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Casson fluid flow over an unsteady stretching surface

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Abstract The unsteady two-dimensional flow of a non-Newtonian fluid over a stretching surface having a prescribed surface temperature is investigated. The Casson fluid model is used to characterise the non-Newtonian fluid behaviour. Similarity transformations are employed to transform the governing partial differential equations into ordinary differential equations. The transformed equations are then solved numerically by shooting method. Exact solution corresponding to momentum equation for steady case is obtained. The flow features and heat transfer characteristics for different values of the governing parameters viz. unsteadiness parameter, Casson parameter and Prandtl number are analysed and discussed in detail. Fluid velocity initially decreases with increasing unsteadiness parameter and temperature decreases significantly due to unsteadiness. The effect of increasing values of the Casson parameter is to suppress the velocity field. But the temperature is enhanced with increasing Casson parameter.

1. Introduction

Flow and heat transfer of a viscous fluid past a stretching sheet is a significant problem in fluid dynamics. The study of the flow over a stretching sheet is also relevant in the field of metallurgy and chemical engineering. The quality of the resulting product depends on the heat transfer rate, and so, the knowledge of the flow and heat transfer properties of the ambient fluid are very much essential [1–3].

In order to obtain a thorough cognition of non-Newtonian fluids and their various applications, it is necessary to study their flow behaviours. Due to their application in industry and technology, few problems in fluid mechanics have enjoyed the attention that has been accorded to the flow which involves non-Newtonian fluids. It is well known that mechanics of non-Newtonian fluids present a special challenge to engineers, physicists and mathematicians. The non-linearity can manifest itself in a variety of ways in many fields, such as food, drilling operations and bio-engineering. The Navier–Stokes theory is inadequate for such fluids, and no single constitutive equation is available in the literature which exhibits the properties of all fluids. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids. Thus, a number of non-Newtonian fluid models have been proposed. In the literature, the vast majority of non-Newtonian fluid models are concerned

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Nomenclature

A	unsteadiness parameter	μ	dynamic viscosity
Pr	Prandtl number	ν	kinematic viscosity
β	Casson parameter	ψ	stream function
η	similarity variable	ρ	density of the fluid
κ	thermal diffusivity	θ	non-dimensional temperature

with simple models like the power law and grade two or three [4–10]. These simple fluid models have shortcomings that render to results not having accordance with fluid flows in the reality. Casson fluid is another fluid model for non-Newtonian fluid. In the literature, the Casson fluid model is sometimes stated to fit rheological data better than general viscoplastic models for many materials [11,12]. Examples of Casson fluid include jelly, tomato sauce, honey, soup and concentrated fruit juices, etc. Human blood can also be treated as Casson fluid. Due to the presence of several substances like, protein, fibrinogen and globulin in an aqueous base plasma, human red blood cells can form a chainlike structure, known as aggregates or rouleaux. If the rouleaux behaves like a plastic solid, then there exists a yield stress that can be identified with the constant yield stress in Casson's fluid [13–15]. The non-linear Casson's constitutive equation has been found to describe accurately the flow curves of suspensions of pigments in lithographic varnishes used for preparation of printing inks [16] and silicon suspensions [17]. The shear stress–shear rate relation given by Casson satisfactorily describes the properties of many polymers [18] over a wide range of shear rates. Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear [19]. Eldabe and Salwa [20] have studied the Casson fluid for the flow between two rotating cylinders, and Boyd et al. [21] investigated the Casson fluid flow for the steady and oscillatory blood flow.

In all these above studies, the flow and temperature fields are considered to be at steady state. However, in some cases, the flow field, heat and mass transfer can be unsteady due to a sudden stretching of the flat sheet or by a step change of the temperature of the sheet. A few papers have been published on the boundary layer flow and heat transfer problems where the stretching force and surface temperature are varying with time. Some authors [22–25] studied the problem for unsteady isothermal stretching surface by using a similarity method to transform governing time-dependent boundary layer equations into a set of ordinary differential equations. Elbashbeshy and Bazid [26] have presented similarity solutions of the boundary layer equations that describe the unsteady flow and heat transfer over an unsteady stretching sheet. Sharidan et al. [27] studied the unsteady flow and heat transfer over a stretching sheet in a viscous and incompressible fluid. Recently, Tsai et al. [28], Ishak et al. [29], Mukhopadhyay [30,31] and Chamkha et al. [32] obtained similarity solutions for unsteady flow and heat transfer over a stretching sheet under different conditions. Hayat and Awais [33] analysed the time-dependent flow over a stretching surface. Bhattacharyya et al. [34] analysed the effects of slip on unsteady boundary layer stagnation point flow past a stretching sheet. Of late, Hayat et al. [35] discussed the three-dimensional flow of Jeffery fluid past a stretching surface.

Despite the overwhelming importance and frequent occurrence of non-Newtonian behaviour in industry and technology, no attempt has been made so far to analyse the Casson fluid flow and heat transfer past a non-isothermal unsteady stretching surface. Motivated by this, an attempt is made in this paper to extend the work of Andersson et al. [22] for non-Newtonian Casson fluid and heat transfer. The present work aims to fill the gap in the existing literature. Similarity solutions are obtained, and the reduced ordinary differential equations are solved numerically using shooting method. Exact solution of momentum equation for steady case is also obtained. The effects of unsteadiness parameter, Casson parameter and Prandtl number on velocity and temperature fields of the fluid are investigated and analysed with the help of their graphical representations.

2. Equations of motion

Consider laminar boundary layer two-dimensional flow and heat transfer of an incompressible, conducting non-Newtonian Casson fluid over an unsteady stretching sheet. The unsteady fluid and heat flows start at $t = 0$. The sheet emerges out of a slit at origin ($x = 0$, $y = 0$) and moves with non-uniform velocity $U(x,t) = cx/(1 - \alpha t)$ [22] where $c > 0, \alpha \geqslant 0$ are constants with dimensions $(\text{time})^{-1}$, c is the initial stretching rate.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is [20,36]

$$\tau_{ij} = \begin{cases} 2(\mu_B + p_y/\sqrt{2\pi})e_{ij}, \pi > \pi_c \\ 2(\mu_B + p_y/\sqrt{2\pi_c})e_{ij}, \pi < \pi_c \end{cases}$$

Here, τ_{ij} is the (i,j)-th component of the stress tensor, $\pi = e_{ij}e_{ij}$ and e_{ij} are the (i,j)-th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, and p_y is the yield stress of the fluid.

So, if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move.

The governing equations of such type of flow are, in the usual notations,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

when the viscous dissipation term in the energy equation is neglected (as the fluid velocity is low). Here, u and υ are the components of velocity, respectively, in the x and y directions, ν is the kinematic viscosity of the fluid, $\beta = \mu_B\sqrt{2\pi_c}/p_y$ is parameter of the Casson fluid, T is the temperature, and κ is the thermal diffusivity of the fluid.

2.1. Boundary conditions

The appropriate boundary conditions for the problem are given by

$$u = U(x,t), \upsilon = 0, T = T_w(x,t) \text{ at } y = 0, \tag{4}$$

$$u \to 0, T \to T_\infty \text{ as } y \to \infty. \tag{5}$$

Here, $T_w\,(x,t) = T_\infty + cx^2 T_0(1-\alpha t)^{-\frac{3}{2}}/(2\nu)$ [22] where T_0 is a (positive or negative; heating or cooling) reference temperature (slit temperature at $x = 0$), T_∞ is the constant free stream temperature. The expressions for $U(x,t)$, $T_w(x,t)$ are valid for time $t < \alpha^{-1}$.

2.2. Method of solution

Introducing $u = \frac{\partial\psi}{\partial y}, \upsilon = -\frac{\partial\psi}{\partial x}$ and $\theta = \frac{T-T_\infty}{T_w-T_\infty}$ (ψ being stream function) and with the help of $\eta = \sqrt{c/\{\nu(1-\alpha t)\}}\,y, \psi = \sqrt{\nu c/(1-\alpha t)}\,x f(\eta),\ \ T = [T_\infty + T_0 c x^2 (1-\alpha t)^{-\frac{3}{2}}/(2\nu)]\theta(\eta)$, the governing equations finally reduce to

$$A\left(\frac{\eta}{2}f'' + f'\right) + f'^2 - ff'' = \left(1 + \frac{1}{\beta}\right)f''', \tag{6}$$

$$\frac{A}{2}(\eta\theta' + 3\theta) + 2f'\theta - f\theta' = \frac{1}{\Pr}\theta'', \tag{7}$$

where $A = \alpha/c$ is the unsteadiness parameter, $\Pr = \nu/\kappa$ is the Prandtl number.

The boundary conditions (4) and (5) then become

$$f' = 1, f = 0, \theta = 1 \text{ at } \eta = 0 \tag{8}$$

$$\text{and } f' \to 0, \theta \to 0 \text{ as } \eta \to \infty. \tag{9}$$

Eqs. (6) and (7) along with boundary conditions (8) and (9) are solved numerically by shooting method [30,31].

Exact solution for steady case:

The exact solution of Eq. (6) subject to the corresponding boundary conditions for steady case i.e. for $A = 0$ is given by $f(\eta) = \sqrt{1+\frac{1}{\beta}}\left(1 - e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right)$.

3. Results and discussions

In order to validate the method used in this study and to judge the accuracy of the present analysis, comparison with available results of Sharidan et al. [27] and Chamkha et al. [32] corresponding to the skin-friction coefficient $f''(0)$ for unsteady flow of viscous incompressible fluid is made (Table 1) and found in excellent agreement. Moreover, to verify the accuracy of the present numerical scheme, a comparison of the results corresponding to the stream function profiles $f(\eta)$ and velocity profiles $f'(\eta)$ with the exact results for steady motion ($A = 0$) is presented in Fig. 1 and found in excellent agreement.

Table 1 The values of $f''(0)$ for various values of unsteadiness parameter A for Newtonian fluid.

A	Sharidan et al. [27]	Chamkha et al. [32]	Present study
0.8	−1.261042	−1.261512	−1.261479
1.2	−1.377722	−1.378052	−1.377850

In order to study the behaviour of velocity and temperature fields for Casson fluid, a comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics, and the results are reported in terms of graphs as shown in Figs. 2–5.

Fig. 2a exhibits the velocity profiles for several values of unsteadiness parameter A. It is seen that the velocity along the sheet decreases initially with the increase in unsteadiness parameter A, and this implies an accompanying reduction of the thickness of the momentum boundary layer near the wall but away from the wall, fluid velocity increases with increasing unsteadiness i.e. away from the wall, the velocity field and the corresponding boundary layer are found to increase with an increase in A. Same type of behaviour has been reported by Mukhopadhyay [32]. $A = 0$ indicates the steady case. Fig. 2b represents the effects of unsteadiness parameter on the temperature distribution. From this figure, it is noticed that the temperature at a particular point is found to decrease significantly with increasing unsteadiness parameter. Same observation can be found from Refs. [27,28]. Rate of heat transfer (from the sheet to the fluid) decreases with increasing A (see also Fig. 5b). As the unsteadiness parameter A increases, less heat is transferred from the sheet to the fluid; hence, the temperature $\theta(\eta)$ decreases (Fig. 2b). Since the fluid flow is caused solely by the stretching sheet, and the sheet surface temperature is higher than free stream temperature, the velocity and temperature decrease with increasing η. It is important to note that the rate of cooling is much faster for higher values of unsteadiness parameter, whereas it may take longer time for cooling during steady flows.

Effects of Casson parameter β on velocity and temperature profiles for both steady and unsteady motion are clearly exhibited in Fig. 3a and b, respectively. For both steady and unsteady motion, same type of behaviour of velocity with increasing β is noted. The effect of increasing values of β is to reduce the velocity, and hence, the boundary layer thickness decreases

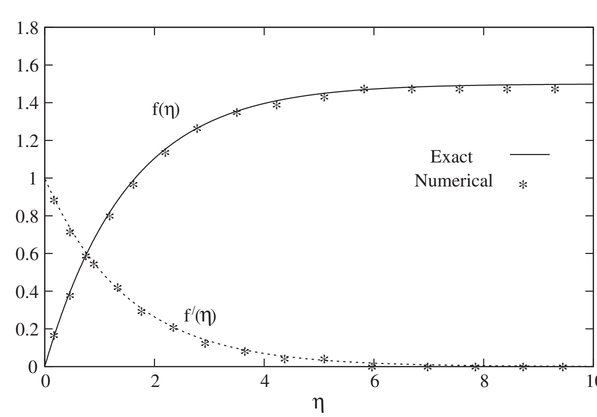

Figure 1 Stream function $f(\eta)$ and velocity $f'(\eta)$ with η for steady motion.

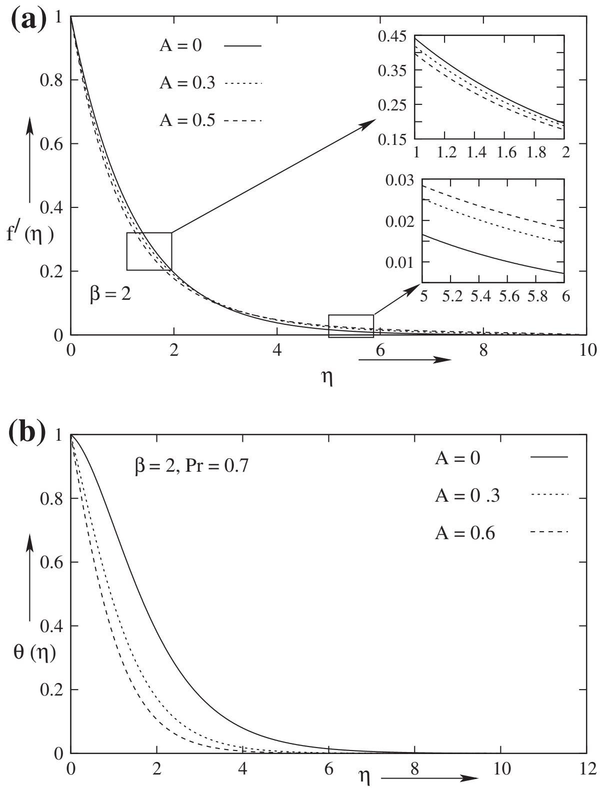

Figure 2 (a) Velocity and (b) temperature profiles for variable unsteadiness parameter A.

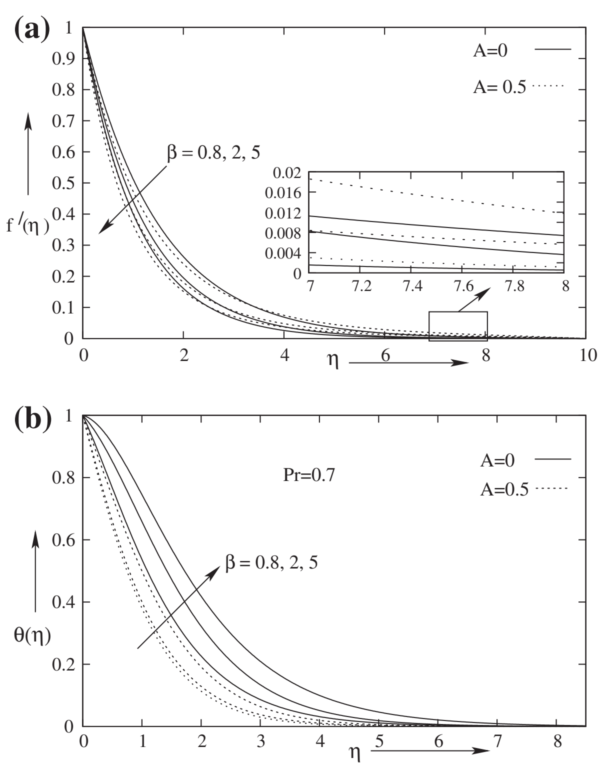

Figure 3 (a) Velocity and (b) temperature profiles for variable values of β for steady and unsteady motion.

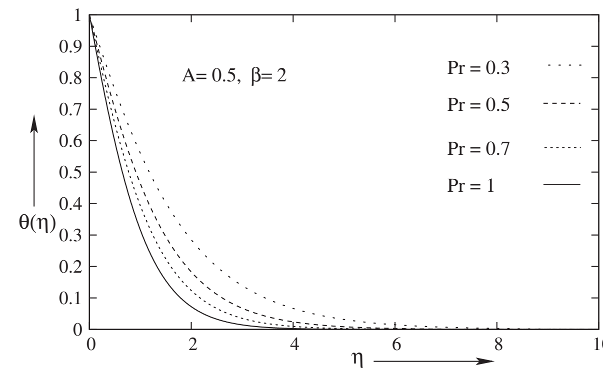

Figure 4 Temperature profiles for variable values of Prandtl number Pr.

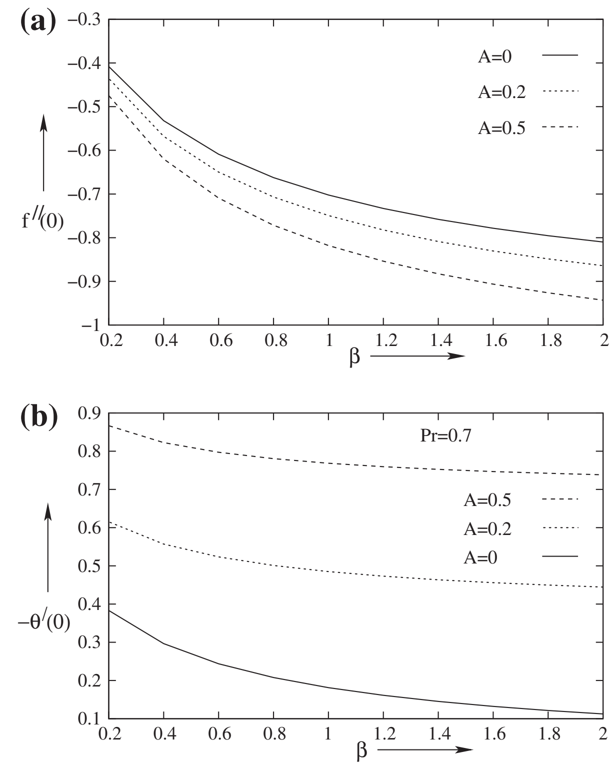

Figure 5 Variation of: (a) $f''(0)$ related to skin-friction and (b) heat transfer coefficient with Casson parameter β for three values of unsteadiness parameter A.

(Fig. 3a). The increasing values of the Casson parameter i.e. the decreasing yield stress (the fluid behaves as Newtonian fluid as Casson parameter becomes large) suppress the velocity field. It is observed that $f'(\eta)$ and the associated boundary layer thickness are decreasing function of β. The velocity curves in Fig. 3a show that the rate of transport is considerably reduced with the increase of β. The effect of increasing β leads to enhance the temperature field for both steady and unsteady motion (Fig. 3b). This effect is more pronounced for steady motion. The thickening of the thermal boundary layer occurs due to increase in the elasticity stress parameter. It can also be seen from Fig. 3a that the momentum boundary layer thickness decreases as β increases and hence induces an increase in the absolute value of the velocity gradient at the surface.

The behaviour of the temperature profiles for the variation of Prandtl number can be found from Fig. 4. It is noted that the temperature decreases with increasing Pr (Fig. 4). Moreover, the thermal boundary layer thickness decreases by increasing Prandtl numbers. Wall temperature gradient is negative for all values of Prandtl number which means that the heat is always transferred from the surface to the ambient fluid. An increase in Prandtl number reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Fluids with lower Prandtl number will possess higher thermal conductivities (and thicker thermal boundary layer structures), so that heat can diffuse from the sheet faster than for higher Pr fluids (thinner boundary layers).

Furthermore, the effects of unsteadiness parameter A and Casson parameter β on velocity gradient at the wall and heat transfer coefficient are presented in Fig. 5a and b. Magnitude of $[f''(0)]$ related to skin-friction coefficient decreases with increasing unsteadiness parameter A and also with Casson parameter β, but the magnitude of heat transfer rate at the surface $[\theta'(0)]$ decreases for β, increases with A. A drop in skin-friction as investigated in this paper has an important implication that in free coating operations, and elastic properties of the coating formulations may be beneficial for the whole process. This means that less force may be needed to pull a moving sheet at a given withdrawal velocity, or equivalently higher withdrawal speeds can be achieved for a given driving force resulting in, increase in the rate of production.

4. Conclusions

The present study provides the numerical solutions for unsteady boundary layer flow of a Casson fluid and heat transfer over a stretching surface. Fluid velocity decreases initially due to increase in unsteadiness parameter. The temperature also decreases significantly in this case. The effect of increasing values of the Casson parameter is to suppress the velocity field, whereas the temperature is enhanced with increasing Casson parameter. Prandtl number can be used to increase the rate of cooling in conducting flows.

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