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A Probabilistic Cellular Automata Model for Highway Traffic Simulation

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Abstract

This work presents a probabilistic model for the microscopic simulation of traffic roads based on Nagel-Schreckenberg's model. Each driver's behavior is described through the combination of a continuous probability function with an anticipatory feature that leads to a counter flow velocity tunning. The simulations developed and described herein give rise to a phase diagram which resembles and enriches the fundamental diagram, in its theoretical as well as for real data.

Keywords:
Cellular Automata, Probabilistic Model, Rejection Technique, Traffic Simulation.

1. Introduction

Traffic flow directly affects the quality of life in modern societies. Traffic jams and their associated generation of pollution, as well as their psychological effects, are some of the reasons why a better understanding of traffic flow has received so much attention in the last decades. In order to study and understand traffic flow's characteristics, many mathematical models, both macroscopic and microscopic, have been employed. Among them, Cellular Automata (CA) methods have been employed with good results, since the dynamics of the CA tries to closely mimic the movement of all vehicles and their interaction. Thus CA models have been applied for managing, describing and understanding traffic characteristics. In [1] a CA model was developed for studying different strategies to control and [2] describes the freeway traffic in North Rhine-Westphalia in a CA model, presenting on-line information about the freeway. Employment of CA models in this context is recent and a fully satisfactory widely applicable model is yet to be developed. Some of the main advantages of CA models are that they are easily implemented, lead to moderate computational cost and keep the basic features of the phenomenon ([3], [4], [5]).

This work has two different goals and contributions for approaching traffic highway through a cellular automata model. The first is to improve the flow-density relation, leading to a transition to jammed flow that closely resembles real data. The second one introduces a time explicit feature to the model and position and velocity updates are decoupled. In this work a probabilistic model, organized as two well defined steps, with anticipatory features and

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explicit in time, is proposed. Initially the model defines the vehicle's velocities explicitly from the previous time step configuration. In the sequel an update step is performed. Specifically, the driver's behavior is modeled according to a continuous probability function. Moreover, during the determination of velocities, the counter flow velocity tunning gives rise to a recursive procedure that provides the tunning of velocities. This is performed in such a way that velocities are adjusted to appropriate values even in the event of a velocity reduction by a leading car.

This work is organized as follows: in Section 2 related work on cellular automata models applied to traffic highways is discussed; in Section 3 some concepts of traffic-flow theory are reviewed; Section 4 describes the proposed algorithm; Section 5 presents some test cases. Concluding remarks about the proposed method are discussed in Section 6.

2. Related work

The class of Cellular Automata models applied for traffic problems is divided in two well defined sets, namely deterministic and probabilistic models. Among the most widely known deterministic models are the Rule-184 [6] and a modified version of Rule-184[7]. This latter introduces more velocity variations besides 0 and 1, which are the values established by the rule-184, supporting up to 5 variation cell per unit time simulation. The random factor introduced by the probabilistic models tries to properly represent the behavior of drivers, as well as to improve the flow-density relation. Nagel and Schreckenberg [8] proposed the successful algorithm that has been widely employed, became the basis for several developments and is referred to as NaSch model. NaSch follows simple basic rules: all drivers attempt to drive in the maximum velocity allowed by the stream of vehicles or the road's speed limit; a driver shall keep the vehicle's velocity or, without any apparent reason and with a small probability, simply reduce it. A subsequent adjustment of each vehicle's speed considers its distance to the one immediately ahead.

Slow-to-start models form a subset of the set of probabilistic models. Their main purpose is to represent the driver's eagerness for restoring his velocity when his vehicle is stopped, i.e., the inertia effect of the vehicles. Although slow-to-start models can represent the meta stability phase, drivers' conservative behavior leads to jammed flow with lower density than empirical data indicate.

Among the slow-to-start, one has the following: models discussed in [9] and [10] consider the space ahead after some simulation time, in order to restore the car's velocity. In [11] and [12] the flow adjustment method defines that a driver considers the space between vehicles ahead and adds the free space between them. The concept adopted in such method is that every driver considers that its leading vehicle (that is, the one immediately ahead) will move in the same velocity as in the previous time frame, thus a model with an anticipatory feature.

The anticipatory policy represents a level of expectancy that a vehicle has of its leader. It is given by the distance between two vehicles plus the potential distance that the leader has for moving. The level of expectancy is quantified by a randomness related with the velocity of the leading vehicle. The other rule of anticipation delineates the situation when a (driver's) vehicle has a high level of expectancy that its leader is going to keep its velocity at next time frame and this does not happen.

3. Traffic-Flow Theory

The behavior of traffic is analysed here on the basis of a set of variables: density describes the number of vehicles per unit length of a highway at some time (see Eq. 1) where n is the number of vehicles as well as L represents a part of the highway.

$$\rho = \frac{n}{L} \tag{1}$$

The average velocity, i.e., the averaged sum of velocities, is established as (Eq. 2)

$$\overline{v} = \frac{\sum_{i=1}^{n} v_i}{n} \tag{2}$$

Flow is defined as the number of vehicles that pass by a specific point of the highway per unit time, as in Eq. 3

$$J = \rho\overline{v} \tag{3}$$

By using Eqs. 2 and 3, J can be re-written as in Eq. 4.

$$J = \frac{\sum_{i=1}^{n} v_i}{L} \tag{4}$$

The equations, as described previously, show how to compute the variables at a particular time. However, when one tries to simulate realistic scenarios, many time steps should be used in order for the complexity of the phenomena to emerge. Thus, the equations shall be re-written for greater adequacy, as in Eq. 5, where m is the number of vehicles that pass at a highway section and T is a time period.

$$J = \frac{m}{T} \tag{5}$$

Moreover, the average velocity is re-written, so as to consider the vehicles that pass at a highway section, as in Eq.6.

$$\overline{v} = \frac{\sum_{i=1}^{m} v_i}{m} \tag{6}$$

In order to obtain the average density at a highway section, Eqs. 5 and 6 are replaced by Eq. 7 , that is

$$\overline{\rho} = \frac{m^2}{T \sum_{i=1}^{m} v_i} \tag{7}$$

The equations above represent the behavior of traffic flow on a highway by means of vehicle counts over long periods of time. Typically, the analysis of traffic flow is performed by constructing the corresponding fundamental diagram. This depicts how flow and density relate. The theoretical model is illustrated in Fig. 1 and real data is presented in Fig. 2.

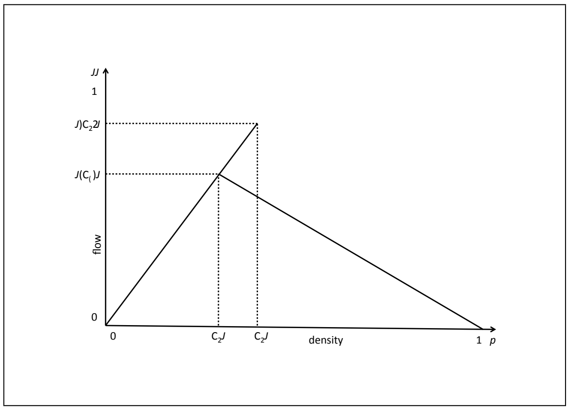

Figure 1: Theoretical fundamental diagram

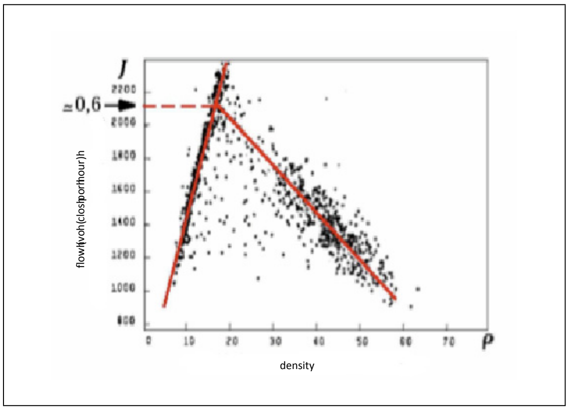

Figure 2: Real data fundamental diagram

The fundamental diagram exhibits three well defined phases, namely: the first one represents the free-flow and corresponds to the region of low to medium density and weak interaction between vehicles. In this phase, the vehicles can move almost at the highway's speed limit, and the flow increases linearly with increasing density, density ranging in $0 \leq \rho \leq c_1$. The second phase presents medium and high density, the flow shall behave either free or jammed, i.e., the phase $c_1 < \rho < c_2$ shows a flow that is not defined only by density, but by the interaction between the vehicles instead. This phase is named the metastable phase. The last phase is $\rho > c_2$, and represents the jammed flow, where an increase in density forces a decrease in the flow.

Therefore, usable models have to represent both qualitatively (mandatory) as well as quantitatively (desirable) the fundamental diagram, making it possible for a coherent analysis of all variables involved in the process to be carried on.

4. The Probabilistic Model

In this section, NaSch's model is briefly reviewed, and its algorithmic description, Algorithm 1, provides a framework for the presentation of the new probabilistic model, which is based on NaSch's model and extends the anticipatory concept proposed by [12].

4.1. Nasch's Model

The set up and discretization used by NaSch's model is depicted as follows. The variables space and time are both discrete so $t \in \mathbf{N}$, $x_i^t \in \mathbf{Z}$; The highway is considered with periodic boundary condition, i.e., the position X is the same as position $X + L$ where L is the length of the circuit. Likewise, the ith vehicle is the same as the $(i + N)$th vehicle, where N is the number of vehicles.

Each cell represents a length space of 7.5 meters and the time evolution scale is measured in seconds. Besides, the variable (v_i^t) denotes the velocity of ith vehicle in the time instant t, in cells per time, while x_i^t denotes the spatial position. The distance between two vehicles in a time instant is represented by d_i^t, and the maximum speed permitted is given by v_{max}.

The probabilistic character of the model is represented by p and p_m. The first is chosen from an uniform distribution and the latter is an initial parameter. They represent the probability for a vehicle to preserve its speed at the next time instant $t + 1$.

Algorithm 1 NaSch's Algorithm

for from the last to the first vehicle **do**
    $v_i^t \leftarrow min[v_i^{t-1} + 1, v_{max}]$
    $d_i^t = x_i^{t-1} - x_{i-1}^{t-1} - 1$
    $p \leftarrow$ randomize number $\in [0, 1]$
    if $v_i^t > d_i^t$ **then**
        $v_i^t \leftarrow d_i^t$
    end if
    if $p \leq p_m$ and $v_i^{t-1} > 0$ **then**
        $v_i^t \leftarrow v_i^{t-1} - 1$
    end if
    $x_i^t \leftarrow x_i^{t-1} + v_i^t$
end for

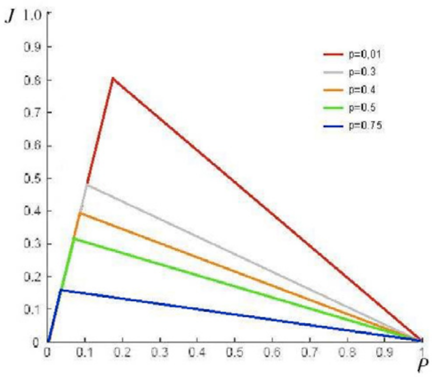

Figure 3: Fundamental diagram of NaSch's model

NaSch's model presents satisfactory results. The Fig 3 shows the effects of different values of p_m. These define in which density the free flow behaviour switches into the jammed one.

4.2. The Proposed Model

The model proposed herein is described by algorithms 2 and 3. The algorithm is explicit in time and its aim is to ensure the correct velocity definition of each vehicle, in order to prevent a cell of CA to be occupied by more than one vehicle, what would characterize an unreal collision (unreal stands for the fact that effective collisions are left out of the model, since one here considers the driver would avoid it by braking more intensely).

The proposed model has two separate stages. In the first, velocities are defined while, in the second, positions for all vehicles are updated. The former is described by lines $2-14$ and the latter is treated by line 17, both of algorithm 3. Velocity definition and position updating are independent tasks. Moreover, they are not tied to the flow direction of the simulation, thus allowing free choice of access to the data structure.

Algorithm 2 The Proposed Algorithm - main algorithm

1: **for all** vehicles **do**
2:     $v_i^t \leftarrow min(v_i^{t-1}+1, v_{max})$
3:     $p \leftarrow$ randomize number $\in [0,1]$
4:     **if** $p \leq p_m$ and $v_i^{t-1} > 0$ **then**
5:         $v_i^t \leftarrow v_i^{t-1} - 1$
6:     **end if**
7:     $\alpha_i^t \leftarrow normal(\mu, \sigma)$
8:     $d_{is}^t \leftarrow (x_i^{t-1} - x_{i+1}^{t-1} - 1) + [v_{i+1}^{t-1} \times (1 - \alpha_{i+1}^{t-1})]$
9:     **if** $v_i^t > d_{is}^t$ **then**
10:         $v_i^t \leftarrow d_{is}^t$
11:     **end if**
12:     **if** $[v_i^{t-1} \times (1 - \alpha_i^t)] > d_{is}^t$ **then**
13:         call Solver Vehicles' Cluster (i)
14:     **end if**
15: **end for**
16: **for all** vehicles **do**
17:     $x_i^t \leftarrow x_i^{t-1} + v_i^t$
18: **end for**

The algorithm 3 is part of Alg. 2 and is responsible for executing the explicit procedure of reverse adjustment of velocity. This procedure is invoked whenever a vehicle reduces its velocity unexpectedly, forcing the algorithm to redefine the velocities of vehicles affected by this fact.

Algorithm 3 The Proposed Algorithm - Solver Vehicles' Cluster algorithm

1: **while** vehicles **do**
2:     $\alpha_i^t \leftarrow normal(\mu, \sigma)$
3:     $d_{is}^t \leftarrow (x_i^{t-1} - x_{i+1}^{t-1} - 1) + [v_{i+1}^t \times (1 - \alpha_{i+1}^t)]$
4:     **if** $v_i^t > d_{is}^t$ **then**
5:         $v_i^t \leftarrow d_{is}^t$
6:     **end if**
7:     **if** $[v_i^t \times (1 - \alpha_i^t)] > d_{is}^t$ **then**
8:         call Solver Vehicle's Cluster (i)
9:     **end if**
10: **end while**

The anticipatory procedure adopted employs the concept of effective distance (d_{is}^t). It considers the distance between two vehicles increased by an expectation term. This term is an expectation level of how much the next vehicle will move analysing its velocity in the previous time instant. It is described by lines 8 and 3 of algorithms 2 and 3, respectively.

The expression is composed by two parts: the first is the distance from the ith vehicle to its follower $(i+1)$th, the second is the velocity of $(i+1)$th vehicle. This is shown by Fig. 4, where one verifies that the leading vehicle is not going to change its velocity at next time instant with probability $\alpha \in [0..1]$. The parameter α stands for the uncertainty that one vehicle will keep its velocity from the previous time instant to the one under computation. It is modeled by a continuous probability function and its choices are implemented based on a Monte Carlo rejection process.

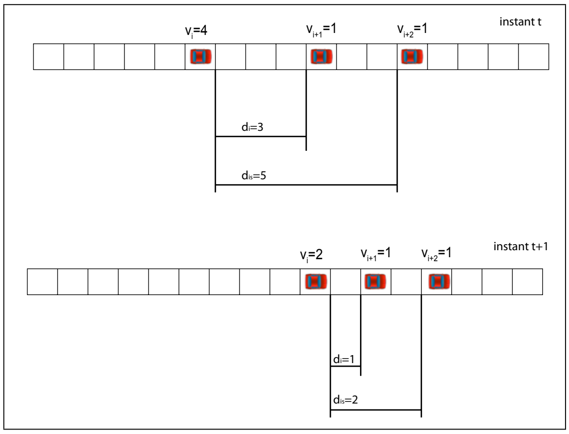

Figure 4: Effective distance scheme

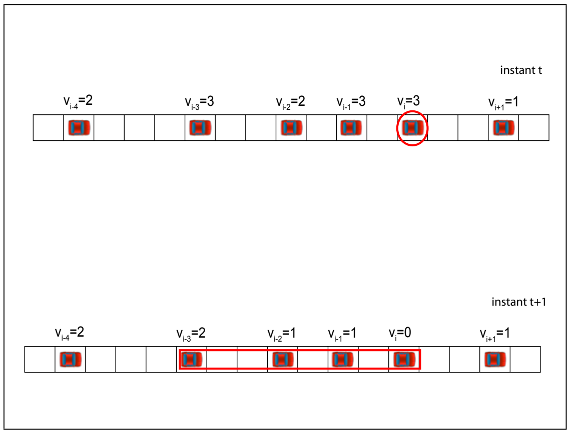

Figure 5: Cluster of vehicles

The rule of explicit reverse adjustment of velocity, in order to avoid unrealistic collision whenever the ith vehicle does not move what is expected or the ith velocity is not the same as in the previous instant, the vehicle signalizes for its follower this fact. It has its velocity re-adjusted and an iterative update procedure is enabled until there are no vehicles affected by such velocity reduction. This set of vehicles is called the influence cluster and illustrated by Fig. 5.

In order to evaluate the number of vehicles that are influenced and have their velocities re-adjusted, the following procedure is adopted:

- Consider the ith vehicle is to be analyzed. One defines how many cells the previous vehicle expects it to move, i.e., $\alpha_i v_i^{t-1}$. Where $\alpha_i = min(\alpha)$, $\forall f(\alpha) \neq 0$, i.e., $\alpha_i = l_i$ the minimum value of $\alpha \neq 0$ and $f(\alpha)$ is a probability distribution function modeling the driver's behavior. This is illustrated in Fig. 6;
- When this vehicle signalizes to the previous, through the expression $\alpha v_i^{t-1} > d_{is}^t$, an iterative process is enabled and repeated until all influenced vehicles have their velocities updated.

The procedure described above is applied when the condition $\alpha_i v_i^{t-1} > d_{is}$ holds. This is used to restrict the number of vehicles that need to have their velocities updated. It divides the highway in clusters, so that the algorithm can separate the vehicles in influence regions, i.e., these regions are composed by clusters of vehicles that will be affected by sudden changes in vehicles' velocities.

The cluster is identified by collecting cluster members along the direction opposing the flow. One begins with a vehicle that does not update its velocity until the vehicle that is not affected by the decrease of the velocities of the vehicles belonging to the cluster, i.e., until the expression $v_i^t \leq d_i^{t-1}$ turns out to be true. Additionally, the signalization process may be propagated as any vehicle in the cluster may move with a velocity which differs from the one it was expected to. This will result in the creation of subclusters inside the main cluster. This is computationally treated by using a recursive strategy. In a cluster where one vehicle does not move as expected, the iterative procedure is applied within all cluster and its subclusters.

4.3. An Approach based on Continuous Probability and The Rejection Technique

In the model presented herein a novel policy for anticipation is proposed. Namely, instead of using fixed probabilities to describe average behavior on highways, a continuous probability function is employed.

The new approach for the probability distribution function proposed in this work represents a more flexible and realistic model for the representation of highway diversity of behavioral responses. Here, the normal probability distribution function is adopted for its convenience and ease of use.

The rejection technique is an essential part of the technique presented herein. It shall be described by algorithm 4:

Algorithm 4 Rejection Rule (μ, σ)

repeat
  $x \leftarrow randomize$
  $y \leftarrow randomize$
  $p(x) \leftarrow \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$
until $y > p_x$
return $\leftarrow x$

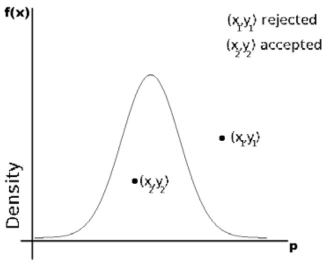

Figure 6: The Monte Carlo rejection technique

This work uses a Monte Carlo rejection technique as illustrated in Fig. 6. The procedure is effective both for discrete as well as continuous probability distribution functions.

5. Testing and Results

The set up conditions for simulation are the same as those employed by Nagel and Schreckenberg's work [8]. Therefore, preliminary tests are performed with circular highways, under periodic boundary condition, with 300 cells. The simulations were carried out using highway densities ranging from 0.01 up to 0.99, and evolving for 10000 time units. The results for the first 1000 units were discarded since transient effects are not the target. Thus, after the simulation evolves for some time steps/units, the long range collective effects which arise from the short term interactions emerge and prevail during the long term operation of the highway.

The average value represents the average speed of the highway, subjected to the maximum allowed value of 5 cells per unit time. Furthermore, the average and standard deviation values must be in the range such that $\int_1^5 f(x, \mu, \sigma)\, dx = 1$ is ensured.

The fundamental diagram is represented by Fig 7 for some values for the uncertainty on a car keeping its speed. Among the exhibitted results for the p_m values, $p_m = 0.25$ closely resembles real data.

Fig. 8 illustrates different values for averages that describe the flow for several levels of expectation of anticipation. For the set of parameters $p_m = 0.25$, $\mu = 3.0$ and $\sigma = 0.25$, the simulated data provide very good agreement with real data.

6. Conclusion

In this work a probabilistic model based on Nagel-Schreckenberg's model is presented. Besides, each driver's behavior is described through the combination of a continuous probability function and counter flow velocity tunning.

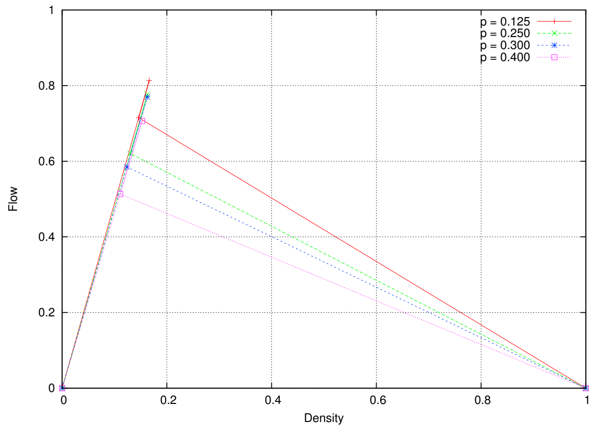

Figure 7: Fundamental diagram - $\mu = 2.5/\sigma = 0.25$

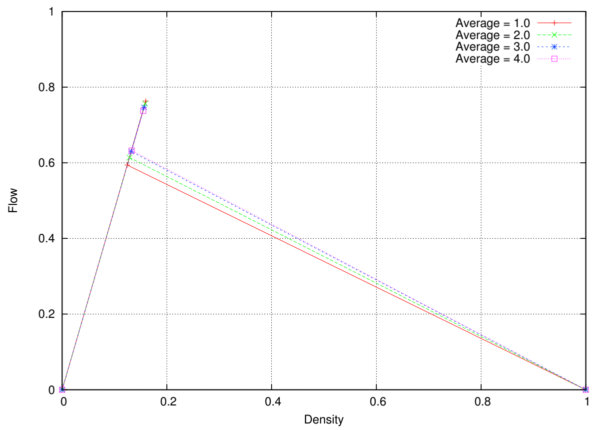

Figure 8: Fundamental diagram - $\sigma = 0.25$

Drivers' behavior may be, in a first attempt, modelled by a uniform distribution. However, some drivers behave more aggressively than others. Thus, the normal distribution is a more appropriate description. Indeed, some drivers behave more aggressively than others. Moreover, the leading vehicle will very often keep its velocity in the next time instants. however, when a vehicle reduces its velocity, as in the real world, it should be indicated for the trailing ones.

The model proposed in this work requires few parameters to set up a simulation. Despite being an explicit method, it is stable. This is due to the fact that the recursive cluster self adjustment method always converges because of the updating sequence. The simulations developed and described herein give rise to a phase diagram which resembles and enriches the fundamental diagram, in its empirical as well as theoretical versions.

Summarizing, the method proposed herein is a fast procedure which appears to appropriately capture the essential features of the real problem, since the phase diagrams become very similar, both qualitatively as well as quantitatively, to empirical data.

7. Acknowlegments

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