Measurement deviations for shear wave velocity of bender element test using time domain, cross-correlation, and frequency domain approaches

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Abstract
Discrepancies in shear wave velocity measurements, attributed to the use of different methods to estimate travel time, for the bender element test were examined for various types of samples in this paper. A series of bender element tests using three different techniques, the time domain (TD), cross-correlation (CC), and frequency domain (FD) techniques, were performed to estimate travel times on identical specimens in triaxial or unconfinned compression apparatuses. Experimental results revealed that the CC technique can provide reasonable values of the travel time by the maximum peak point for soft soils, while it is difficult to apply to sandy or artificially treated soils because of the unclear arrival point. From both, experimental investigation and numerical analysis of CC, the frequency dependence of the travel time is demonstrated. As a result of travel time estimation using frequency sweep signals, strong dispersion and scattering of phase and group velocities due to the non-linearity of the test system are indicated in the FD technique. The FD technique broadly underestimates the shear wave velocity compared with the TD technique, a trend shown in most soil samples. Shear wave velocities obtained from the CC and FD techniques are also compared with those obtained from the TD technique.

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Keywords: Bender element test; Shear wave velocity; Time domain; Frequency domain; Cross-correlation

1. Introduction

Shear wave velocity measurement using the bender element, which was originally introduced by Shirley and Hampton (1978) and was widely spread to many researchers by Dyvik and Madshus (1985), is a practical method which came to be embedded in laboratory test apparatuses in the mid 1980s. Bender elements are well-known as low-cost and simple tools to measure the shear wave velocity of soil in a laboratory (e.g., Gu et al., 2013; Seng and Tanaka, 2012). In early use of the bender element test, the travel time of the shear wave was determined by the distance between characteristic points of the transmitted and received waveforms, and thence the shear wave velocity $V_s$ was evaluated as

\[ V_s = \frac{L}{\Delta t} \]

where $L$ and $\Delta t$ are the travel distance and travel time of the shear wave, respectively. This approach has recently been labeled the time-domain (TD) technique. Due to the problem of the near-field effect frequently masking the characteristic point corresponding to the arrival time, the original method has been closely investigated by many researchers to increase the precision and reliability of $V_s$ measurements (Sanchez-Salinero et al., 1986; Brignoli et al., 1996; Arroyo et al., 2003; Arulnathan et al., 1998). Consequently, a general consensus has been reached concerning the following characteristics of TD analysis: the tip-to-tip distance

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between bender elements is reasonable for the travel distance $L$; one cycle of a sinusoid makes reading the travel time simpler compared to a rectangular wave; the peak-to-peak distance between the transmitted and received signals has a reasonable travel time; and higher sinusoid frequency, i.e., higher travel distance to wavelength ratio $L/\lambda$, can lessen the distortion of the received waveform caused by the near-field effect. Yamashita et al. (2009) demonstrated the effect of these improvements on shear modulus evaluation and laid the foundation for the 2011 establishment of the Japanese Geotechnical Society’s standard concerning the measurement of the shear wave velocity using bender elements.

Nevertheless, the fundamental problem of determining the first arrival point in a given received waveform is not entirely solved. Depending on the conditions, such as the tested soil sample dimensions, the saturation, and the stress level, the occurrence of ambiguous waves due to the near-field effect is still difficult to prevent and complicates the detection of the first shear wave arrival. The cross-correlation (CC) technique (Viggiani and Atkinson, 1995) can be considered an alternative approach even though many researchers have commented on its limitations (Viggiani and Atkinson, 1997; Santamarina and Fam, 1997; Gajo et al., 1997; Wang et al., 2007; da Fonseca et al., 2009). Airey and Mohsin (2013) re-examine the claims made by these authors and demonstrate that CC between single- and multi-cycle sine input signals and their received signals reflects a reasonable travel time, if the input frequency and characteristic point corresponding to arrival are chosen appropriately.

The frequency domain (FD) technique, an alternate method for travel time estimation based on the phase shift between the transmitted and received signals, was also introduced in the last decade (Greening and Nash, 2004). This method provides two objective criteria for travel time determination, the secant and the tangent of phase spectra, which yield two shear wave velocities called phase and group velocities, respectively. However, both velocities obtained from FD analysis have been found to be slower than those from the TD technique even when the applied frequency range is carefully chosen (Greening and Nash, 2004; da Fonseca et al., 2009; Alvarado and Coop, 2012; Styler and Howie, 2013).

This paper comprehensively re-examines the results of the bender element test based on the TD, CC, and FD techniques to understand their characteristics, particularly the frequency dependence of the $V_s$ measurement. A series of bender element tests using these techniques are carried out on a variety of soil samples with a wide spectrum of stiffness values. In the following section, the characteristics of the CC technique are described based on both experimental and analytical investigations. CCs are identified experimentally and numerically to reveal the frequency dependence of the CC technique. This is followed by an assessment of the phase and group velocities measured by the FD technique, including identification of the cross-spectrum using frequency sweep inputs. Finally, these velocities are compared with that from the TD technique, which is based on the standard of the Japanese Geotechnical Society.

### 2. Laboratory equipments and experimental procedures

Ten types of soil samples with a wide range of stiffness parameters were prepared to compare the difference in travel times obtained from the TD, CC, and FD techniques on identical specimens under identical conditions. The physical properties of the samples are shown in Table 1. Appropriate preparation methods were applied for each sample: air pluvia-

| Sample name                  | Soil particle density $\rho_s$ $\left(t/m^3\right)$ | Maximum void ratio $\varepsilon_{max}$ | Minimum void ratio $\varepsilon_{min}$ | Effective consolidation stress $\sigma_0$ (kPa) | Void ratios $\rho_s$ $\left(t/m^3\right)$ | Liquid limit $w_L$ (%) | Plastic limit $w_p$ (%) | Void ratios $\rho_s$ $\left(t/m^3\right)$ | Effective consolidation stress $\sigma_0$ (kPa) | Void ratios $\rho_s$ $\left(t/m^3\right)$ | Ignition loss $L_i$ (%) | Initial water content $\omega_i$ (%) | Void ratios $\rho_s$ $\left(t/m^3\right)$ | Effective consolidation stress $\sigma_0$ (kPa) | Void ratios $\rho_s$ $\left(t/m^3\right)$ |
|------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| Toyoura sand (dry)           | 2.65                             | 0.98                            | 0.62                            | 25, 50, 100, 200, 400           | 0.69, 0.69, 0.69, 0.69, 0.69   | 0.92                            | 0.69                            | 0.69                            | 0.69                            | 0.69                            | 0.69                            | 0.69                            | 0.69                            | 0.69                            | 0.69                            |
| Inagi sand (dry)             | 2.72                             | 1.27                            | 0.77                            | 15, 25, 50, 100, 200            | 1.03, 1.01, 0.99, 0.99, 0.92   | 1.03                            | 1.01                            | 0.99                            | 0.99                            | 0.99                            | 0.99                            | 0.99                            | 0.99                            | 0.99                            | 0.99                            |
| Glass bead $d=2$ mm (dry)   | 2.49                             | 0.68                            | 0.59                            | 10, 50, 200                     | 0.59, 0.59, 0.59                | 0.59                            | 0.59                            | 0.59                            | 0.59                            | 0.59                            | 0.59                            | 0.59                            | 0.59                            | 0.59                            | 0.59                            |
| Glass bead $d=0.4$ mm (dry) | 2.49                             | 0.70                            | 0.53                            | 10, 50, 200                     | 0.55, 0.55, 0.54                | 0.55                            | 0.55                            | 0.55                            | 0.55                            | 0.55                            | 0.55                            | 0.55                            | 0.55                            | 0.55                            | 0.55                            |
| Pisa clay                    | 2.77                             | 81                              | 25                              | 20, 150, 450                    | 1.64, 1.56, 1.27               | 1.64                            | 1.56                            | 1.27                            | 1.27                            | 1.27                            | 1.27                            | 1.27                            | 1.27                            | 1.27                            | 1.27                            |
| NSF clay                     | 2.73                             | 67                              | 35                              | 88                             | 1.29                            | 1.29                            | 1.29                            | 1.29                            | 1.29                            | 1.29                            | 1.29                            | 1.29                            | 1.29                            | 1.29                            | 1.29                            | 1.29                            |
| Artificially treated soil (base soil) | 2.56                             | 86                              | 36                              | Unconfined                      | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               |
| Etsu organic clay            | 2.34                             | 22.4                            | 288                             | 17, 38                         | 4.68, 2.93                      | 4.68                            | 2.93                            | 2.93                            | 2.93                            | 2.93                            | 2.93                            | 2.93                            | 2.93                            | 2.93                            | 2.93                            | 2.93                            | 2.93                            |
| Etsu peat                    | 1.99                             | 46.5                            | 384                             | 34, 55                          | 6.76, 5.49                      | 6.76                            | 5.49                            | 5.49                            | 5.49                            | 5.49                            | 5.49                            | 5.49                            | 5.49                            | 5.49                            | 5.49                            | 5.49                            | 5.49                            |
| Akita peat                   | 1.64                             | 76.5                            | 630                             | 17–300                          | 6.56–2.95                       | 6.56                            | 2.95                            | 2.95                            | 2.95                            | 2.95                            | 2.95                            | 2.95                            | 2.95                            | 2.95                            | 2.95                            | 2.95                            | 2.95                            | 2.95                            |

Table 1
Physical properties of tested soil samples and states of specimens during $V_s$ measurement.
tion for sandy soil or glass beads, pulling out of a thin walled tube or preconsolidation cell and trimming to given dimensions for clayey or peaty soil, or curing in a mold for artificially treated soil (Kataoka et al., 2013). The dimensions of the specimens are 75 mm diameter and 150 mm height for peat and organic clay, and 50 mm diameter and 100 mm height for other samples.

Apart from the artificially treated soil, specimens are isotropically consolidated at a prescribed stress in a triaxial cell, where the transmitter and receiver elements are installed in the top cap and pedestal, respectively. At the end of primary consolidation, the bender element tests are implemented under consolidation stress. Certain measurements using single-period sinusoids and frequency sweep signals are carried out so that $V_s$ in identical specimens can be simultaneously evaluated by the TD, CC, and FD techniques. Frequencies for single-period sinusoids were selected from the range of 1 to 100 kHz corresponding to the stiffness of each sample. For artificially treated soil specimens, the same procedures were performed under unconfined conditions in unconfined compression apparatuses. The dimensions of the bender elements are as follows: 12 mm in length (i.e., the length embedded in top cap or pedestal and soil specimen are 7 and 5 mm, respectively), 10 mm in width, and 0.5 mm in thickness. A substantial length of 5 mm was inserted into the specimen. An epoxy resin coating 0.5 mm in thickness was applied for waterproofing and insulation. The setup for the bender element test consisted of a pair of bender elements installed in triaxial or unconfined compression apparatuses, a function generator (WF1974 or WF1945, NF Corporation), and a digital oscilloscope (DS-5554, Iwatsu Test Instruments Corporation or DL1740, Yokogawa Corporation). The sampling frequencies of the function generator and the digital oscilloscope are 30 and 500 MHz, respectively. Data for the transmitted and received signals were recorded by the oscilloscope. The analysis for travel time determination with each technique was carried out after each test was finished.

Triaxial cyclic loading tests were performed on some samples after $V_s$ measurements in order to confirm the reliability of the TD analysis. The amplitude of the axial strain was 0.005% and the number of cycles was 11.

3. Techniques of travel time estimation

3.1. Time domain technique

In the TD technique, a high value of $L/\lambda$ reduces signal distortion in the received wave and simplifies the detection of the arrival time. Using the fundamental relation $V_s = f_T \lambda$ and Eq. (1), the $L/\lambda$ ratio can be written as

$$\frac{L}{\lambda} = \frac{f_T}{f} \Delta t,$$

where $f_T$ is the input signal frequency to drive the transmitter element. Given a constant $\Delta t$, $L/\lambda$ can only be controlled by adjusting the frequency. In practical situations, variations in the shear wave velocity due to using different soil samples or stress levels results in major variations in the travel time $\Delta t$, if the height of the specimen is fixed. This means that a satisfactory frequency for a given value of $L/\lambda$ varies across a wide spectrum of frequencies, depending on the type of soil or the applied stress level.

In this study, appropriate frequencies are found according to the Japanese Geotechnical Society’s standard (Japanese Geotechnical Society, 2011), then the travel time of the shear wave is estimated: (1) for every data set, travel times $\Delta t_{TD}$ and $\Delta t_{TDp}$ are evaluated by the start-to-start and peak-to-peak distances of transmitted and received waves, respectively. (2) Data with $L/\lambda$ ratios less than two are excluded to prevent the near-field effect. (3) Results with errors between $\Delta t_{TD}$ and $\Delta t_{TDp}$ of over 3% are excluded. (4) Finally, $\Delta t_{TD}$ is given by

$$\Delta t_{TD} = \frac{\Delta t_{TDs} + \Delta t_{TDp}}{2}.$$

When more than two datasets are applicable, the mean value of $\Delta t_{TD}$ is adopted.

3.2. Cross-correlation technique

The CC function is given by

$$cc_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t) y(t+\tau) dt,$$

where $x(t)$ and $y(t)$ are the transmitted and received signals, respectively, and $T$ is the recording time. Eq. (4) can also be written as an alternative expression of the cross-spectrum between the transmitted and received waves in the FD

$$CC_{xy}(f) = X(f)Y(f)^*,$$

where $CC_{xy}(f)$, $X(f)$ and $Y(f)$ are the Fourier transforms of $cc_{xy}(\tau)$, $x(t)$ and $y(t)$, respectively, and the asterisk denotes the complex conjugate. CC’s are easily given by calculating the inverse Fourier transform of $CC_{xy}(f)$ in Eq. (5). The travel time of the shear wave is defined as the time corresponding to the maximum and first peak point of CC. The former is denoted by $\Delta t_{CC1}$ and the latter by $\Delta t_{CC2}$. $\Delta t_{CC2}$ is manually determined from multiple signals recorded with different input frequencies, while $\Delta t_{CC1}$ is automatically determined. Generally, $\Delta t_{CC1}$ does not provide a reasonable travel time when sandy soil is tested (Airey and Mohsin, 2013; Yamashita et al., 2009). $\Delta t_{CC1}$ and $\Delta t_{CC2}$ are essentially identical when the maximum peak point is consistent with the first peak.

3.3. Frequency domain technique

When applying the FD technique to estimate the travel time, a frequency sweep signal is commonly used because of its wide frequency spectrum. The travel time is then obtained from the phase difference of the cross-spectrum between the transmitted and received waves. Two kinds of travel times, i.e., $\Delta t_{ph}$ and $\Delta t_{gr}$ corresponding to the phase and group velocities, respectively, can be defined by the slope of the phase spectrum, $\Delta t_{ph}$, given by the secant of the spectrum at frequency $f$, is described as

$$\Delta t_{ph} = \frac{1}{2\pi f} \phi.$$

$$\Delta t_{TD} = \frac{\Delta t_{TDs} + \Delta t_{TDp}}{2}.$$
where $\phi$ is the unwrapped phase of the spectrum. $\Delta t_{gr}$, defined as the tangent of the spectrum at frequency $f$, is given by

$$
\Delta t_{gr} = -\frac{1}{2\pi f} \frac{d\phi}{df}. \tag{7}
$$

Eqs. (6) and (7) indicate that $\Delta t_{gr}$ requires the absolute phase shift of transmitted and received waves for estimation, while $\Delta t_{ph}$ can be obtained from the relative phase shift. Offsets to the absolute phase can be evaluated by the combined identification of TD and FD analysis (Styler and Howie, 2013), with the results of TD analysis from Eq. (3), or the $\pi$-point technique (Greening and Nash, 2004).

Three types of sweep signals are used: linear sine sweep (e.g., Greening and Nash, 2004; da Fonseca et al., 2009; Styler and Howie, 2013), log sine sweep, and time-stretched pulse (TSP) (Aoshima, 1981). In discrete time, each signal can be written as follows. For linear sweep signals

$$
x_{lin}(t) = \begin{cases} 
A \sin 2\pi \left( f_0 + \frac{t}{t_f} \Delta f \right) t, & 0 \leq t \leq t_f \\
0, & t_f \leq t \leq N \Delta t \\
t = k \Delta t, & (k = 0, 1, 2, \ldots, N) \\
\Delta t = f_s^{-1},
\end{cases} \tag{8}
$$

where $x_{lin}(t)$ is the time history of the linear sweep, $A$ is the amplitude, $f_0$ and $\Delta f$ are the origin and width of the frequency sweep, respectively, $t_f$ is the duration of the frequency sweep, $N$ is the number of data points, $\Delta t$ is the discrete time, and $f_s$ is the sampling frequency. For log sweep signals

$$
x_{log}(t) = \begin{cases} 
A \sin 2\pi \left( 10 \left( \frac{t}{t_f} \log \left( 1 + \frac{\Delta f}{f_0} \right) \right) \right) f_0 t, & 0 \leq t \leq t_f \\
0, & t_f \leq t \leq N \Delta t \\
t = k \Delta t, & (k = 0, 1, 2, \ldots, N) \\
\Delta t = f_s^{-1},
\end{cases} \tag{9}
$$

where $x_{log}(t)$ is the time history of the log sweep. For TSP,

$$
X_{TSP}(k) = \begin{cases} 
A \exp \left( \frac{4mck^2}{N^2} \right), & 0 \leq k \leq N/2 \\
X^*(N-k), & \frac{N}{2} + 1 \leq k \leq N \\
f(k) = \frac{k}{N \Delta t} (k \leq N/2)
\end{cases}
$$

where $X_{TSP}(k)$ is the Fourier transform of TSP, $m$ is a parameter relating to the degree of sweeping, and $f(k)$ is the discrete frequency. The frequency increases linearly with time in Eq. (8) and exponentially in Eq. (9), while the phase shifts are proportional to square of the frequency in Eq. (10). Fig. 1 shows the amplitudes of the Fourier spectra of the sweep signals, exhibiting remarkable amplitude decay in the high frequency range for the log sweep signals, while maintaining the amplitudes for the linear sweep and TSP signals. The
Fig. 4. Typical results of bender element test on soft soils.

Fig. 5. Typical results of bender element test on sandy soils.
values of the parameters in Eqs. (8)–(10) are summarized in Table 2, and the frequency characteristics of the sweep signals varying with the parameters are shown in Figs. 2 and 3.

4. Results and discussion

4.1. Estimation of travel time and shear wave velocity using cross-correlation

Typical input and received waveforms of the bender element test on various soil types with a wide spectrum of input frequencies are shown in Figs. 4–6. Results that do not fulfill the criteria for travel time estimation in the TD (Section 3.1) are also represented here. Arrival points based on the start-to-start and peak-to-peak distances are represented only for fulfilled received waveforms. CC’s are shown below each set of waveforms. For soft soils, waveforms of CC’s show clear peak points. The first peaks are consistent with the maximum peaks, except when the input frequency is high. Travel times can be uniquely determined in accordance with the CC technique, even though the travel time $\Delta t_{CC}$ increases for Ebetsu peat (2.42–2.47 ms), Ebetsu organic clay (3.11–3.18 ms), and Pisa clay (0.636–0.680 ms).

In contrast, the maximum and first peaks do not coincide for sandy soils (Fig. 5). Similar oscillations in the beginnings of the waveforms are also observed for glass beads and artificially treated soil (Fig. 6). These oscillations are derived from ambiguous peaks observed in the beginning of the received signal. This behavior, commonly observed in sandy soils (Airey and Mohsin, 2013; Yamashita et al., 2009; Gu et al., 2013), is due to reflected P-waves and does not represent the arrival of a shear wave (Lee and Santamarina, 2005). The effect of the reflected wave is more pronounced for sandy soils than for soft soils because of long reverberations and low damping. Since the correct peak corresponding to the arrival point migrates as stress state or the travel distance varies (Airey and Mohsin, 2013), it is difficult to detect it only from the CC waveform. Considering the similarity between the CC and received waveforms, it is more direct and practical to use the peak-to-peak distance $\Delta t_{TDp}$ instead of $\Delta t_{CC}$. Note that since CC reduces signal noise (Figs. 4–6), the use of CC can be advantageous if the received wave has high noise levels.

In Figs. 7–9, shear wave velocities $V_{sCC}$ obtained from CC are compared with velocities $V_{sTD}$ obtained from TD analysis for various samples, with velocities fulfilling previously defined criteria (Section 3.1). Travel times for $V_{sCC}$ estimation are determined from the maximum peak point ($\Delta t_{CC}$). In addition to results from the observed data, the values of $V_{sCC}$ calculated by the waveform reconstruction method (Ogino et al., 2010) are shown. These interpolate the observed data and help to clarify the relationship between the variation of $V_{sCC}$ and the frequency variation. Values of shear moduli calculated from $V_{sTD}$ are shown in Table 3 and compared to those obtained from other tests or well-established equations in order to confirm the reliability of TD analysis, which is in agreement with results for soft and sandy soils. The value of $V_{sTD}$ for artificially treated soil is also reported to be consistent with PS logging results, though less than soft and sandy soils (Kataoka et al., 2013).

For soft soils and glass beads, the mean values of $V_{sCC}$ are nearly equal to those of $V_{sTD}$. The discrepancy is less than 10%
with respect to $V_{sCC}$ and is acceptable considering the scatter of $V_{sTD}$. However, the difference cannot be disregarded for sandy or artificially treated soils, for which $\Delta t_{CC1}$ and $\Delta t_{CC2}$ do not coincide. The CC technique yields unreasonable values, significantly underestimating the shear wave velocity. Values of $V_{sCC}$ are smaller than those from the empirical equations for Ottawa and Toyoura sands at comparable stress and void ratios (Hardin and Richart, 1963; Iwasaki and Tatsuoka, 1977).

## 4.2. Correlation of travel time in cross-correlation and time domain techniques

Although the peak of CC provides a travel time close to the TD estimation when the maximum and first peaks coincide, most of mean values of $V_{sCC}$ are smaller than those of $V_{sTD}$ (Figs. 7–9). Moreover, the behavior of the observed and

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### Table 3

Comparison of shear moduli $G$ between bender element test (TD technique) and other tests.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\sigma_0$ (kPa)</th>
<th>$\rho_t$ (t/m$^3$)</th>
<th>$V_{sTD}$ (m/s)</th>
<th>$G_{TD}$ (MPa)</th>
<th>$G$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ebetsu peat</td>
<td>34</td>
<td>1.15</td>
<td>36.6</td>
<td>1.54</td>
<td>1.56$^a$</td>
</tr>
<tr>
<td>Ebetsu organic clay</td>
<td>17</td>
<td>1.24</td>
<td>30.4</td>
<td>1.14</td>
<td>1.14$^a$</td>
</tr>
<tr>
<td>Pisa clay</td>
<td>150</td>
<td>1.69</td>
<td>118</td>
<td>23.8</td>
<td>20.4$^a$</td>
</tr>
<tr>
<td>Toyoura sand</td>
<td>200</td>
<td>1.56</td>
<td>302</td>
<td>143</td>
<td>149$^b$</td>
</tr>
</tbody>
</table>

$^a$Values of $G$ are obtained from cyclic loading test.

$^b$Value of $G$ is obtained from empirical equation by Iwasaki and Tatsuoka (1977).

Moreover, a discontinuity in the shear wave velocity can be seen at 25 kHz for Toyoura sand and artificially treated soil due to a jump of the maximum peak to the next peak as the frequency of transmitted signal varies. This means that the CC technique does not work as an alternative to the TD technique, at least for relatively stiff samples. CC analysis is effective only when the first peak of CC is consistent with the maximum peak, as with clayey or peaty soil.
calculated data of $V_{sCC}$, which slightly decreases as frequency increases, differs from that of $V_{sTD}$ obtained from peak-to-peak estimation. Numerical simulation using an oscillation model can provide helpful information about the behavior of the input and received signals of the bender element test. This simulation can qualitatively evaluate the position of the peak of CC and hence the frequency dependence of CC analysis. Employing a damped free vibration given by Eq. (11) as the transfer function of the receiver bender element, the received signal $y(t)$ is described by Eq. (12) (Viggiani and Atkinson, 1997; Wang et al., 2007; Styler and Howie, 2013).

$$h(t) = \frac{\exp(-2\pi f_n h t)}{2\pi m f_n \sqrt{1 - h^2}} \sin 2\pi f_n \sqrt{1 - h^2} t,$$

$$y(t) = \text{IFFT}[\text{FFT}[x(t)] \cdot H_s(f) \cdot \text{FFT}[h(t)]]$$

where $f_n$ is the natural frequency, $h$ is the damping ratio, $m$ is the system mass, $x(t)$ is the input signal, $H_s(f)$ is the transfer function of soil tested, and FFT and IFFT represent the Fourier and inverse transforms, respectively. Values of $f_n$ and $h$ are configured so that spectrum of $y(t)$ matches that of the observed received signal. Spectrum matching, with $f_n = 1$ kHz and $h = 0.2$ for Ebetsu peat (Fig. 4), can be seen to reproduce observed waveforms (Fig. 10). Predominant frequencies $f_1$ can be determined as the peaks of the spectra (represented by arrows). Values of $f_0$ stay at 1 kHz, equal to $f_n$, while $f_T$ varies dramatically. Note that the near-field effect and waves reflected at boundaries are not considered here for simplicity, hence $H_s(f) = 1$ and $m = 1$ are adopted. Single-cycle sinusoids $x(t)$ yield received signals $y(t)$ (Fig. 11). So that its start position is consistent with that of the observed received waveforms (Fig. 4), $y(t)$ is rotated by 2.25 ms. Every received signal exhibits a maximum at the first peak. $\Delta t_{TDp}$ can be easily estimated by the distance between the peaks in $x(t)$ and $y(t)$. CC and hence $\Delta t_{CC1}$ are also yielded by Eq. (5).

Fig. 12 shows the variation in $\Delta t_{CC1}$ and $\Delta t_{TDp}$ with frequency compared with observed data. Frequency $f_T$ is normalized with respect to $f_0$. $\Delta t_{CC1}$ and $\Delta t_{TDp}$ exhibit frequency dependence, but are in agreement at $f_T/f_R = 1$, where the predominant frequencies of the transmitted and received signals coincide. $\Delta t_{CC1}$ is greater than $\Delta t_{TDp}$ in the range of $f_T/f_R > 1$. The simulation results are roughly reproduced by the observed data. Fig. 13 shows the difference between $\Delta t_{CC1}$ and $\Delta t_{TD}$ normalized with respect to the predominant period of the received signal $1/f_R$. The model shows that the difference $\Delta t_{CC1} - \Delta t_{TD}$ is generally greater than zero and reaches a maximum of approximately 0.15, regardless of the value of $f_n$, and the observed data support this. Therefore, the CC technique usage involves overestimation of the travel time compared to the TD technique. As the resulting error depends on the overall travel time, it is more obvious when the predominant period is large or when the overall travel time is small. Therefore, the error may be neglected when tests are performed with a short travel distance, such as in oedometer and direct shear tests, or involve the use of extremely soft soil, such as highly organic soil.

### 4.3 Estimation of travel time and shear wave velocity using frequency domain technique

Typical waveforms of sweep inputs and the resulting received signals are shown in Fig. 14. As in Fig. 1, each type of sweep has different frequency characteristics and waveforms. Figs. 15–17 show a variety of unwrapped phase angles of cross-spectra between the two signals and their coherence functions within 20 or 50 kHz for various samples. Note that each position of the
phase spectrum is corrected by a combined TD and FD method based on previous research (Styler and Howie, 2013). The phase spectra yield travel times \( \Delta t_{ph} \) and \( \Delta t_{gr} \) as Eqs. (6) and (7). The use of the coherence function simplifies travel time estimation in the FD analysis. Styler and Howie (2013) and Fonseca et al. (2009) demonstrate that a reasonable frequency range for \( \Delta t_{ph} \) and \( \Delta t_{gr} \) estimation is associated with the range with a high coherence value. The upper frequency limit of the reasonable range is determined by a marked drop in coherence (Figs. 15–17), even though the following difficulties are found: in artificially treated soil, high coherence can be found between 25 and 40 kHz, and the phase angle shows unreasonable behavior despite the

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**Fig. 13.** Difference in travel times of time domain and cross-correlation techniques.

**Fig. 14.** Typical waveforms of frequency swept signals: (a) linear sweep, (b) log sweep, and (c) TSP.

**Fig. 15.** Coherence \( \gamma \) and phase angle \( \phi \) of cross-spectra of sweep input and received signals for soft soils.
high coherence range in soft soils. The phase angles exhibit a higher linearity (or less dispersion) below the frequency limit than in the range above it and are hardly affected by the input sweep signal type, despite the difference in frequency characteristics. Moreover, the location of the upper frequency limit varies dramatically with the soil sample. The values are relatively low in soft soils and over 20 kHz for sandy soil or artificially treated samples. This likely reflects the frequency characteristics of impedance between input signals and transmitter elements, or between transmitter elements and media. Given that the FD technique using a frequency sweep is essentially equivalent to identifying the transfer function, it is quite reasonable that the cross-spectrum and its coherence strongly reflect characteristics of the transfer function.

Figs. 16–20 illustrate variation in the phase velocity $V_{ph}$ in different soil samples. The phase velocity exhibits significant frequency dependence (or strong dispersion) in all samples. The effects of the dispersion, which is attributed to soil medium-bender element coupling and the apparatus geometry, are not negligible. In particular, $V_{ph}$ is sensitive to variations in the phase angle in the low frequency range. The significant scattering then reduces and stabilizes toward the upper frequency limit, except with artificially treated soil. Consequently, the steady values of $V_{ph}$ at the upper frequency limit are nearly equal to values of $V_{TD}$ for all soil samples, except artificially treated soil. The effect of input sweep type is negligible. Since the discrepancy is small, it could be acceptable for Pisa clay or Inagi sand; however, it is unacceptable for Toyoura sand. Moreover, the estimate of $V_{ph}$ is not realistic for artificially treated soil. Taking into account the correspondence between results of the TD technique and other tests (Table 3), $V_{ph}$ is likely to underestimate the shear wave velocity, and hence the stiffness of soils.

Similar results are yielded for group velocity $V_{gr}$. Figs. 21–23 show variation in $V_{gr}$ and are comparable to Figs. 18–20. As with the phase velocity, the group velocity is widely scattered from $V_{TD}$, whereas the type of sweep signal negligibly affects the frequency–velocity relationship. The values of $V_{gr}$ within the upper frequency limit are generally below the value of $V_{TD}$, indicating that $V_{gr}$ also tends to be underestimated compared to that of $V_{TD}$, even though the scattering due to dispersion is considerable. Scattering of $V_{gr}$ from $V_{TD}$ is less pronounced in soft soils, indicating that the dispersion effect is likely a serious problem in relatively stiff samples. The scattering can be improved by choosing a wider fitting range to estimate the tangent of the phase spectrum (Fonseca et al., 2009). The slope of the fitted line approaches the travel time that derives the mean value of the group velocity for the overall frequency range, as the fitting range is expanded. If the system is ideal and non-dispersive, the group velocity, which is obtained from the phase spectrum making a straight line passing through the origin, is constant.

Fig. 16. Coherence $\gamma$ and phase angle $\phi$ of cross-spectra of sweep input and received signals for sandy soils.

Fig. 17. Coherence $\gamma$ and phase angle $\phi$ of cross-spectra of sweep input and received signals for artificially treated soil or artificial granular material.
Fig. 18. Frequency dependence of phase velocity $V_{ph}$ for soft soils.

Fig. 19. Frequency dependence of phase velocity $V_{ph}$ for sandy soils.

Fig. 20. Frequency dependence of phase velocity $V_{ph}$ for artificially treated soil or artificial granular material.

Fig. 21. Variation of group velocity $V_{gr}$ for soft soils.
and equal to the phase velocity. Mean values of $V_{gr}$ obtained from the best fit of the overall frequency range below the upper frequency limit are represented in Figs. 21–23 and summarized in Table 4. The values are stably smaller than those from the TD technique and close to those of $V_{ph}$. The discrepancy between $V_{ph}$ and $V_{gr}$ is up to 11%. This implies that the FD technique is still helpful in travel time estimation for most soil samples if the upper frequency limit is appropriately determined, even if it alone cannot provide a reasonable value.

4.4. Comparison of shear wave velocities determined by various techniques

The results of $V_s$ measurements obtained from the CC technique are compared with those from the TD technique (Fig. 24). Travel time estimation for $V_{sCC}$ is based on the maximum peak point in CC. Overall, data are plotted below the solid line of $V_{sCC} = V_{sTD}$, indicating that the CC technique estimates the shear wave velocity as slightly lower than the TD technique. While the discrepancy is less than 10% for soft soils, it exceeds 10%, and in some cases 20% for sandy and artificially treated soils. This is due to inconsistencies between the first and maximum peaks, and implies that if the maximum peak appears after the first peak point, this leads to significant underestimation of the shear wave velocity in these soils. As a result of the linear regression analysis applied to the data, the following relationship between $V_{sTD}$ and $V_{sCC}$ is derived:

$$V_{sCC} = 0.836V_{sTD}^{1.03}$$

(13)

The data of $V_{sCC}$ corrected by applying the first peak of CC is also shown in the figure, suggesting the validity of the CC technique for these samples, even though it involves the same difficulty in detecting the arrival point as the TD analysis.

Figs. 25 and 26 compare shear wave velocities determined by the TD and FD techniques, showing the phase and group velocities, respectively. The $V_{ph}$ and $V_{gr}$ data show a trend: the FD technique estimates the shear wave velocity to be lower than the TD technique. The discrepancy is up to 20%, with some outlying samples exhibiting considerable dispersion. If these data are excluded, the best fits of $V_{ph}$ and $V_{gr}$ are given as follows:

$$V_{ph} = 0.885V_{sTD}^{1.01}$$

(14)

$$V_{gr} = 0.733V_{sTD}^{1.04}$$

(15)

5. Conclusions

In order to examine the discrepancies in the shear wave velocity measurements caused by different methods to estimate the travel time, a series of bender element tests using the TD, CC, and FD techniques were performed on various soil samples. From the results for 10 types of samples having wide spectrum of stiffness values, the following comprehensive conclusions were obtained.

The CCs between single sine inputs and the resulting received signals were divided into two types in accordance with the location of the maximum peak point, i.e., whether it was consistent with the first peak or not. In cases where these peaks were consistent, mainly with relatively soft soils, the CC technique provided reasonable travel times. The maximum peak of CC tended to underestimate the shear wave velocity otherwise. The first peak point, the detection of which requires manual judgment, delivered reasonable results in these cases. Thus, the CC technique can work effectively only when the maximum and first peaks coincide.
As a result of detailed examination using a numerical oscillation model, the frequency dependence of the travel time was demonstrated for the CC and TD techniques. Although the travel time determined by the maximum peak of CC was equal to that obtained from the peak-to-peak distance in the TD, only when the frequency of the sine input signal coincided with the predominant frequency of the received signal, they were fundamentally inconsistent. The value of the travel time obtained from the CC technique was larger than that from the TD technique based on the peak-to-peak distance, when the input frequency was greater than the predominant frequency of the received signal. The discrepancy between TD and CC analysis came close to approximately 0.15 times the predominant frequency of the received signal. The applicability of the FD technique was also examined. In the FD, the unwrapped phase of the cross-spectrum between the input sweep and the resulting received signals exhibited a broadly linear relationship in a plot of phase versus frequency within the high coherence frequency range. The upper limit of the frequency range varied, mainly depending on the soil stiffness. The shape of the cross-spectrum was minimally affected by the type of input sweep signal.

Table 4
Summary of mean values of shear wave velocities (m/s).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Ebetsu peat</th>
<th>Akita peat</th>
<th>Ebetsu organic clay</th>
<th>Pisa clay</th>
<th>NSF clay</th>
<th>Inagi sand</th>
<th>Toyoura sand</th>
<th>Glass bead 0.4 mm</th>
<th>Glass bead 2 mm</th>
<th>Artificially treated soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consolidation stress (kPa)</td>
<td>40</td>
<td>60</td>
<td>20</td>
<td>150</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>Unconfined</td>
</tr>
<tr>
<td>$V_{STD}$</td>
<td>36.6</td>
<td>54.0</td>
<td>30.4</td>
<td>119</td>
<td>80.3</td>
<td>203</td>
<td>302</td>
<td>321</td>
<td>301</td>
<td>324</td>
</tr>
<tr>
<td>$V_{CC}$</td>
<td>33.9</td>
<td>50.8</td>
<td>29.5</td>
<td>119</td>
<td>77.2</td>
<td>175</td>
<td>235</td>
<td>314</td>
<td>296</td>
<td>242</td>
</tr>
<tr>
<td>$V_{ph}$</td>
<td>30.0</td>
<td>48.9</td>
<td>25.3</td>
<td>119</td>
<td>57.1</td>
<td>202</td>
<td>218</td>
<td>296</td>
<td>285</td>
<td>---</td>
</tr>
<tr>
<td>$V_{gr}$(^a)</td>
<td>31.5</td>
<td>48.0</td>
<td>24.7</td>
<td>113</td>
<td>63.7</td>
<td>189</td>
<td>197</td>
<td>288</td>
<td>291</td>
<td>264</td>
</tr>
<tr>
<td>Discrepancy between $V_{ph}$ and $V_{gr}$</td>
<td>4.9%</td>
<td>1.9%</td>
<td>2.5%</td>
<td>5.2%</td>
<td>10.9%</td>
<td>6.6%</td>
<td>10.1%</td>
<td>2.7%</td>
<td>2.9%</td>
<td>---</td>
</tr>
</tbody>
</table>

\(^a\) $V_{ph}$ is estimated at the upper frequency limit.
\(^b\) $V_{gr}$ is estimated by the best fit of the whole range below the upper frequency limit.
Even though the phase velocities evaluated by FD analysis were dispersed because of the non-linearity of the test system, the values became steady toward the upper frequency limit for most types of soil samples. Overall, the steady value of the phase velocity agreed with the shear wave velocity given by the TD technique; however, significant discrepancies were observed for a few relatively stiff samples. The group velocity exhibited a trend of underestimation of the shear wave velocity for every type of sample. The overall trend of underestimation, especially for stiff samples remained, if the fitting range to estimate the travel time was expanded.

On the basis of statistical analysis for various types of samples, relationships among shear wave velocities obtained from the TD, CC, and FD techniques were quantitatively evaluated, the results of which are given by Eqs. (13) to (15). The CC technique underestimated the shear wave velocity by 10% or less when the first peak was adopted, and the FD technique underestimated the shear wave velocity by 20% or less in the case of both the phase and group velocities.

Acknowledgments

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References


