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# Radial Displacements of an Infinite Liquid Saturated Porous Medium with Cylindrical Cavity

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**Abstract**—This paper deals with radial displacement fields in solid and liquid parts of a liquid-saturated porous medium with cylindrical cavity subjected to an arbitrary time dependent force. The Laplace transform technique is used to solve the problem. A particular case of impulsive force is discussed and closed form solutions are obtained. As a special case, results of classical elasticity are derived. 

**Keywords**—Radial displacement fields, Cylindrical capacity.

## 1. INTRODUCTION

The problem of the disturbance in an elastic medium containing a cavity due to arbitrary stresses on the cavity is of great importance, particularly as a model of an earthquake source. On the other hand, the propagation of elastic waves in a liquid-saturated porous medium has been a subject of continued interest due to its importance in seismology and geophysics. Chakraborty [1] studied the problem of the disturbance in an isotropic elastic infinite slab of finite thickness due to forces applied on the inner surface of a cylindrical cavity. Vodicka [2] discussed the problem of radial vibrations of an infinite medium with a cylindrical cavity. Thiruvenkatachar and Viswanathan [3] investigated the dynamic response of an elastic half-space with cylindrical cavity to time dependent surface tractions over the boundary of the cavity. In this paper, we consider the problem of radial displacement of an unbounded liquid-saturated porous medium due to a cylindrical cavity whose boundary is subjected to an arbitrary time dependent force. A particular case of impulsive force is discussed with the closed form solution. Results of classical elasticity are derived as a special case.

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## 2. BASIC EQUATIONS

In the absence of dissipation, the field equations for the liquid-saturated porous solid, are given by Biot [4,5],

$$N\nabla^2\mathbf{u} + \operatorname{grad}\{(D+N)e + Q\varepsilon\} = \frac{\partial^2}{\partial t^2}(\rho_{11}\mathbf{u} + \rho_{12}\mathbf{U}), \tag{2.1}$$

$$\operatorname{grad}\{Qe + R\varepsilon\} = \frac{\partial^2}{\partial t^2}(\rho_{12}\mathbf{u} + \rho_{22}\mathbf{U}), \tag{2.2}$$

where $D$, $N$, $Q$,and $R$ are the elastic constants for the solid-liquid aggregate: $\rho_{11}$, $\rho_{12}$, $\rho_{22}$ are dynamical coefficients. $\mathbf{u}$ and $\mathbf{U}$ are the displacements in the solid and liquid parts, respectively, and the corresponding dilatations are given by

$$e = \operatorname{div}\mathbf{U}, \qquad \varepsilon = \operatorname{div}\mathbf{U}. \tag{2.3a,b}$$

The stresses in the solid $\sigma_{ij}$ and in the liquid $\sigma$ are given by

$$\sigma_{ij} = (De + Q\varepsilon)\delta_{ij} + 2N\varepsilon_{ij}, \tag{2.4}$$

$$\sigma = Qe + R\varepsilon, \tag{2.5}$$

where $\delta_{ij}$ is the Kronecker delta, and

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{2.6}$$

## 3. FORMULATION AND SOLUTION OF THE PROBLEM

We consider an isotropic, homogeneous, liquid-saturated porous medium of infinite extent with a cavity of the form of circular cylinder of radius $a$. The surface of the cylindrical cavity is assumed to be acted upon by time-dependent pressure $f(t)$. We take the cylindrical polar coordinates $(r, \theta, z)$, with origin on the axis of cylinder and $z$-axis coinciding with it. We consider the case of radial symmetry, and assume that all quantities depend upon the radial coordinate $r$ and $t$ only. Therefore, the displacements in the solid and liquid parts can be written as

$$\mathbf{u} = u(r,t)\hat{e}_r, \tag{3.1}$$

$$\mathbf{U} = U(r,t)\hat{e}_r. \tag{3.2}$$

With the help of equations (3.1) and (3.2), equations (2.1) and (2.2), reduce to

$$\rho\nabla_1^2 u + Q\nabla_1^2 U = \frac{\partial^2}{\partial t^2}(\rho_{11}u + \rho_{12}U), \tag{3.3}$$

$$Q\nabla_1^2 u + R\nabla_1^2 U = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}U), \tag{3.4}$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}. \tag{3.5}$$

We assume that the initial displacements and their corresponding velocities are zero throughout the medium, that is,

$$\left.\begin{aligned} u(r,0) = u_t(r,0) = 0 \\ U(r,0) = U_t(r,0) = 0 \end{aligned}\right\} \quad \text{for } r > a. \tag{3.6}$$

The radiation condition imply that

$$\lim_{r\to\infty} u(r,t) = \lim_{r\to\infty} U(r,t) = 0, \qquad \text{for all } t > 0. \tag{3.7}$$

We define the potentials $\phi(r,t)$ and $\psi(r,t)$ by

$$u = \frac{\partial \phi}{\partial r} \quad \text{and} \quad U = \frac{\partial \psi}{\partial r}. \tag{3.8a,b}$$

Substituting equations (3.8a,b), in equations (3.3) and (3.4), we obtain the coupled equations

$$P\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right\}\phi + Q\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right\}\psi = \rho_{11}\frac{\partial^2 \phi}{\partial t^2} + \rho_{12}\frac{\partial^2 \psi}{\partial t^2}, \tag{3.9}$$

$$Q\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right\}\phi + R\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right\}\psi = \rho_{12}\frac{\partial^2 \phi}{\partial t^2} + \rho_{22}\frac{\partial^2 \psi}{\partial t^2}. \tag{3.10}$$

If $\phi$ or $\psi$ is eliminated from these equations, both $\phi$ and $\psi$ satisfy the same equation

$$\left\{A\nabla_2^4 - B\frac{\partial^2}{\partial t^2}\nabla_2^2 + C\frac{\partial^4}{\partial t^4}\right\}(\phi,\psi) = 0, \tag{3.11a,b}$$

where

$$\begin{aligned} \nabla_2^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial^2}{\partial r}, \qquad A = PR - Q^2, \qquad B = P\rho_{22} + R\rho_{11} - 2Q\rho_{12}, \\ C = \rho_{11}\rho_{22} - \rho_{12}^2, \qquad \text{and} \qquad P = D + 2N. \end{aligned} \tag{3.12}$$

Application of the Laplace transform to equations (3.11a,b) with respect to $t$ gives the solutions of the transformed equations satisfying the radiation condition

$$\overline{\phi} = A_1 K_0(ps_1 r) + A_2 K_0(ps_2 r), \tag{3.13}$$

$$\overline{\psi} = E_1 K_0(ps_1 r) + E_2 K_0(ps_2 r), \tag{3.14}$$

where $K_0(z)$ are the modified Bessel functions and $p$ is the Laplace transform variable, and

$$s_1^2 = \frac{1}{\alpha_1^2} = \frac{B - \sqrt{B^2 - 4AC}}{2A}, \qquad s_2^2 = \frac{1}{\alpha_2^2} = \frac{1}{\alpha_2^2} = \frac{B + \sqrt{B^2 - 4AC}}{2A}, \tag{3.15a,b}$$

and $\alpha_1, \alpha_2$ are the velocities of fast $P$ (or $P_f$) wave and slow $P$ (or $P_s$) wave respectively; $A_1$, $A_2$, $E_1$, and $E_2$ are arbitrary constants.

Application of the Laplace transform to equation (3.9) and (3.10), and the use of equations (3.13) and (3.14) yields

$$E_j = m_j A_j, \qquad (j = 1, 2), \tag{3.16}$$

where

$$m_j = \frac{Ps_j^2 - \rho_{11}}{\rho_{12} - Qs_j^2} = \frac{Qs_j^2 - \rho_{12}}{\rho_{22} - Rs_j^2}, \qquad (j = 1, 2). \tag{3.17}$$

With the help of equations (3.8a,b), (3.13), (3.14) and (3.16), we obtain the Laplace transformed solutions

$$\overline{u}(r,p) = -[A_1 ps_1 K_1(ps_1 r) + a_2 ps_2 K_1(ps_2 r)], \tag{3.18}$$

$$\overline{U}(r,p) = -[m_1 A_1 ps_1 K_1(ps_1 r) + m_2 A_2 ps_2 K_1(ps_2 r)]. \tag{3.19}$$

# 4. BOUNDARY CONDITIONS

Deresiewicz and Skalak [6] formulated the boundary conditions appropriate for the boundaries of liquid-saturated porous solid.

The boundary conditions for the present problem are given by

$$\text{(i)} \qquad \sigma_r r = -f(t), \quad r = a, \quad t > 0, \tag{4.1}$$

$$\text{(ii)} \qquad \sigma = 0, \qquad r = a, \quad t > 0, \tag{4.2}$$

where

$$\sigma_{rr} = P\,\frac{\partial u}{\partial r} + D\,\frac{u}{r} + Q\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right), \tag{4.3}$$

$$\sigma = Q\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + \left(R\,\frac{\partial U}{\partial r} + \frac{U}{r}\right). \tag{4.4}$$

Applying Laplace transform to the boundary conditions (4.1) and (4.4), and making use of equations (3.18) and (3.19), we get the following equations:

$$\begin{aligned}&\left\{(P + Qm_1)p^2 s_1^2 a K_0(ps_1a) + 2Nps_1K_1(ps_1a)\right\} A_1\\ &\quad + \left\{(P + Qm_2)p^2 s_2^2 a K_0(ps_2a) + 2Nps_2K_1(ps_2a)\right\} A_2 = -a\overline{f}(p),\end{aligned} \tag{4.5}$$

$$\left\{(Q + Rm_1)p^2 s_1^2 a K_0(ps_1a)\right\} A_1 + \left\{(Q + Rm_2)p^2 s_2^2 K_0(ps_2a)\right\} A_2 = 0. \tag{4.6}$$

Solving equations (4.5) and (4.6) gives

$$A_1 = -\frac{1}{\Delta}\left\{a\overline{f}(p)(Q + m_2R)p^2 s_2^2\right\} K_0(pas_2), \tag{4.7}$$

$$A_2 = \frac{1}{\Delta}\left\{a\overline{f}(p)(Q + m_1R)p^2 s_1^2\right\} K_0(pas_1), \tag{4.8}$$

where

$$\begin{aligned}\Delta &= s_1 s_2\left(B_1p^4 + B_2p^3\right),\\ B_1 &= as_1s_2K_0(ps_1a)K_0(ps_2a)A(m_2 - m_1),\\ B_2 &= 2N[Q\{s_2K_0(ps_2a)K_1(ps_1a) - s_1K_0(ps_1a)K_1(ps_2a)\}\\ &\quad + R\{m_2s_2K_0(ps_2a)K_1(ps_1a) - m_1s_1K_0(ps_1a)K_1(ps_2a)\}].\end{aligned} \tag{4.9}$$

Substituting the values of $A_1, A_2$ from equations (4.7) and (4.8) in equations (3.18) and (3.19) gives

$$\overline{u} = a\left[\overline{f}(p)\,\overline{w}_1(r,p) - \overline{f}(p)\,\overline{w}_2(r,p)\right], \tag{4.10}$$

$$\overline{U} = a\left[m_1\overline{f}(p)\,\overline{w}_1(r,p) - m_2\overline{f}(p)\,\overline{w}_2(r,p)\right], \tag{4.11}$$

where

$$\overline{w}_1(r,p) = \frac{(Q + m_2R)s_2K_0(ps_2a)}{B_1p + B_2}\,K_1(ps_1r), \tag{4.12}$$

$$\overline{w}_2(r,p) = \frac{(Q + m_1R)s_1K_0(ps_1a)}{B_1p + B_2}\,K_1(ps_2r). \tag{4.13}$$

Making use of the convolution theorem of the Laplace transform, we obtain from (4.10) and (4.11)

$$u(r,t) = a[f(t) * w_1(r,t) - f(t) * w_2(t,r)], \tag{4.14}$$

$$U(r,t) = a[m_1f(t) * w_1(r,t) - m_2f(t) * w_2(r,t)], \tag{4.15}$$

where $*$ denotes the convolution operation and $w_j(r,t), (j = 1,2)$ are the functions whose transforms are $\overline{w}_j(r,p), (j = 1,2)$, respectively.

The solutions $u(r,t)$, $U(r,t)$ given by (4.14) and (4.15) are known if $w_1(r,t)$ and $w_2(r,t)$ are known. Thus, the problem reduces to the determination of $w_1(r,t)$ and $w_2(r,t)$.

## 5. EVALUATION OF $w_1(r,t)$ AND $w_2(r,t)$

Applying the inverse Laplace transform to equation (4.12) gives

$$w_1(r,t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{pt}\,\overline{w}_1(r,p)\,dp, \tag{5.1}$$

and with the help of asymptotic relation, we get

$$e^{pt}\,\overline{w}_1(r,p) \sim \frac{a}{2N}\,\frac{(Q+m_2R)s_2}{\{(Q+m_2R)s_2-(Q+m_1R)s_1\}}\,\frac{(a/r)^{1/2}}{(\alpha p+1)}\exp\left\{p\left(t-\frac{r-a}{\alpha_1}\right)\right\},$$

where

$$\alpha_1 = \frac{a}{2N}\,\frac{s_1s_2A(m_2-m_1)}{\{(Q+m_2R)s_2-(Q+m_1R)s_1\}}.$$

Therefore, we have two different expressions for $w_1(r,t)$ corresponding to the cases, $t < (r-a)/\alpha_1$ and $t > (r-a)/\alpha_1$.

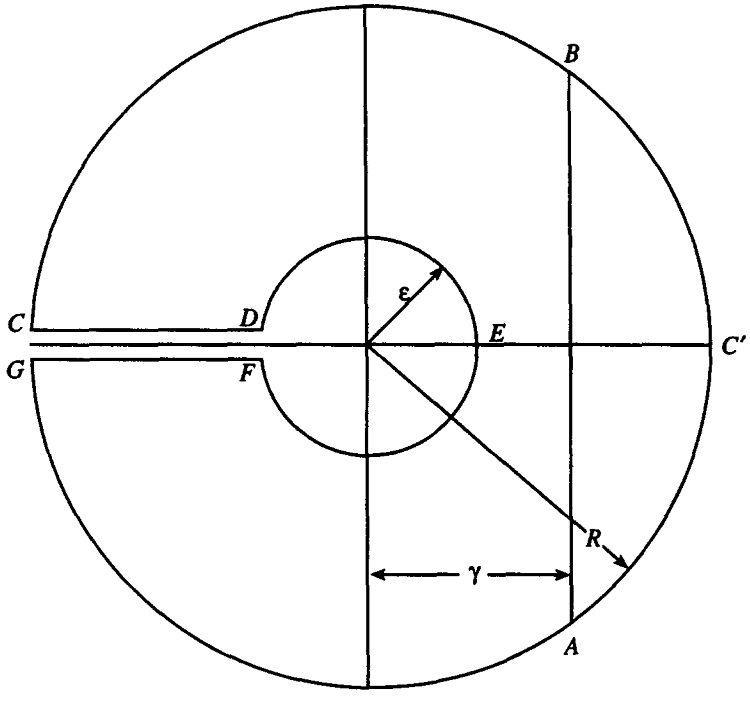


Figure 1.

If $t < (r-a)/\alpha_1$, we have by Cauchy's theorem (see Figure 1)

$$w_1(r,t) = 0, \qquad t < \frac{r-a}{\alpha_1}, \tag{5.2}$$

since the integral over $BC'A$ vanishes as $R \to \infty$.

For the case of $t > (r-a)/\alpha_1$, we take the contour $ABCDEFGA$ of Figure 1. The integrals over the arcs $BC$, $GA$ and $DEF$ tend to zero as $R \to \infty$, $\varepsilon \to 0$ and there are no poles of the integrand within the contour. Consequently,

$$\int_{\gamma-i\infty}^{\gamma+i\infty} e^{pt}\,\overline{w}_1(r,p)\,dp = \lim\left[\int_{DC} e^{pt}\,\overline{w}_1(r,p)\,dp - \int_{FG} e^{pt}\,\overline{w}_1(r,p)\,dp\right], \tag{5.3}$$

the limit being taken for $R \to \infty$ and $\varepsilon \to 0$.

On $DC$ and $FG$, we put $p = \zeta_1 e^{i\pi}$ and $p = \zeta_1 e^{-i\pi}$, respectively, and equation (5.3) becomes

$$\begin{aligned}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{pt}\,\overline{w}_1(r,p)\,dp &= \int_0^\infty e^{-\zeta_1 t}\left\{\overline{w}_1\left(r,\zeta_1 e^{-i\pi}\right) - \overline{w}_1\left(r,\zeta_1 e^{i\pi}\right)\right\}\,d\zeta_1,\\ &= 2i\int_0^\infty e^{-\zeta_1 t}\mathrm{Im}\left[\overline{w}_1\left(r,\zeta_1 e^{-i\pi}\right)\right]d\zeta_1,\end{aligned} \tag{5.4}$$

where $\mathrm{Im}(z)$ denotes the imaginary part.

In equations (4.12) and (4.13), we make use of the following results:

$$\begin{aligned}K_0(se^{-i\pi}) &= K_0(s) + i\pi I_0(s),\\ K_1(se^{-i\pi}) &= -K_1(s) + i\pi I_1(s).\end{aligned} \tag{5.5a,b}$$

Then, we have

$$\overline{w}_1(r,\zeta_1 e^{-i\pi}) = \left(\frac{c+id}{e+if}\right),$$

where

$$c = -a(Q+m_2R)s_2\left\{K_0(\zeta_1 s_2 a)K_1(\zeta_1 s_1 r) + \pi^2 i_0(\zeta_1 s_2 a)I_1(\zeta_1 s_1 r)\right\},$$

$$d = a\pi(Q+m_2R)s_2\left\{K_0(\zeta_1 s_2 a)I_1(\zeta_1 s_1 r) - I_0(\zeta_1 s_2 a)K_1(\zeta_1 s_1 r)\right\},$$

$$\begin{aligned}e = &-as_1s_2A(m_2-m_1)\zeta_1\left\{K_0(\zeta_1 s_1 a)K_0(s_1 s_2 a) - \pi^2 I_0(\zeta_1 s_1 a)I_0(\zeta_1 s_2 a)\right\}\\ &- 2N(Q+m_2R)s_2\left\{K_0(\zeta_1 s_2 a)K_1(\zeta_1 s_1 a) + \pi^2 I_0(\zeta_1 s_2 a)I_1(\zeta_1 s_1 a)\right\}\\ &+ 2N(Q+m_1R)s_1\left\{K_0(\zeta_1 s_1 a)K_1(\zeta_1 s_2 a) + \pi^2 I_0(\zeta_1 s_1 a)I_1(\zeta_1 s_2 a)\right\},\end{aligned}$$

$$\begin{aligned}f = &-a\pi s_1s_2A(m_2-m_1)\zeta_1\left\{K_0(\zeta_1 s_1 a)I_0(\zeta_1 s_2 a) + K_0(\zeta_1 s_2 a)I_0(\zeta_1 s_1 a)\right\}\\ &+ 2N\pi(Q+m_2R)s_2\left\{K_0(\zeta_1 s_2 a)I_1(\zeta_1 s_1 a) - I_0(\zeta_1 s_2 a)K_1(\zeta_1 s_1 a)\right\}\\ &- 2N\pi(Q+m_1R)s_1\left\{K_0(\zeta_1 s_1 a)I_1(\zeta_1 s_2 a) - I_0(\zeta_1 s_1 a)K_1(\zeta_1 s_2 a)\right\}.\end{aligned}$$

Therefore,

$$\mathrm{Im}\left[\overline{w}_1\left(r,\zeta_1 e^{-i\pi}\right)\right] = \frac{Z_1(r,\zeta_1)}{N_1(\zeta_1)}, \tag{5.6}$$

where

$$Z_1(r,\zeta_1) = de - cf, \qquad N_1(\zeta_1) = e^2 + f^2. \tag{5.7a,b}$$

Substituting (5.4) in (5.1) and making use of equation (5.6) gives

$$w_1(r,t) = \frac{1}{\pi}\int_0^\infty \exp(-t\zeta_1)\,\frac{Z_1(r,\zeta_1)}{N_1(\zeta_1)}\,d\zeta_1, \qquad t > \frac{r-a}{\alpha_1}. \tag{5.8}$$

With the help of equations (5.2) and (5.8), we obtain the expression for $w_1(r,t)$ as

$$w_1(r,t) = \frac{1}{\pi}\cdot \mathrm{H}\left(t - \frac{r-a}{\alpha_1}\right)\int_0^\infty \exp(-t\zeta_1)\,\frac{Z_1(r,\zeta_1)}{N_1(\zeta_1)}\,d\zeta_1, \tag{5.9}$$

and $\mathrm{H}(x)$ is the Heaviside unit step function.

Similarly, we obtain the expression for $w_2(r,t)$ as

$$w_2(r,t) = \frac{1}{\pi}\cdot \mathrm{H}\left(t - \frac{r-a}{\alpha_2}\right)\int_0^\infty e^{-\zeta_2 t}\,\frac{Z_2(r,\zeta_2)}{N_2(\zeta_2)}\,d\zeta_2. \tag{5.10}$$

Substituting equations (5.9) and (5.10), in equations (4.14) and (4.15), and using the convolution theorem, we obtain the displacement fields in the integral form

$$\begin{aligned} u(r,t) = \frac{a}{\pi} \Bigg[ & \mathrm{H}\left(t - \frac{r-a}{\alpha_1}\right) \int_{(r-a)/\alpha_1}^{t} f(t-\tau)\, d\tau \int_0^\infty e^{-\zeta_1 \tau} \frac{Z_1(r,\zeta_1)}{N_1(\zeta_1)}\, d\zeta_1 \\ & - \mathrm{H}\left(t - \frac{r-a}{\alpha_2}\right) \int_{(r-a)/\alpha_2}^{t} f(t-\tau)\, d\tau \int_0^\infty e^{-\zeta_2 \tau} \frac{Z_2(r,\zeta_2)}{N_2(\zeta_2)}\, d\zeta_2 \Bigg], \end{aligned} \tag{5.11}$$

$$\begin{aligned} U(r,t) = \frac{a}{\pi} \Bigg[ & m_1 \mathrm{H}\left(t - \frac{r-a}{\alpha_1}\right) \int_{(r-a)/\alpha_1}^{t} f(t-\tau)\, d\tau \int_0^\infty e^{-\zeta_1 \tau} \frac{Z_1(r,\zeta_1)}{N_1(\zeta_1)}\, d\zeta_1 \\ & - m_2 \mathrm{H}\left(t - \frac{r-a}{\alpha_2}\right) \int_{(r-a)/\alpha_2}^{t} f(t-\tau)\, d\tau \int_0^\infty e^{-\zeta_2 \tau} \frac{Z_2(r,\zeta_2)}{N_2(\zeta_2)}\, d\zeta_2 \Bigg]. \end{aligned} \tag{5.12}$$

## 6. SPECIAL CASE

We consider the disturbance produced by an impulsive force at the boundary $r = a$ as

$$f(t) = F\delta(t), \tag{6.1}$$

where $\delta(t)$ is the Dirac delta function and $F$ is the constant magnitude of the force and the Laplace transform of $f(t)$ is $\overline{f}(p) = F$.

Therefore, equations (4.10) and (4.11) becomes

$$\overline{u}(r,p) = aF\left[\overline{w}_1(r,p) - \overline{w}_2(r,p)\right], \tag{6.2}$$

$$\overline{U}(r,p) = aF\left[m_1\overline{w}_1(r,p) - m_2\overline{w}_2(r,p)\right]. \tag{6.3}$$

Thus, using the inverse $w_j(r,t)$, $(j = 1,2)$, of $\overline{w}_j(r,p)$, $(j = 1,2)$, as given by equations (5.9) and (5.10), we obtain the displacement fields as

$$\begin{aligned} u(r,t) = \frac{aF}{\pi} \Bigg[ & \mathrm{H}\left(t - \frac{r-a}{\alpha_1}\right) \int_0^\infty e^{-\zeta_1 t} \frac{Z_1(r,\zeta_1)}{N_1(\zeta_1)}\, d\zeta_1 \\ & - \mathrm{H}\left(t - \frac{r-a}{\alpha_2}\right) \int_0^\infty e^{-\zeta_2 t} \frac{Z_2(r,\zeta_2)}{N_2(\zeta_2)}\, d\zeta_2 \Bigg], \end{aligned} \tag{6.4}$$

$$\begin{aligned} U(r,t) = \frac{aF}{\pi} \Bigg[ & m_1 \mathrm{H}\left(t - \frac{r-a}{\alpha_1}\right) \int_0^\infty e^{-\zeta_1 t} \frac{Z_1(r,\zeta_1)}{N_1(\zeta_1)}\, d\zeta_1 \\ & - m_2 \mathrm{H}\left(t - \frac{r-a}{\alpha_2}\right) \int_0^\infty e^{-\zeta_2 t} \frac{Z_2(r,\zeta_2)}{N_2(\zeta_2)}\, d\zeta_2 \Bigg]. \end{aligned} \tag{6.5}$$

In the limit as $Q/R \to 0$, $\rho_{12}/\rho 22 \to 0$, we obtain the expressions for displacement in the solid part, $u(r,t)$ due to [6], whereas displacement in liquid part, $U(r,t)$ vanishes.

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