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# Synthetic error modeling for NC machine tools based on intelligent technology

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**Abstract**

The precision of machine tools is greatly constrained by errors either built into the machine tools or occurring on a periodic basis on account of temperature changes or variation in cutting forces, so it is essential to obtain these errors, and then eliminate or compensate for them. However, the interaction between many factors inducing errors, such as the heat source, thermal expansion coefficient, the machine system configuration and the running environment, creates complex behavior of a machine tool, and also makes synthetic error prediction difficult with traditional mathematics. Therefore, several modeling methods based on non-classical mathematics have been presented in recent years. The intelligent technology methods of neural network, support vector machines, Bayesian networks are the effective modeling and forecasting methods for machine errors. All these three methods were introduced in briefly in the paper, and the characteristics of them were discussed. A series of experiments were carried out to evaluate their merits and defects. Finally, some important conclusions about how to use these methods in different situations were provided. The works in this paper make a special summary of the error modeling with intelligent technology, and provide a useful guidance to further research on error compensation of NC machine tools.


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*Keywords:* synthetic error modeling; neural network; support vector machines; Bayesian networks; intelligent technology

## 1. Introduction

Machine errors including cutting-force induced, fixture dependent and thermal errors, which we name synthetic error here to distinguish it from each kind of individual error, significantly affect the precision of machine tools, so it is essential to obtain these errors, and then eliminate or compensate for them[1]. In the prediction of machine errors, most current research has focused on arriving at a direct correlation between the monitoring parameters and error data obtained through experiments. Once a correlation is established, the model is then used to forecast the error under any other condition based on the monitoring data. [2]

However, some difficulties have all along remained in modeling for the influence of properties of synthetic error. Since the accuracy of a machine tool is affected by the overall effect of the various error sources mentioned earlier, the error compensation system should take into consideration the interaction between these sources rather than consider each error in isolation[1]. This situation leads to two problems: one is that the synthetic error becomes a time-varying nonlinear and non-stationary process, and the other is that the various interactions make it hard to understand the relationships among various factors. These problems affect the model accuracy directly, and limit the use of traditional fitting methods in modeling errors. Therefore, some modeling methods based on non-classical mathematics have been presented in recent years. The intelligent technology methods of neural network, support vector machines, Bayesian networks are the effective ones of them for synthetic errors.

This paper seeks to present these intelligent technology methods in synthetic error modeling and is organized as follows: in the next section, some important work has been done in this field is introduced; In Section 3, the three modeling method based on neural network,

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support vector machines and Bayesian networks are introduced respectively; The comparison experiment is shown in Section 4 and the result is analyzed; Finally, some conclusions are presented in Section 5.

## 2. Previous Work

Various intelligence techniques have been employed in the modeling of machine errors in recent 15 years. Artificial neural network (ANN) is the most important one of them. Yang et al.[3] proposed a cerebellar model articulation controller (CMAC) neural network to correlate machine thermal errors to temperature measurements through large amount of experiments and data analysis techniques. Chen[4] used a three-layer ANN with a supervised back propagation training algorithm and the 'sigmoid' activation function to map the calibrated thermal errors to the temperature measurements. However, the results were found to be unacceptable where input conditions were significantly different from the data set used to train the network. A neural network based on artificial resonance theory (ART-map) was used to predict and compensate the tool point errors of a 3-axis machining center by Mize and Ziegert[5] using discrete temperature readings from the machine's structure as input. Fu and Chen[6] presented an artificial neural network method which was combined with fuzzy logic to predict thermal error. A new method of principal component analysis was presented to identify the structure and parameters of the fuzzy model. Then, the artificial neural network method was used to overcome the problem which the fuzzy model could not resolve, because the effective data number was smaller than the consequence parameter number. Yang and Ni[7] used the dynamic neural network model to track nonlinear time-varying machine tool errors under various thermal conditions. Study has shown that the dynamic models have the advantage over static models in terms of model robustness to different working conditions, since the dynamics and the nonlinearity of the thermal-elastic process are captured using the temporal association networks.

Support vector machine (SVM) is another useful method to model errors in machine tools for its good performance in classification and regression. Ramesh et al.[8] developed a model using support vector machines to classify the error based on operating conditions. Once classified, the error was then predicted based on the temperature states. Lin et al.[9] presented a modeling methodology using the least squares support vector machine (LS-SVM) model to track nonlinear time-varying spindle thermal error under certain conditions. Experiments on spindle thermal deformation were conducted to evaluate the model performance in terms of model estimation accuracy and robustness. The comparison has indicated that the LS-SVM performs better than other modeling methods, such as multi-variable least squares regression analysis, in terms of model accuracy and robustness. Recently, He et al.[10] used SVM to predict the volumetric errors and also acquired high modeling accuracy.

Bayesian networks (BN) is a probabilistic representation for uncertain relationships and is useful for modeling real-world problems such as diagnosis, forecasting, manufacturing control, etc., wherein there exist multiple cause and effect dependency relations[11]. It is a relatively new technique for thermal errors modeling. Ramesh et al.[2] presented a hybrid SVM-BN model to account for the specific conditions inducing thermal errors. The experimental data was first classified using a BN model with a rule-based system. Once the classification had been effected, the error was predicted using a SVM model. Yao et al.[12] described causal relationships of factors inducing thermal deformation by graph theory and estimated the thermal error by Bayesian statistical techniques. Due to the effective combination of domain knowledge and sampled data, the BN method could adapt to the change of running state of machine, and obtain satisfactory prediction accuracy.

## 3. Intelligent Techniques for Synthetic Error Modeling

### *3.1. Artificial neural network*

The artificial neural network is a kind of multiple nonlinear model in which the coefficients are called weights which are evaluated by training with an iterative technique called back-propagation. Thus, an ANN is appropriate for a system having a nonlinear relationship between multiple inputs and multiple outputs. There are three types of layer for the neural network, i.e. input layer, hidden layer, and output layer. For the synthetic error application, the input layer is designed to consist of sensor monitoring data such as temperature and force, and the output layer of the spindle errors.

The hidden layers represent intermediate nodes, which are determined so that the neural model has the least errors. After the hidden layers are determined by testing the output layers, the output, $y^*$, can be calculated as Eq.(1)[6].

$$y^* = (\sum_{i=1}^{q} w^i \bullet \theta_i^T \bullet \varphi) / \sum_{i=1}^{q} w^i . \tag{1}$$

Where,

$w^i = A_1^i(x_i) \wedge \cdots \wedge A_k^i(x_k) = \prod_{j=1}^{k} A_j^i(x_j)$, $x_i$ ($i$=1, 2, …,

$q$) are the antecedent input variables of fuzzy set, $A_j^i$ is the value of $x_j$,

$\theta_i = [P_0^i, P_1^i, \cdots P_k^i]^T$ , $P_k^i$ are the parameters of output variables,

$\varphi = [1, x_1, \cdots, x_k]^T$ .

Then, the fuzzy-neural network for spindle error $y^*$ can be constructed as Fig. 1.

When the structure of the neural network is determined, some special data groups should be chosen to train this model. When the model precision is not enough, the fuzzy neural network should be trained a lot of times until the precision is satisfied.

### 3.2. Least squares support vector machine

SVM can be applied to regression by the introduction of an alternative loss function and the results appear to be very encouraging. In SVM, the basic idea is to map the data into a higher-dimensional feature space via a nonlinear mapping and then to do regression in this space. The optimization problem of the least squares support vector machine (LS-SVM) which is a modification of Vapnik SVM is formulated as Eq.(2)[13]:

$$\left.\begin{aligned} &\min J(\omega,\xi) = \frac{1}{2}\omega^{\mathrm{T}}\omega + \gamma\frac{1}{2}\sum_{i=1}^{l}\xi_i^2, \\ &s.t. \quad y_i = \omega^T\phi(\mathbf{x}_i) + b + \xi_i, \quad i = 1,\dots,l. \end{aligned}\right\} \tag{2}$$

Where $x_i$ is the input vector, $y_i$ is the desired value, $\varphi$ is the set of mappings of input features, and $\omega$, $\gamma$ and $b$ are coefficients.

We define the Lagrangian as:

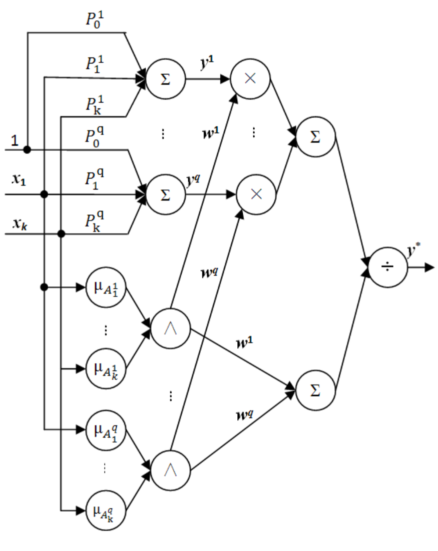


Fig. 1. Structure of the fuzzy neural network

$$L = \frac{1}{2}\omega^T\omega + \gamma\frac{1}{2}\sum_{i=1}^{l}\xi_i^2 - \sum_{i=1}^{l}\alpha_i(\omega^T\phi(\mathbf{x}_i) + b + \xi_i - y_i). \tag{3}$$

Where $\alpha_i$ are Lagrange multipliers. By the optimality conditions

$$\begin{cases} \dfrac{\partial L}{\partial \omega} = 0 \rightarrow \omega = \sum_{i=1}^{l}\alpha_i\phi(\mathbf{x}_i), \\ \dfrac{\partial L}{\partial b} = 0 \rightarrow \sum_{i=1}^{l}\alpha_i = 0, \\ \dfrac{\partial L}{\partial \xi_i} = 0 \rightarrow \alpha_i = \gamma\xi_i; i = 1,\dots l, \\ \dfrac{\partial L}{\partial \alpha} = 0 \rightarrow y_i = \omega^T\phi(\mathbf{x}_i) + b + \xi_i, i = 1,\dots,l. \end{cases} \tag{4}$$

Then, optimization problem can be rewritten as

$$\begin{bmatrix} 0 & 1 & \dots & 1 \\ 1 & K(\mathbf{x}_1,\mathbf{x}_1)+1/\gamma & \dots & K(\mathbf{x}_1,\mathbf{x}_l) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & K(\mathbf{x}_l,\mathbf{x}_1) & \dots & K(\mathbf{x}_l,\mathbf{x}_l)+1/\gamma \end{bmatrix}\begin{bmatrix} b \\ \alpha_1 \\ \vdots \\ \alpha_l \end{bmatrix} = \begin{bmatrix} 0 \\ y_1 \\ \vdots \\ y_l \end{bmatrix} \tag{5}$$

Finally, nonlinear function takes the form:

$$f(x) = \sum_{i=1}^{l}\alpha_i K(\mathbf{x},\mathbf{x}_i) + b. \tag{6}$$

This LS-SVM regression leads to solving a set of linear equations, which is for many practitioners in different areas. Especially, the solution by solving a linear system is instead of quadratic programming. It can decrease the model algorithm complexity and shorten the computing time greatly.

### 3.3. Bayesian networks

A BN is a graphical model consisting of nodes which represent causes and effects in real-world situations and a set of edges each of which connects two nodes. If there exists a causal relationship between any two of the nodes, the edge would be directional leading from the cause variable to the effect variable. For such directed edges, the edge is said to be from parent to child. A BN is also called a 'directed acyclic graph' for this reason as all the edges of the graph point in a particular direction and there is no way to start from one node, travel along a set of directed edges in the proper direction and arrive at the same node.

A BN for a set of variables $\boldsymbol{X}=\{X_1, \dots, X_n\}$ consists of a network $S$ that encodes a set of conditional independence assertions about variables in $\boldsymbol{X}$ and a set $\boldsymbol{P}$

of local probability distributions associated with each variable[11]. Together, these components define the joint probability distribution for $\boldsymbol{X}$.

For a joint probability distribution, from the chain rule of probability, we have Eq.(7)

$$P(X_1,\cdots,X_n)=\prod_{i=1}^{n}P(X_i\mid X_1,X_2,\cdots,X_{i-1}). \tag{7}$$

For every $X_i$ there will be some subset $\pi(X_i)\subseteq\{X_1, X_2, ..., X_{i-1}\}$ such that $X_i$ and $\{X_1, ..., X_{i-1}\}\backslash\pi(X_i)$ are conditionally independent. That is, for any $X_i$,

$$P(X_i\mid X_1,\ ...,\ X_{i-1})=P(X_i\mid\pi(X_i)), \tag{8}$$

Where $X_i$ denotes the variable and $\pi(X_i)$ represents the parents of node $X_i$. Combining Eqs.(7) and (8),

$$P(X_1,\cdots,X_n)=\prod_{i=1}^{n}P(X_i\mid\pi(X_i)). \tag{9}$$

Now, we assume that the number of nodes in a BN is $n$, the average number of parent nodes is $m$, and each variable owns $k$ values on average. Then, the computation complexity of Eq.(9) is $nk^m$, while it is $k^n-1$ to Eq.(7). Obviously, the time cost will decline by using Eq.(9) when $m$ is far smaller than $n$.

In general, the computation of a probability of a constructed model is known as probabilistic inference. In this section, probabilistic inference in BNs is described briefly.

To a BN whose structure has been fixed, if there are $N$ cases in sampled data, the inference can be calculated as follows[14]:

$$\begin{aligned}&P(x_{N+1}\mid D,S^h)\\&=\int P(x_{N+1}\mid\theta_s,D,S^h)P(\theta_s\mid D,S^h)d\theta_s\\&=\prod_{i=1}^{n}\prod_{j=1}^{q_i}\frac{N'_{ijk}+N_{ijk}}{N'_{ijk}+N_{ijk}}.\end{aligned} \tag{10}$$

Especially, if there is only one variable $X_i$ to be calculated, Eq.(10) can be simplified as

$$\hat{X}_i=x_i^{k_0}, \tag{11}$$

Where $\hat{X}_i$ is the prediction value of $X_i$, and $k_0$ should satisfy

$$\theta_{ij_0k_0}=\max_k\{\theta_{ijk}\}=\max_k\{P(x_i^k\mid\pi(X_i)^j,S^h)\}. \tag{12}$$

Fig.2. Synthetic error measurement

With the help of Bayesian statistics, prior knowledge of a domain can be combined well with sampled data. Accordingly, modeling of synthetic errors in machine tools will be implemented by two steps: (1) Construct network from prior knowledge. Here we should identify the factors that are of critical importance in inducing machine errors, analyze the interrelation among them, then describe all this information by building a directed acyclic graph, and finally assess prior probability of each variable in graph. (2) Conduct probabilities learning and inference in the network based on sampled data. The parameters of network will be refined, and the result of modeling can be calculated by probabilistic inference.

## 4. Experiment and Discussion

### *4.1. Experiment setup*

Without loss of generality, this paper presents models for a high speed spindle of 10 KW. The models developed for this type of spindle are transferable to other spindles. The experimental setup that is used to measure synthetic error is shown in Fig. 2.

Spindle error in radial and axial direction is respectively measured using two CCD laser displacement sensors (LK150H, KEYENCE), as shown in Fig. 2. To model and forecast synthetic error with three intelligent methods discussed above, there are three major factors inducing errors are taken into account:

1. he temperatures of several key parts of the machine tool and the entironment, which monitored by thermal sensors (OS18B20, DALLAS);
2. Volumetric error which measured by laser interferometer with vector diagonal step method;
3. Machining parameters including rotation speed of spindle and the feed rate.

### *4.2. Results and discussion*

In experiments, temperatures are measured every 1 second during motion, using numerical thermal sensors that are attached to the spindle and other positions. The machine has been running for 2.5 hours and then stops to cool down for 2 hours. A set of data from displacement sensors is recorded for every 1 minute.

For models comparison, three models are built using the same model training data. The modeling results for axial deformation from three methods and the actual value of machine are shown in Fig.3, and the results for radial deformation are shown in Fig.4.

Furthermore, the modeling results are compared based on the mean absolute percentage error (MAPE) as defined below:

$$MAPE = 100\frac{\sum_{i=1}^{n}\left|(d_i - \hat{d}_i)/\hat{d}_i\right|}{n}, \quad n = 150, \qquad (13)$$

Where $d_i$ is the modeling value, while is the actual value.

The comparison in MAPE and calculating time among the three methods are shown in Table 1. Results disclose that:

1. All these three intelligent technology methods give quite a good modeling accuracy in error prediction. The MAPE of LS-SVM, for comparison, is a little better than ANN and BN methods.

2. The LS-SVM model has distinct advantages over ANN and BN models in calculating time. The ANN model spends about 22 times as LS-SVM for it needs a relatively long time to train the network to achieve good prediction accuracy. The BN model calculates the probability of every error state by combining the experience and the sample data without network training, so it is faster than ANN and slower than LS-SVM method.

3. In terms of model's understandability, the BN method describes causal relationships of factors inducing synthetic error by graph theory, so it helps users to gain understanding about a problem domain readily. Modeling with ANN or LS-SVM may need more professional knowledge.

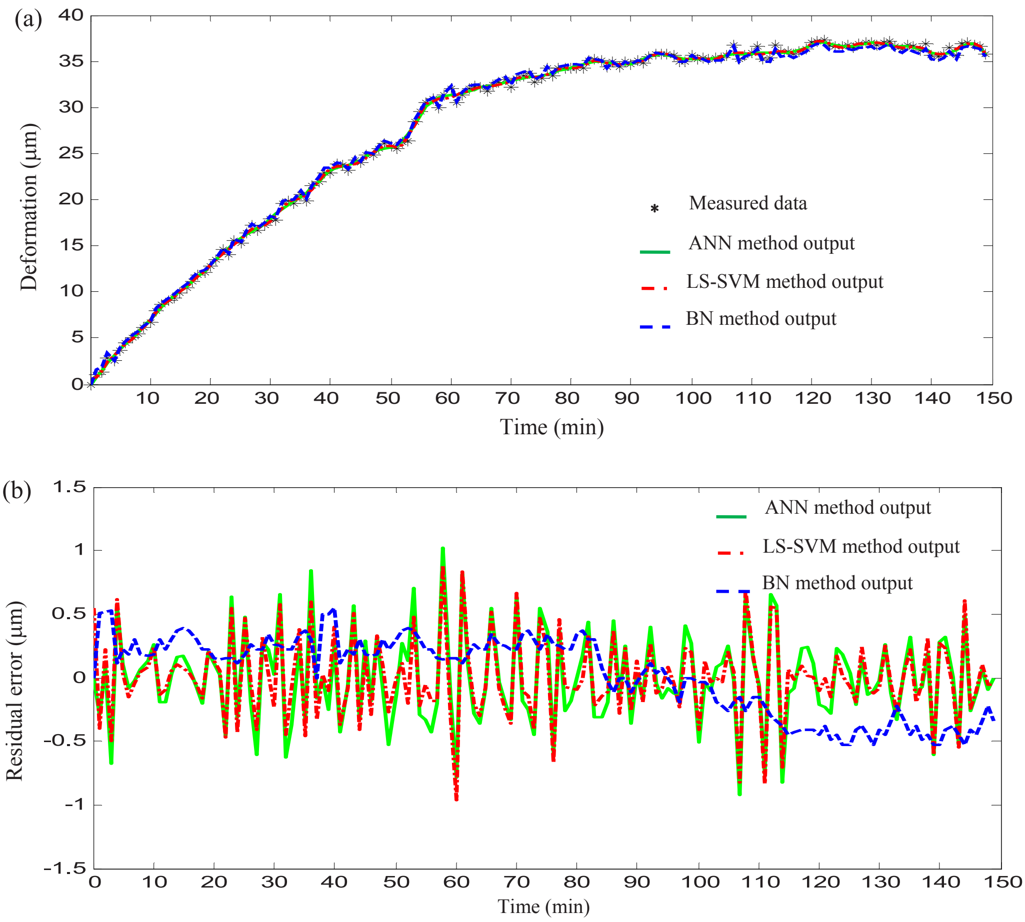


Fig.3. Modeling comparison results in axial direction. (a) Deformation; (b) Residual error

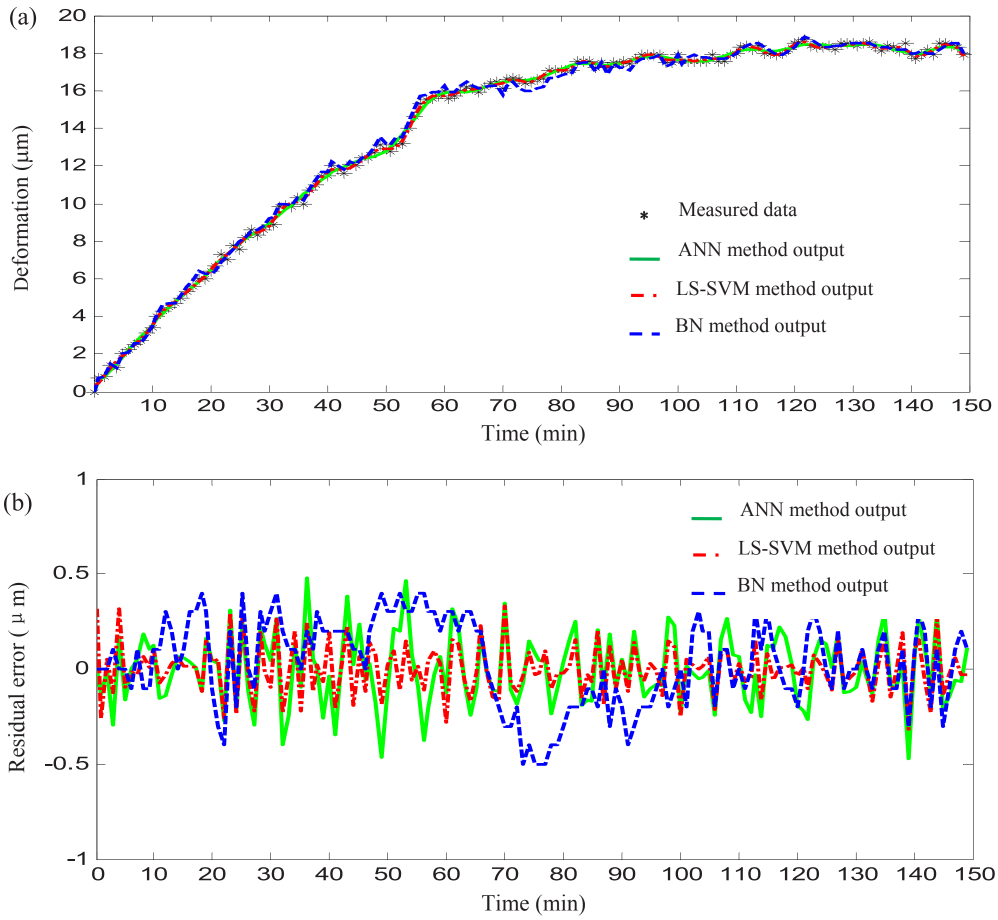


Fig.4. Modeling comparison results in radial direction. (a) Deformation; (b) Residual error

To sum up the above arguments，the LS-SVM model has comprehensive performance in forecast accuracy and calculating time, so it is extremely suitable for modeling synthetic error of NC machine tools on-line. Furthermore, taking the understandability of model into consideration, using the BN and LS-SVM methods together will be a perfect solution to modeling complex synthetic error behavior of a machine tool.

Table 1. Comparison of MAPE and calculating time among the three methods

| Method | *MAPE* (%) | | *Calculating Time* (*s*) | |
|---|---|---|---|---|
| | Axial direction | Radial direction | Axial direction | Radial direction |
| ANN | 1.60 | 1.58 | 67.119 | 58.636 |
| SVM | 1.56 | 1.34 | 2.829 | 2.806 |
| BN | 1.86 | 1.64 | 19.925 | 17.261 |

## 5. Conclusion

This paper demonstrates three intelligent technology methods of artificial neural network, support vector machines and Bayesian networks to model and forecast synthetic errors of NC machine tools. The experiments were carried out to evaluate their merits and defects. The comparison result shows that all these three intellgent methods can work well in machine error modeling, how to use them depends on the requirements of the application. Especially, the LS-SVM model has comprehensive performance in high modeling accuracy and short calculating time, while BN model has better understandability than others. Therefore, using the BN and LS-SVM methods together may be a perfect solution to synthetic error modeling.

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