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The methodology for calculating deflections of statically indeterminate reinforced concrete beams (based on nonlinear deformation model)

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Abstract

The paper presents an advanced methodology for calculating deflections of statically indeterminate reinforced concrete curved beams with allowance for discrete cracking. The theoretical approach based on the basic principles of nonlinear deformation model and takes into account nonlinear concrete and reinforcement behavior. The methodology for calculating deflections of statically indeterminate curved reinforced concrete beams introduced as a computational algorithm worked out based on "MathCAD-15" computer system. The numerical modelling of indeterminate curved reinforced concrete beams exposed to short duration uniform loading shown as a finite element model in PC "LIRA-SAPR 2014". Taking into account the results of their theoretical research the authors compare calculation data based on the existing domestic and foreign specification documents with the numerical experiment based upon the authors' own method. The suggested method can be successfully used for calculation of deflections both of statically indeterminate reinforced curved structures made of ultra-high strength concrete (HSC and HPC types) and of normal or over-reinforced structures.

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Keywords: Deflections; Diagrams of concrete deformation; Curved members; Evenly distributed load; Statically indeterminate members

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1. Introduction

The methodology for calculating deflections of statically indeterminate curved reinforced concrete flexural members is inextricably connected with the definition of real stress and strain state in any section of the member, considering along the whole structure, cracking detection as well as physical properties of reinforced concrete. Thus, the accuracy and convergence of research results depend on the introduction to the calculation method of realistic deformation models of concrete and reinforcement.

The purpose of this work, as continuation of research conducted [1-13], is to improve methods for calculating statically indeterminate flexural curved reinforced concrete beams deflections under uniformly distributed loading, taking into account the discrete cracking and the nonlinear behavior of reinforcement and concrete.

The paper aims to find solutions for the following:

- To analyze methodologies of calculating deflections and factors on which deflections depend (published in both Russian and foreign regulatory documents, standards and research papers);
- To model a numerical experiment construct and analyze these methodologies of calculating deflections using “Lira-SAPR 2014 R3” software system;
- To carry out field tests on statistically determined pre-stressed curved reinforced concrete beams under short duration uniform loading;
- To compare results of curved reinforced concrete beams deflections calculated during field tests, results calculated by methodologies given in Russian and foreign regulatory documents and results calculated by numeric modelling using “Lira-SAPR 2014 R3” software system.

Nomenclature

σ_b (ε_{bx})	compressed concrete stress
$\sigma_{bt}(\varepsilon_{bty})$	tensile concrete stress
$f_s(\varepsilon_s)$	tensile reinforcement stress
f_{sc} (ε_{sc})	compressed reinforcement stress
ε_{bx}	current value of strain in compressed concrete
ε_{by}	current value of strain in tensile concrete
ε_{sc}	current value of strain in compressed reinforcement
ε_s	current value of strain in tensile reinforcement
ε_{b2}	maximum value of strain of compressed concrete
ε_{bt2}	maximum value of strain of tensile concrete
x	present altitude of compressive zone of the concrete
y	present altitude of tensile zone of the concrete
k	altitude of compressive zone of the concrete
t	altitude of tensile zone of the concrete
h	overall depth of section
b	width of section
a_1 and a_2	depth of concrete cover of tensile and compressive reinforcement
A_s and A_{sc}	tensile and compressive reinforcement area

2. Advanced calculation methodology for statically indeterminate curved reinforced concrete elements and its description

There are many different methods to determine the deformation dependence of compressed and tensile concrete and reinforcement described in domestic [2, 15, 16, 20, 22, 23] and foreign [2, 10, 21, 24, 25] scientific works and specification documents. On the basis of the deformation diagram proposed in SGASU [15] an advanced method for calculating deflection of flexural curved reinforced concrete structures was developed. As a basis of concrete tensile strain diagram, we accepted the diagram of G.V. Murashkin [15] using experimental research of G.A.

Smolyago [20]. Such methods as EuroCode [24, 25] and Wang et al [21] accepted as deformation diagrams of compressed and tensile reinforcement.

In this case, there are following assumptions accepted:

- We consider a concrete curved double-span whole beam under evenly distributed load;
- Spans of such are beam are equal;
- Curvature distribution made in a sinusoidal manner of variable amplitude.
- Sinusoidal law characteristics are determined by calculating the height of the compressed zone in the area of cracks and blocks between cracks;
- Curvature R - in a section with cracks is determined by the equilibrium condition in the compressed concrete efforts and tensile reinforcement;
- Curvature R1 - in a section in the middle of the block between cracks- is determined from the condition that strain values of tensile concrete don't exceed the limit (if $\varepsilon_{bt}= \varepsilon_{bt2}$), and the hypothesis of plane sections is applied only to compressed concrete.

Agorithm for determining the deflection of reinforced concrete flexural structures exposed to short-term duration uniform loading has been implemented in «MathCAD 15».

Equilibrium equations in a block section without cracks will be as follows (1):

$$\begin{cases} \int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot b\cdot dx - \int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot b\cdot dy - f_s\left[\frac{\varepsilon_{bt}}{t}\cdot(h_0-k)\right]\cdot A_s + \\ + f_{sc}\left[\frac{\varepsilon_b}{k}\cdot(k-a_2)\right]\cdot A_{sc} = 0 \;\; ; \\ \int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot b\cdot dx\cdot\left[\left(\frac{\int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot x\cdot dx}{\int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot dx}\right)+h_0-k\right]+ \\ + f_s\left[\frac{\varepsilon_b}{k}\cdot(k-a_2)\right]\cdot A_{sc}\cdot(h_0-a_1-a_2)- \\ -\int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot b\cdot dy\cdot\left(h\;\;0-k-\frac{\int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot y\cdot dy}{\int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot dy}\right)-M_u = 0, \end{cases} \tag{1}$$

By solving the equations (1) we obtain the values of k and ε_b considering nonlinear deformation of concrete and reinforcement for tension and compression with the appropriate strain diagrams. Hence, the curvature *R*defined by the formula (2):

$$R = \frac{\varepsilon_b}{k} \tag{2}$$

Stress and strain state in the middle of the block separated by cracks described by the formula (3):

$$\int_0^{k1} \sigma_b\left(\frac{\varepsilon_{b1}}{k_1}\cdot x\right)\cdot b\cdot dx - \int_0^{H-k1}\sigma_{bt}\left(\frac{\varepsilon_{bt2}}{h-k_1}\cdot y\right)\cdot b\cdot dy - f_s\left[\frac{\varepsilon_{b1}}{k_1}\cdot\left(h_0-k_1\right)\right]\cdot A_s + $$
$$+ f_{sc}\left[\frac{\varepsilon_{b1}}{k_1}\cdot\left(k_1-a_2\right)\right]\cdot A_{sc} = 0 \quad (3)$$

In this work, we used the distance value between cracks l_s, obtained by EuroCode 2 method [24, 25].

The total deflection value includes bending moment effect FM_{max} and shearing force deflection FM_{max} (equation 4):

$$F = FM_{\max} + FQ_{\max} \quad (4)$$

3. Numerical modelling and experimental research of curved pre-stressed concrete elements

As reliability evaluation of suggested methods, numerical experiment has been conducted in nonlinear formulation for the finite element model in PC " Lira-SAPR 2014 R3".

The computational model consists of flat quadrilateral finite elements, the upper, lower longitudinal, and lateral reinforcement in the form of rod finite elements. Cracks were modeled by jointing nodes. The distance between cracks was taken based on the calculation procedure of EuroCode 2 [24, 25].

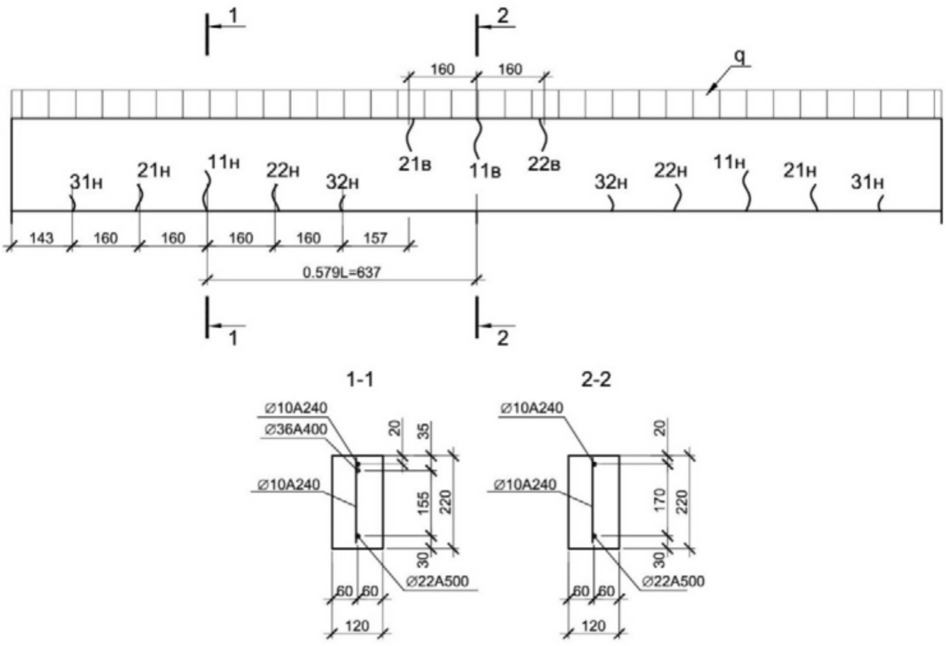

Fig. 1. A design model of a reinforced concrete beam.

For theoretical calculations the authors considered a double-spanned concrete beam with 20×200(h) mm cross section and L=1100 mm design span. The samples taken were made of B30 concrete. Reinforcement was made of plane frames: longitudinal reinforcement of a span is Ø22 mm A500; on supporting structures – Ø36 mm A400. Figure 1 shows a design model of a reinforced concrete beam.

In order to estimate the reliability of obtained results, there was a comparison conducted with methods of calculation based on specification documents SP 63.13330.2012 [22], guides to SP 52-101-2003 [23], ACI 318M-08 [26], MC2010 [25] and also with results of nonlinear analysis in the finite element model with discrete cracks pattern in PC " Lira-SAPR 2014 R3". Calculation results are given in Table 1. Convergence estimate of results is given in Table 2.

Table 1. Theoretical and experimental research results summary.

Series marking	Beam deflection according to the used methodology, mm					
	The authors' methodology	SP 63.13330.2012	Guidelinesfor SP 52-101-2003	FIB: MC2010	ACI 318M-08	“Lira-SAPR 2014 R3” PC
Ob 4	1.891	2.282	1.713	1.819	1.486	1.781

Table 2. Theoretical and experimental research results summary.

Series marking	Deflection deviation in the used methodology out of numerical experiment value in “Lira-SAPR 2014 R3” PC, %				
	The authors' methodology	SP 63.13330.2012	Guidelinesfor SP 52-101-2003	FIB: MC2010	ACI 318M-08
Ob 4	6.17	28.13	-3.81	2.13	-16.56

4. Conclusions

Based on the results of the conducted theoretical and experimental research it is possible to draw the following conclusion: the deflection value of a statically indeterminate curved reinforced concrete beam under evenly distributed load calculated by the authors' methodology is similar to that of a numerical experiment. The research proves that non-linear properties of concrete and reinforcement influence accuracy of deflections calculation.

Further research with account of other works [1-13] will be aimed at conducting experimental field tests in order to confirm the suggested method of calculation. The authors also plan to develop a refined methodology for calculating deflections of reinforced concrete curved beams under evenly distributed load and including crack formation [1], non-linear diagrams of concrete deflection [15,16], loading duration [14, 15] and unequal strength along the cross-sectional height [18, 19].

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