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Crack detection in cantilever beam by frequency based method

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Abstract

During the last few decades, intense research on the detection of crack using the vibration based techniques has been done and various approaches have been developed by researchers. In the present paper, detection of the crack presence on the surface of beam-type structural element using natural frequency is presented. First two natural frequencies of the cracked beam have been obtained experimentally and used for detection of crack location and size. Detected crack locations and size are compared with the actual results and found to be in good agreement. Also, the effect of the crack location and the crack depth on the natural frequency is presented.

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Keywords: Crack detection, natural frequency, vibration based inspection, Non-Destructive Testing (NDT), Transverse Vibration.

Nomenclature

a	Crack depth (m)
A	Cross-sectional area (m^2)
b	Beam Width (m)
h	Height of the beam (m)
L	Beam Length (m)
K_t	Rotational Spring Stiffness (N/m)
L_1	Crack Location (m)
c	Compliance (m/N)
E	Modulus of Elasticity (N/m^2)
I	Moment of Inertia (m^4)
t	Time (s)
ρ	Mass density (kg/m^3)
y	Displacement (m)
$f(a/h)$	Dimension-less Local Compliance function
Greek symbols	
λ	Frequency parameter
β	Normalized crack location
ω_c	Natural frequency of free vibration of the cracked beam
ω	Natural frequency of free vibration of the un-cracked beam

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1. Introduction

All structures are prone to damage, may be due to over-stressing in operation or due to extreme environmental conditions or due to any accidental event. Crack present in the component may grow during service and may result in the component failure once they grow beyond a critical limit. It is desirable to investigate the damage occurred in the structure at the early stage to protect the structure from possible catastrophic failures. There are various Non-destructive techniques (NDTs) available for the detection of the discontinuities in the structural components and the mechanical components. They are efficient but time consuming, expensive and laborious, particularly for slender beam like components. The crack present in the component imparts local flexibility to the component and reduces the natural frequency of free vibration of the component. The local damage also affects the mode shapes of the vibration of the component. Thus, Vibration Based Inspection (VBI) can be a potential method for crack detection. Though, there has been an intense study on crack detection through vibration based inspection, there is a need to develop an effective and economically appropriate approach.

Rizos et al. [1] have represented the crack as rotational spring in modal analysis for a cantilever beam having a rectangular cross section as shown in Fig. 1.

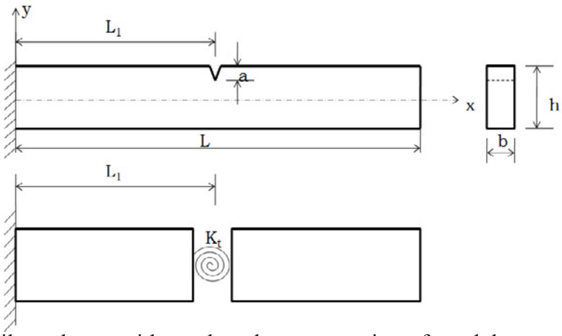

Fig. 1 Cantilever beam with crack and representation of crack by rotational spring

Dimarogonas and Paipetis [2] defined the torsional spring constant 'K_t' in the vicinity of the cracked section of a beam when a lateral crack of uniform depth exists, from crack energy function,

$$c = \frac{5.346 \cdot h \cdot f(a/h)}{EI} \tag{1}$$

$$K_t = 1/c \tag{2}$$

$$f(a/h) = 1.8624(a/h)^2 - 3.95(a/h)^3 + 16.375(a/h)^4 - 37.226(a/h)^5 + 76.81(a/h)^6 - 126.9(a/h)^7 + 172(a/h)^8 - 143.97(a/h)^9 + 66.56(a/h)^{10} \tag{3}$$

where, 'E' is the modulus of elasticity, 'I' is the moment of inertia, 'a' is the crack depth, 'h' is the beam height, 'c' is compliance and 'K_t' is the rotational spring stiffness, $f(a/h)$ is dimension-less local compliance function.

The crack or damage in the structure reduces stiffness and increases damping (Adams et al. [3]). The natural frequency is affected by change in stiffness and this can be the basis of the frequency-based damage detection (FBDD) technique.

Chondros and Dimarogonas [4], Liang et al. [5-6] etc. used the rotational spring approach and established the relationship between the spring stiffness K, crack location β and natural frequency of particular modes in the form,

$$K = \frac{|\Delta_2(\beta,\lambda)|}{|\Delta_1(\beta,\lambda)|} \tag{4}$$

Here, $|\Delta_1(\beta,\lambda)|$ and $|\Delta_2(\beta,\lambda)|$ can be obtained from the characteristic equations of the system.

In this paper, the crack is modelled as rotational spring and effect of crack location and the crack size on natural frequency is studied. Also, the strategy for the crack detection is presented. The results are compared with experimental results and found to be in close agreement.

2. Analytical solution for free vibration of beam

The governing equation of transverse vibration of the beam with uniform cross-section without any crack can be given as,

$$\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 y(x,t)}{\partial t^2} = 0, \tag{5}$$

where, 'x'-axis is aligned with the beam axis and is located at the left end of the beam, 't' is the time, 'ρ' is beam material density and 'EI' is the modulus of rigidity. Here, shear deformation and the rotational inertia have been neglected.

For free vibration, Eq. (5) can be written as,

$$\frac{\partial^4 y(\beta)}{\partial x^4} + \lambda^4 y(\beta) = 0, \tag{6}$$

where, $\beta = x/L$ is normalized crack location and 'λ' is non-dimensional frequency parameter given as,

$$\lambda^4 = \frac{\omega^2 \rho A L}{EI}, \tag{7}$$

where, 'ω' is the natural frequency of free-vibration, 'L' is the length of the beam.

Mode shapes of the vibration can be given by,

$$Y(R) = A_1 \sin(\lambda\beta) + A_2 \cos(\lambda\beta) + A_3 \sinh(\lambda\beta) + A_4 \cosh(\lambda\beta) \tag{8}$$

3. Analysis of beam with crack using Rotational Spring Approach

If the crack is assumed to be open and of uniform depth, the mode of vibration of the left and right side of the crack of the beam is given by,

$$Y_1(R) = A_4 \sin(\lambda\beta) + A_3 \cos(\lambda\beta) + A_2 \sinh(\lambda\beta) + A_1 \cosh(\lambda\beta), \tag{9}$$

$$Y_2(R) = B_4 \sin(\lambda\beta) + B_3 \cos(\lambda\beta) + B_2 \sinh(\lambda\beta) + B_1 \cosh(\lambda\beta), \tag{10}$$

where, $Y_1(R)$ and $Y_2(R)$ are harmonic modes of vibration of the left and right side of the crack of the beam respectively. A_i and B_i are the constants where i = 1, 2, 3, 4. Here, $\beta = x/L$ and $R = L_1/L$ = normalized crack position, 'λ' is the frequency parameter.

Frequency parameter 'λ' is given by,

$$\lambda^4 = \frac{\omega^2 \rho A L}{EI}, \tag{11}$$

where, 'ω' is the natural frequency of free-vibration, 'ρ' is material density, 'A' is cross-sectional area, 'λ' is frequency parameter, 'x' is coordinate along the beam, 'E' is Young's Modulus and 'I' is moment of inertia.

Applying the boundary conditions based on end condition on the beam and the compatibility condition along with continuity conditions at the rotational spring, eight characteristic equations can be derived. Solving for non-trivial solution of the characteristic equations, crack damage location can be located with the use of first two natural frequencies.

4. Crack detection in the cantilever beam by Rotational Spring Approach

At the crack location, the continuity of the displacement, moment and shear force and compatibility condition i.e., jump in the slope can be given as,

$$Y_1(R) = Y_2(R) \tag{12}$$

$$Y_1''(R) = Y_2''(R) \tag{13}$$

$$Y_1'''(R) = Y_2'''(R) \tag{14}$$

$$Y_1'(R) + \left(\frac{EI}{K_r}\right) Y_1''(R) = Y_2'(R) \tag{15}$$

Boundary conditions for the cantilever beam are as follows:

At Fixed End : Displacement $Y_1(R) = 0$ and slope $Y_1'(R) = 0$ at $\beta = 0$,

At Free End : Moment $Y_2''(R) = 0$ and shear force $Y_2'''(R) = 0$ at $\beta = 1$.

Applying these eight conditions to mode shape equations [Eqs. (9)-(10)], eight characteristic equations can be obtained. Having the value of first two natural frequencies of vibration of cracked beam, crack present in the beam can be detected.

The characteristic equation will be of the form,

$$A \cdot B = 0, \tag{16}$$

where,

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \cosh(\lambda\beta) & \sinh(\lambda\beta) & \cos(\lambda\beta) & \sin(\lambda\beta) & -\cosh(\lambda\beta) & -\sinh(\lambda\beta) & -\cos(\lambda\beta) & -\sin(\lambda\beta) \\ \sinh(\lambda\beta) & \cosh(\lambda\beta) & \sin(\lambda\beta) & -\cos(\lambda\beta) & -\sinh(\lambda\beta) & -\cosh(\lambda\beta) & -\sin(\lambda\beta) & \cos(\lambda\beta) \\ \cosh(\lambda\beta) & \sinh(\lambda\beta) & -\cos(\lambda\beta) & -\sin(\lambda\beta) & -\cosh(\lambda\beta) & -\sinh(\lambda\beta) & \cos(\lambda\beta) & \sin(\lambda\beta) \\ 0 & 0 & 0 & 0 & \cosh(\lambda) & \sinh(\lambda) & -\cos(\lambda) & -\sin(\lambda) \\ 0 & 0 & 0 & 0 & \sinh(\lambda) & \cosh(\lambda) & \sin(\lambda) & -\cos(\lambda) \\ \frac{K}{\lambda}\sinh(\lambda\beta)+\cosh(\lambda\beta) & \frac{K}{\lambda}\cosh(\lambda\beta)+\sinh(\lambda\beta) & -\frac{K}{\lambda}\sin(\lambda\beta)-\cos(\lambda\beta) & -\frac{K}{\lambda}\cos(\lambda\beta)-\sin(\lambda\beta) & -\frac{K}{\lambda}\sinh(\lambda\beta) & -\frac{K}{\lambda}\cosh(\lambda\beta) & \frac{K}{\lambda}\sin(\lambda\beta) & \frac{K}{\lambda}\cos(\lambda\beta) \end{bmatrix} \tag{17}$$

$$B = [A_1 \quad A_2 \quad A_3 \quad A_4 \quad B_1 \quad B_2 \quad B_3 \quad B_4]' \tag{18}$$

For the non-trivial solution, condition $|A| = 0$ must be satisfied. Location of the crack on the component can be obtained by finding the crack location satisfying this condition.

5. Results and Discussion

The natural frequency is greatly affected by crack depth and crack location. To illustrate this, the condition $|A| = 0$ is incorporated for different crack depth, crack location and frequency. Fig. 2 shows the effect of the crack location on the first natural frequency for various crack depths whereas Fig. 3 shows the effect of the crack depth ratio on the first natural frequency for various crack locations. It can be observed that the crack of a particular size present near the fixed end reduces the natural frequency significantly higher than the crack of the same size present closer to the free end. It can also be observed that the higher crack depth ratio has more effect on the normalized natural frequency than that of smaller ratio.

A contour diagram of natural frequency for each crack location and depth is prepared and presented in the Fig. 4. From this diagram, the depth of the crack can be read from its cracked natural frequency, if the position of the crack is known.

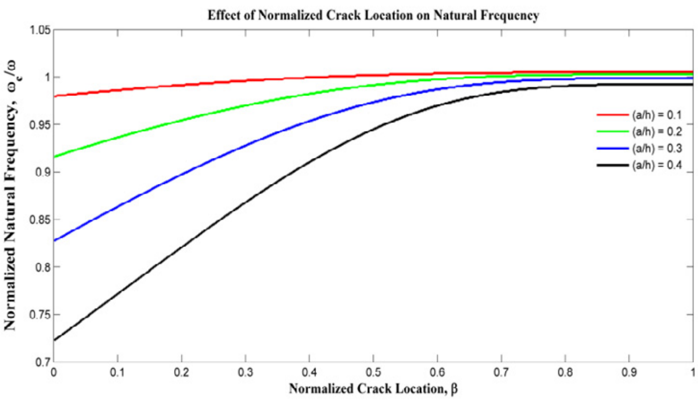

Fig. 2 Effect of Normalized Crack Location on the Normalized Natural Frequency

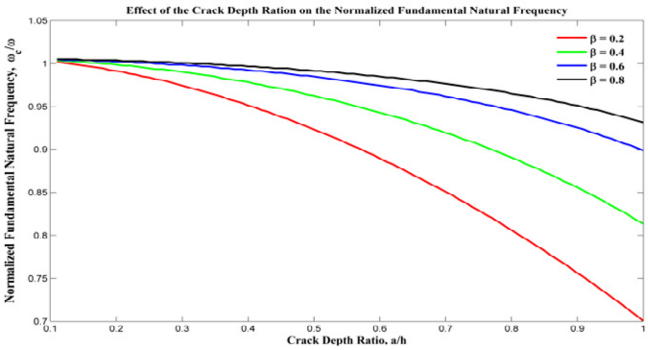

Fig. 3 Effect of the Crack Depth Ratio on the Normalized Natural Frequency

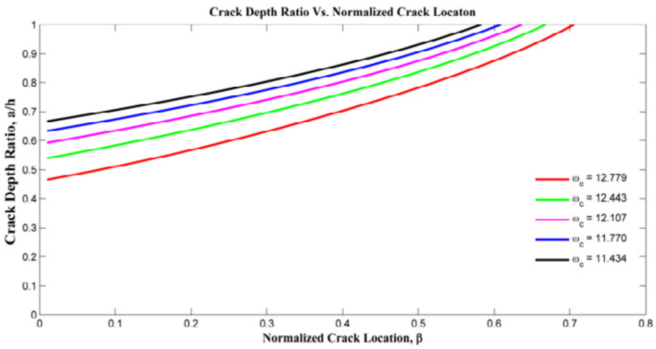

Fig. 4 Contours of Crack Depth Ratio to Normalized Crack Locations for Various Normalized Natural Frequency of Cantilever Beam with Single Crack

The experimental setup is developed for obtaining natural frequencies of cracked and uncracked beam (Refer Fig. 5). The beam dimensions and its material properties are as follows: Length (L) = 0.78 m, width (b) = 0.04 m and depth (h) = 0.01 m, mass density (ρ) = 7860 kg/m^3 and modulus of elasticity (E) = 210 GPa.

The cracks are developed using wire-cut electro-discharge machining and natural frequencies are measured using LMS make FFT analyzer.

The experimental natural frequencies of the different cantilever beam are then utilized in the mathematical formulation presented here, to predict the crack location and size. Corresponding to first and second natural frequencies, the variation of normalized crack depth (a/h) with normalized location (x/L) is obtained, as shown in the Fig. 6. The intersection of these two graphs will precisely predict the crack location and the size. The Fig. 6 is prepared for the natural frequencies corresponding to $a/h = 0.5$ and $\beta = 0.5$. The predicted location and the crack size are $\beta = 0.52$ and $a/h = 0.48$, respectively. For different crack locations, natural frequencies are obtained and crack locations and the crack sizes are predicted as mentioned in the Table 1.

Fig. 5 Experimental set up for crack detection of a cantilever beam with single crack

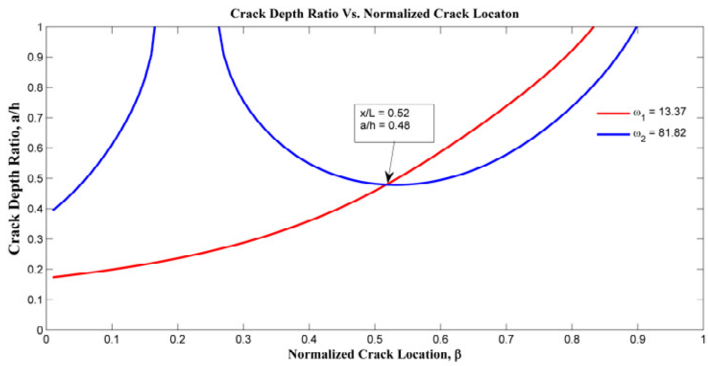

Fig. 6 Detection of the crack in Cantilever beam with single crack

Normalized crack location ($\beta = x/L$)	Crack Depth Ratio *(a/h)*	Natural Frequency, ω_1 (Hz)	Natural Frequency, ω_1 (Hz)	Predicted location *(x/L)* (present method)	Difference (%)	Predicted crack depth *(a/h)* (present method)	Difference (%)
No Crack	No Crack	13.45	84.30	-	-	-	-
0.1	0.5	12.8	82.89	0.0948	-0.52	0.5	0
0.2	0.5	13.02	84.3	0.2141	1.41	0.5	0
0.3	0.5	13.15	83.3	0.3291	2.91	0.52	2
0.4	0.5	13.28	82.87	0.3842	-1.58	0.46	-4
0.5	0.5	13.37	81.82	0.5145	1.45	0.48	-2
0.6	0.5	13.4	81.93	0.5806	-1.94	0.48	-2
0.7	0.5	13.43	82.5	0.6788	-2.12	0.5	0

Table 1 Crack detection on experimental setup cantilever beam

6. Conclusions

Using this approach, damage detection can be done using natural frequency. The followings are the conclusions made from the present study:

- The present method to detect crack location and size is fast and efficient.
- Crack with larger crack depth ratio (a/h) imparts greater reductions in natural frequency than that of the smaller crack depth ratio. Hence, the accuracy of results improves as crack depth increases.
- Crack present near to fixed end imparts greater reductions in natural frequency than that to present at away from the fixed end.

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