}$, which is satisfied by the experimental results within a few percent accuracy, $\theta_{\rm sol} + \theta_C = 45.4^\circ \pm 1.7^\circ$ [2–4]. This quark–lepton complementarity (QLC) relation has been simply interpreted as an evidence for certain quark–lepton symmetry or quark–lepton unification as shown in Refs. [1,5,6]. But, it can be an accidental phenomenon as pointed out in Refs. [6,7]. Thus, it is worthwhile to find the possible alternatives to the grand unification origin of the deviation of the solar mixing from the maximal mixing.

In this Letter, we propose a scenario that the mass splitting between the first generation of the heavy Majorana neutrino and the other two generations of degenerate heavy neutrinos in the seesaw framework is responsible for the deviation of the solar mixing angle from the maximal mixing, while keeping the maximal mixing between the tau and muon neutrinos as it is. The maximal atmospheric neutrino mixing and the smallness of U_{e3} may be the trace of the original “bi-maximal” mixing which is presumably supposed to be achieved by some underlying flavor symmetries, and thus the best possible approach to the problem is to start in the limit of the maximal mixing with $U_{e3} = 0$, and understand how the deviation of the solar mixing from the maximal is realized. In our scenario, the primitive “bi-maximal” neutrino mixing is generated only from the neutrino Dirac Yukawa matrix by taking a diagonal

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form of three degenerate heavy Majorana neutrinos in a basis where the charged lepton mass matrix is real and diagonal. As will be shown, the deviation of the solar mixing can then be generated from breakdown of the degeneracy of the heavy Majorana neutrino masses between the first and the other two generations. The main point in this scenario is that the deviation can be expressed in terms of the ratio between two heavy Majorana neutrino masses. On top of the scenario, we will also show that the tiny breaking of the degeneracy of the two heavy Majorana neutrinos will lead to the small mixing angle θ_{13} in the PMNS matrix and the very small deviation of the atmospheric neutrino mixing angle from the maximal mixing. We will propose that the origin of the tiny mass splitting among three heavy Majorana neutrinos is the effective dimension-five operators whose structures are governed by discrete flavor symmetry, and show that the desirable mass splitting responsible for the deviation of maximal mixing of solar neutrinos and non-zero U_{e3} is related with the vacuum structures of some scalar fields introduced in the dimension-five operators.

Before proceeding to our scenario, we wish to motivate one scheme that leads to exact "bi-maximal" mixing in the framework of the seesaw mechanism. We study in a basis where the charged lepton mass matrix is real and diagonal. The light neutrino mass matrix M_ν diagonalized by U_{bimax} is given through the seesaw mechanism by

$$M_\nu = M_D^T M_R^{-1} M_D = U_{\text{bimax}} M_\nu^{\text{diag}} U_{\text{bimax}}^T, \tag{1}$$

where $M_D = Y_D v/\sqrt{2}$ with electroweak vacuum expectation value v and the neutrino Dirac Yukawa matrix Y_D, and M_R is a mass matrix of heavy Majorana neutrinos. The mixing matrix U_{bimax} denotes the "bi-maximal" mixing matrix [8]:

$$U_{\text{bimax}} = U_{23}\left(\frac{\pi}{4}\right)U_{12}\left(\frac{\pi}{4}\right) = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{2}$$

Then, the "bi-maximal" mixing can be achieved by one of the three possible ways as follows:

- Taking Y_D diagonalized by U_{bimax} and $M_R = M \cdot I$ with the identity matrix I and a common mass scale M;
- Taking $Y_D = y \cdot I$ and M_R diagonalized by U_{bimax};
- Taking "bi-maximal" mixing pattern from the combination of the nontrivial Y_D and M_R.

For the third case, there may exist various origins of the deviation of the solar mixing depending on possible combinations, and some of which have been discussed before [9]. For the other two cases, the modification of the trivial sectors proportional to the unit matrix can be in charge for the origin of the deviation from the maximal mixing. However, since the second case may lead to the undesirable deviation of the atmospheric mixing aside from the deviation of the solar mixing as one can easily see, we only focus on the first case in this Letter. In the first case, the "bi-maximal" mixing can be achieved by taking the symmetric matrix Y_D with specific form. As an example, we present a detailed model of Y_D leading to the "bi-maximal" mixing, while keeping $M_R = M \cdot I$ based on the discrete symmetry $A_4 \otimes Z_2$ [10]. Let the three families of leptons and singlet heavy neutrinos be denoted by $(\nu_i, l_i)_L$, l_{iR}, N_{iR} for $i = 1, 2, 3$. In this convention, $\bar{l}_{iL} l_{jR}$ and $\bar{\nu}_{iL} N_{jR}$ are Dirac mass terms for charged leptons and neutrinos. Under the discrete symmetry $A_4 \otimes Z_2$, the 3 families of leptons transform as $(\nu_i, l_i)_L \sim (\mathbf{3}, +)$, $N_{iR} \sim (\mathbf{3}, +)$, $l_{iR} \sim (\mathbf{1}, -), (\mathbf{1}', -), (\mathbf{1}'', -)$. We introduce Higgs scalar sectors consisted of seven Higgs doublets $\Phi_i \sim (\mathbf{1}, -), (\mathbf{1}', -), (\mathbf{1}'', -)$, $\phi \sim (\mathbf{1}, +)$, $\sigma_i \sim (\mathbf{3}, +)$. From the assignment, the $A_4 \otimes Z_2$ invariant Dirac Yukawa interactions for charged lepton sector, $\bar{l}_{iL} l_{iR} \Phi_j$, leads to a diagonal mass matrix with 3 independent entries each as shown in Ref. [11]. For the mass matrix of the heavy Majorana neutrinos, we can take $M N_{iR} N_{iR}$ with common mass scale M because of A_4 symmetry, i.e., $\mathbf{3} \times \mathbf{3} \sim \mathbf{1}$. The Dirac Yukawa matrix for the neutrino sector, which is invariant under $A_4 \otimes Z_2$ and diagonalized by the "bi-maximal" mixing matrix, can be obtained from the interaction Lagrangian as follows:

$$Y_D = h_1(\bar{\nu}_1 N_1 + \bar{\nu}_2 N_2 + \bar{\nu}_3 N_3)\phi + h_2(\bar{\nu}_1 N_2 \sigma_3 + \bar{\nu}_2 N_3 \sigma_1 + \bar{\nu}_3 N_1 \sigma_2) + h_3(\bar{N}_1 \nu_2 \sigma_3 + \bar{N}_2 \nu_3 \sigma_1 + \bar{N}_3 \nu_1 \sigma_2) + \text{h.c.} \tag{3}$$

In order to achieve the symmetric form of the Dirac Yukawa matrix, we require $h_2 = h_3$. The vacuum expectation values for the neutral components of Higgs sector σ_i^0 can be determined by the Higgs potential invariant under A_4,

$$\begin{aligned} V = {} & m^2 \sigma_i^\dagger \sigma_i + \frac{1}{2}\lambda_1\left(\sigma_i^\dagger \sigma_i\right)^2 + \lambda_2\left(\sigma_1^\dagger \sigma_1 + \omega^2 \sigma_2^\dagger \sigma_2 + \omega \sigma_3^\dagger \sigma_3\right)\left(\sigma_1^\dagger \sigma_1 + \omega \sigma_2^\dagger \sigma_2 + \omega^2 \sigma_3^\dagger \sigma_3\right) \\ & + \lambda_3\left[\left(\sigma_2^\dagger \sigma_3\right)\left(\sigma_3^\dagger \sigma_2\right) + \left(\sigma_3^\dagger \sigma_1\right)\left(\sigma_1^\dagger \sigma_3\right) + \left(\sigma_1^\dagger \sigma_2\right)\left(\sigma_2^\dagger \sigma_1\right)\right] + \left\{\frac{1}{2}\lambda_4\left[\left(\sigma_2^\dagger \sigma_3\right)^2 + \left(\sigma_3^\dagger \sigma_1\right)^2 + \left(\sigma_1^\dagger \sigma_2\right)^2\right] + \text{h.c.}\right\}, \end{aligned} \tag{4}$$

where $\omega = e^{2\pi/3}$. Taking $\langle \sigma_1^0 \rangle = 0$ and $\langle \sigma_2^0 \rangle = \langle \sigma_3^0 \rangle = v$ with $v = \sqrt{\frac{-m^2}{2\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}}$ as well as non-vanishing $\langle \phi^0 \rangle$ for the Higgs sector ϕ, we can achieve the final form of the Dirac Yukawa matrix given as follows,

$$Y_D = \begin{pmatrix} a & b & b \\ b & a & 0 \\ b & 0 & a \end{pmatrix}. \tag{5}$$

Defining $Y_D^{\text{diag}} = \text{diag}(x, y, z)$, the neutrino Dirac Yukawa matrix Y_D diagonalized by U_{bimax} is generally given in the symmetric matrix form by

$$Y_D = U_{\text{bimax}} Y_D^{\text{diag}} U_{\text{bimax}}^T. \tag{6}$$

Here, we consider the case of non-zero values for x and y, which is crucial to our purpose.

In order to achieve the observed deviation of the solar neutrino mixing from the maximal mixing, we take into account the mass splitting between the first generation of the heavy Majorana neutrino and the other two degenerate ones, for which the mass matrix is given by $M_R = M_R^{\text{diag}} = (M_1, M_2, M_2)$, which results from the breaking of A_4 in the heavy neutrino sector and reflects separation of $N_{iR} \sim N_{1R}(\mathbf{1}) \oplus N_{(2,3)R}(\mathbf{2})$ under S_3 symmetry. Then, the light neutrino mass matrix M_ν is presented as follows:

$$\begin{aligned} M_\nu &= U_{\text{bimax}} \begin{pmatrix} x & & \\ & y & \\ & & z \end{pmatrix} U_{\text{bimax}}^T \begin{pmatrix} M_1^{-1} & & \\ & M_2^{-1} & \\ & & M_2^{-1} \end{pmatrix} U_{\text{bimax}} \begin{pmatrix} x & & \\ & y & \\ & & z \end{pmatrix} U_{\text{bimax}}^T \\ &= U_{\text{bimax}} \begin{pmatrix} x & & \\ & y & \\ & & z \end{pmatrix} U_{12}^T\left(\frac{\pi}{4}\right) \begin{pmatrix} M_1^{-1} & & \\ & M_2^{-1} & \\ & & M_2^{-1} \end{pmatrix} U_{12}\left(\frac{\pi}{4}\right) \begin{pmatrix} x & & \\ & y & \\ & & z \end{pmatrix} U_{\text{bimax}}^T \\ &= U_{\text{bimax}} M'_\nu U_{\text{bimax}}^T, \end{aligned} \tag{7}$$

where the mass matrix M'_ν is given by

$$M'_\nu = \begin{pmatrix} \frac{x^2}{2M_1M_2}(M_1+M_2) & \frac{xy}{2M_1M_2}(M_1-M_2) & 0 \\ \frac{xy}{2M_1M_2}(M_1-M_2) & \frac{y^2}{2M_1M_2}(M_1+M_2) & 0 \\ 0 & 0 & \frac{z^2}{M_2} \end{pmatrix}. \tag{8}$$

Then, the matrix M'_ν can be diagonalized by $U_{12}(\theta)$, and after diagonalizing M'_ν, we can obtain the mixing angle θ and three neutrino mass eigenvalues as follows:

$$\tan 2\theta = \frac{2xy(M_2 - M_1)}{(x^2 - y^2)(M_1 + M_2)}, \tag{9}$$

$$\begin{aligned} m_{\nu_1} &= \frac{1}{2M_1M_2}\left[\left(c^2x^2 + s^2y^2\right)(M_1+M_2) + 2csxy(M_1 - M_2)\right], \\ m_{\nu_2} &= \frac{1}{2M_1M_2}\left[\left(s^2x^2 + c^2y^2\right)(M_1+M_2) - 2csxy(M_1 - M_2)\right], \\ m_{\nu_3} &= \frac{z^2}{M_2}, \end{aligned} \tag{10}$$

where $c = \cos\theta$, $s = \sin\theta$. Comparing the mixing matrix $U_{12}(\theta)$ with $U_{12}(\pi/4)$ in U_{bimax}, we can get the solar mixing angle θ_{sol} which deviates as much as the value of θ from the maximal mixing. Note that the value of θ should be negative in order to achieve the desirable deviation of the solar neutrino mixing. We can argue that the generation of the mixing angle θ due to the splitting between M_1 and M_2 in seesaw mechanism may be the origin of the deviation of the solar mixing angle from the maximal mixing in the case of non-zero x and y. However, since we do not have yet any information on the values of M_1 and M_2, we cannot immediately test whether the difference between M_1 and M_2 is really compatible with the deviation of the solar mixing angle from the maximal mixing, but we can make numerical estimate for the size of the ratio of M_1 to M_2, which accommodates the deviation of the solar mixing based on the experimental results for the neutrino oscillation. From the numerical results, we can also predict the magnitude of the effective Majorana neutrino mass m_{ee}, which is the neutrino-exchange amplitude for the neutrinoless double beta decay.

For our purpose, let us define two parameters κ and ω as follows:

$$\kappa \equiv \frac{y}{x}, \qquad \omega \equiv \frac{M_1}{M_2}. \tag{11}$$

Then, the expressions for θ and m_{ν_i} are given as follows,

$$\tan 2\theta = \frac{2\kappa(1-\omega)}{(1-\kappa^2)(1+\omega)}, \tag{12}$$

$$m_{\nu_1} = \frac{x^2}{2M_1}\left[\left(c^2 + s^2\kappa^2\right)(1+\omega) + 2cs\kappa(\omega - 1)\right],$$

$$m_{\nu_2} = \frac{x^2}{2M_1}\left[\left(s^2 + c^2\kappa^2\right)(1+\omega) - 2cs\kappa(\omega - 1)\right],$$

$$m_{\nu_3} = \frac{z^2}{M_2}. \tag{13}$$

In addition, the effective Majorana neutrino mass m_{ee} is presented by

$$m_{ee} = \frac{x^2}{4M_1}\left[(1+\kappa)^2 + \omega(1-\kappa)^2\right]. \tag{14}$$

As shown in Eq. (12), the non-vanishing value of the mixing angle θ can arise when ω is deviated from one, which indicates the splitting between M_1 and M_2. In fact, the present experimental results are not enough to determine all the parameters introduced. But, if we fix one neutrino mass eigenvalue by hand, we can determine several independent parameters as well as the magnitude of m_{ee} from Eqs. (12)–(14). For our numerical calculation, we set the parameter θ, Δm^2_{21} and Δm^2_{32} to be 13°, 8×10^{-5} eV^2, 2.5×10^{-3} eV^2, respectively. Those numbers correspond to the best fit values for the measurements of the deviation of the solar mixing angle from the maximal mixing, the mass-squared differences of the solar and atmospheric neutrino oscillations, respectively. By fixing m_{ν_1} as an input parameter, we can determine the parameter set $(\kappa, \omega, \frac{x^2}{M_1}, \frac{z^2}{M_2})$ for normal hierarchy $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$ through the relations (11)–(13).

In Table 1, we present our numerical results for normal hierarchy. From the Table 1, we can see that the values of κ and ω approach to one as m_{ν_1} increases up to of order 0.1 eV, and one needs fine-tuning to obtain the parameter set satisfying the relations above for the case of such a large $m_{\nu_1} \sim 0.1$ eV. As m_{ν_1} goes down, the value of κ rapidly increases whereas that of ω decreases. We can also predict the size of the amplitude of the neutrinoless double beta decay m_{ee} as a function of m_{ν_1}, which is presented in the last column of Table 1. If the neutrinoless double beta decay will be measured in near future, we will be able to determine three neutrino mass eigenvalues and the parameters introduced in Eqs. (12)–(14). For inverted hierarchy $m_{\nu_3} < m_{\nu_1} < m_{\nu_2}$, the numerical results are presented in Table 2. In this case, contrary to the normal hierarchical case, we take m_{ν_3} as an input.

Next, to generate non-vanishing U_{e3}, on top of the above scenario, we consider an interesting possibility that the breaking of the degeneracy between the second and the third generation masses in the heavy Majorana neutrino sector, i.e., $M_R = \mathrm{diag}(M_1, M_2, M_3)$, can be an origin of the generation of non-vanishing U_{e3}. We remark that the value of U_{e3} goes to zero in the limit of $M_2 = M_3$ in this scenario. The effective light Majorana neutrino mass matrix is given by

$$M_\nu = U_{\mathrm{bimax}} \begin{pmatrix} x & & \\ & y & \\ & & z \end{pmatrix} U^T_{\mathrm{bimax}} \begin{pmatrix} M_1^{-1} & & \\ & M_2^{-1} & \\ & & M_3^{-1} \end{pmatrix} U_{\mathrm{bimax}} \begin{pmatrix} x & & \\ & y & \\ & & z \end{pmatrix} U^T_{\mathrm{bimax}} \tag{15}$$

$$= U_{\mathrm{bimax}} M'_\nu U^T_{\mathrm{bimax}}. \tag{16}$$

Assuming that the mass splitting between M_2 and M_3 is small enough to accommodate the tiny U_{e3}, the mixing matrix, which diagonalizes the neutrino mass matrix M_ν, can be approximately given by

$$U \simeq U_{23}\left(\frac{\pi}{4}\right) U_{12}\left(\frac{\pi}{4}\right) \begin{pmatrix} \cos\sigma & \sin\sigma & \delta \\ -\sin\sigma & \cos\sigma & \eta \\ -\delta & -\eta & 1 \end{pmatrix}, \tag{17}$$

where the mixing angle σ corresponds to the $(1,2)$ rotation of 2×2 submatrix of M'_ν. The mixing angle σ is presented by

$$\tan\sigma \simeq \frac{2\kappa(1-\omega-\varepsilon)}{(1-\kappa^2)(1+\omega+\varepsilon)}, \tag{18}$$

where $\varepsilon = M_1/M_3$, and ω, κ are given earlier. This mixing angle σ is responsible for the deviation of the solar mixing angle from the maximal mixing. We note that non-vanishing value of σ is possible even when $\omega = 1$, i.e., $(M_1 = M_2)$, but this case is undesirable because it leads to negative σ which *positively* contributes to θ_{12}. The mixing angle σ is zero when $\omega + \varepsilon = 1$, but it corresponds to the large hierarchy among three heavy Majorana masses, which is far beyond our purpose. The mixing elements δ

Table 1
All numbers corresponding to the mass parameters are given in the unit eV for normal hierarchy

m_{ν_1} (input)	κ	ω	$\frac{x^2}{M_1}$	$\frac{z^2}{M_2}$	m_{ee}
0.005	1.298	0.772	0.003	0.051	0.009
0.01	1.118	0.897	0.006	0.052	0.013
0.05	1.006	0.994	0.025	0.071	0.051
0.1	1.002	0.998	0.050	0.112	0.101

Table 2
All numbers corresponding to the mass parameters are given in the unit eV for inverted hierarchy

m_{ν_3} (input)	κ	ω	$\frac{x^2}{M_1}$	$\frac{z^2}{M_2}$	m_{ee}
0.005	1.672	0.585	0.011	0.010	0.041
0.01	1.569	0.630	0.013	0.013	0.042
0.05	1.137	0.881	0.029	0.051	0.066
0.1	1.044	0.959	0.052	0.100	0.109

Table 3
The numerical results for the ratio M_2/M_3 and the prediction for the bound on $|U_{e3}|$ for the normal hierarchical case and the inverted hierarchical case

$m_{\nu_{1(3)}}$ (input)	M_2/M_3	$\|U_{e3}\|$	M_2/M_3	$\|U_{e3}\|$
0.005	0.55–1.29	(< 0.015)	0.44–1.36	(< 0.025)
0.01	0.72–1.16	(< 0.0007)	0.59–1.27	(< 0.015)
0.05	0.94–1.04	(< 0.086)	0.93–1.04	(< 0.083)
0.1	0.98–1.01		0.98–1.01	

and η are given by

$$\delta = c_1\left(-\frac{1}{M_2}+\frac{1}{M_3}\right), \qquad \eta = c_2\left(-\frac{1}{M_2}+\frac{1}{M_3}\right), \tag{19}$$

where c_1 and c_2 are presented in terms of three light neutrino mass eigenvalues and the parameters $\kappa, \omega, \varepsilon$. Then, the mixing element U_{e3} and the deviation of the atmospheric mixing from the maximal mixing are simply presented in terms of σ and η as follows,

$$|U_{e3}| \simeq \frac{1}{2}|\delta - \eta|, \tag{20}$$

$$\delta \sin\theta_{23} \simeq \frac{1}{2}(\delta + \eta). \tag{21}$$

Imposing the bound on $|U_{e3}|$ of CHOOZ experiment, $|U_{e3}| < 0.2$, and the result of $\sin^2\theta_{23}$ from atmospheric neutrino data, $\sin^2\theta_{23} = 0.44(1^{+0.41}_{-0.22})$ at 2σ [12], we can determine the allowed regions of the ratio M_2/M_3.

In Table 3, we present the numerical results for the ratio M_2/M_3 and the prediction for the bound on $|U_{e3}|$. The second and third columns correspond to the normal hierarchical case, whereas the fourth and fifth columns to the inverted hierarchy. We find that the result for $\delta \sin^2\theta_{23}$ constrains M_2/M_3 more severely than the bound on $|U_{e3}|$ for $m_{\nu_{1(3)}} < 0.05$ eV. But for $m_{\nu_{1(3)}} \sim 0.1$ eV, both $\delta \sin^2\theta_{23}$ and $|U_{e3}|$ from neutrino data severely constrain the allowed region of M_2/M_3. The values in the columns for $|U_{e3}|$ indicate the predictions for the upper bound. As shown in Table 3, the allowed region for M_2/M_3 gets narrowed as $m_{\nu_{1(3)}}$ increases, and it becomes nearly one for $m_{\nu_{1(3)}} \geqslant 0.1$ eV. This implies that such large values of $m_{\nu_{1(3)}}$ lead to moderately degenerate light neutrino spectrum realized by almost degenerate heavy Majorana neutrinos.

It is very instructive to see how the breaking of the three-fold degeneracy of the heavy Majorana neutrinos responsible for the deviation of maximal mixing of solar neutrinos and non-zero U_{e3} can arise. In this Letter, we propose that the origin of the breaking is the effective dimension-five operators whose structures are governed by $A_4 \otimes Z_2$ flavor symmetry, and show that the desirable mass splitting is related with the vacuum structures of some scalar fields in the dimension-five operators: Introducing $SU(2)_L$ singlet scalar field φ_i ($i = 1, 2, 3$) transforming as $(\mathbf{3}, +)$ under $A_4 \otimes Z_2$, we consider the following dimension-five operators

$$M_S^{-1}(N\varphi)(N\varphi) = M_S^{-1}\big[(N_1\varphi_2)(N_1\varphi_2) + (N_2\varphi_3)(N_2\varphi_3) + (N_3\varphi_1)(N_3\varphi_1)\big], \tag{22}$$

where M_S is a large effective mass and $(N\varphi)$ forms a triplet under A_4. In fact, these effective dimension-five operators can be generated by integrating out heavy singlet fermion $S_i \sim (\mathbf{3}, +)$ in the Yukawa sector given by $N_i\varphi_j S_k$ at a high energy scale M_S. As φ_i acquires non-zero vacuum expectation values (VEVs), we obtain additional contributions to the heavy Majorana neutrino masses. The VEVs of φ_i are developed from the scalar potential given by the same form of Eq. (4), but replacing σ_i with φ_i. Then, the desired vacuum alignment $\langle\varphi_i\rangle = (V_\varphi, 0, V_\varphi)$ generated from the scalar potential leads to the contributions to the heavy Majorana masses, $\delta M(N_2N_2 + N_3N_3)$ with $\delta M \sim V_\varphi^2/M_S$. This additional contribution δM gives rise to the mass splitting between N_1 and $N_{2(3)}$, which is responsible for the deviation of θ_{sol} from the maximal mixing. The breaking of the degeneracy between N_2 and N_3 can also be achieved by considering a dimension-five operator $M_{S'}^{-1}(N_i\xi_i)(N_j\xi_j)$ composed of N_i and another Higgs singlet ξ_i transforming as $(\mathbf{3}, +)$, whose VEV is developed as $\langle\xi\rangle = (0, 0, b)$ after transforming into the 3rd position. Then, an extra contribution to the heavy Majorana mass term $\sim (b^2/M_{S'})N_3N_3$ can be generated, which is responsible for the possible generation of the non-vanishing U_{e3}.

Finally we note that there could be radiative corrections to neutrino mass matrix which can lead to some modification of our results. However, non-negligible renormalization effects can be expected only in the case of degenerate light neutrino spectrum.

The numerical results for $m_{\nu_1(3)} = 0.1$ eV in the tables may be significantly modified due to possible renormalization effects, but the detailed investigation on the renormalization effects is not our main interest in this work and we will leave it for the future work.

In summary, we have proposed a scenario that the mass splitting between the first generation of the heavy Majorana neutrino and the other two degenerate ones in the seesaw framework is responsible for the deviation of the solar mixing angle from the maximal mixing, while keeping the maximal mixing between the tau and muon neutrinos as it is. Our scheme is based on the assumption that nature presumably started with "bi-maximal" neutrino mixing and then it has been deviated somehow. We have considered the case that the "bi-maximal" mixing is achieved only from the neutrino Dirac Yukawa matrix by taking a diagonal form of three degenerate heavy Majorana neutrinos in a basis where the charged lepton mass matrix is real and diagonal. Allowing the mass splitting between the first and the other two generations of the heavy Majorana neutrinos, we could obtain the deviation of the solar mixing angle from the maximal. In addition, we have shown that the tiny breaking of the degeneracy of the two heavy Majorana neutrinos leads to the small mixing angle θ_{e3} in the PMNS matrix and the very small deviation of the atmospheric neutrino mixing angle from the maximal mixing. We have also considered the effective dimension-five operators as the origin of the mass splitting among the heavy Majorana neutrinos.

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