Dynamic failure of ductile materials

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Abstract

The failure of ductile materials subject to high loading rates is notably affected by material inertia. We analyze how strain localization and fracture are influenced by inertia through selected topics comprising dynamic necking, fragmentation, adiabatic shear banding and dynamic damage by micro-voiding. A multiscale modeling of the behavior of voided visco-plastic materials is proposed that extends classical models by including microscale inertia. Applications to spalling and dynamic fracture reveal that microscale inertia has first order effects on results.

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1. Introduction

Dynamic failure of ductile materials is involved in a wide range of applications including the optimization of fast manufacturing processes and the security of structures exposed to impact, explosive loading or crash events. At high loading rates, tiny fluctuations in the plastic flow field induce important acceleration of material particles. Thus, significant inertia effects are taking place at the macroscopic level and sometimes also at the level of microscopic deformation mechanisms. Naturally, when subjected to dynamic loading the behavior of metallic materials is quite distinct from that observed under quasi-static conditions. The flow stress has higher rate dependence. The load bearing capacity is altered by thermal softening due to adiabatic heating resulting from plastic work. At last, inertia effects and material

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properties interact in a complex way which confers a dynamic signature to the patterning of plastic flow localization and to the process of internal damage.

Strain localization, which is often the precursor of failure of ductile materials, is the result of plastic flow instability, an outcome of geometrical or material softening. The patterning of strain localization is observed to be quite sensitive to the loading rate. A single neck is usually seen in a bar under quasi-static tensile loading while multiple necking is triggered at high stretching rates. Multiple necking is a true manifestation of inertia effects.

A comprehensive overview of the whole field of dynamic ductile failure is not attempted here. Selected topics comprise the analysis of: (1) dynamic necking and fragmentation, (2) adiabatic shear banding, (3) dynamic damage by micro-voiding with applications to spalling and dynamic crack growth.

We do not tackle here the difficulties attached to the characterization of the constitutive law at very high loading rates and large deformations. Rather, we focus on the role of inertia in the process of dynamic failure. The following questions are addressed:

(1) Which role is played by inertia in the development of flow localization?
(2) How does inertia influence the overall material ductility?
(3) Are inertia effects significant at the level of microscale damage mechanisms?
(4) How can microscale inertia be integrated in a multiscale modeling of dynamic fracture?

2. Dynamic necking

Experiments on metal rings and shells subject to intense stretching rates of the order of $10^3 \text{s}^{-1}$ reveal that the fragmentation process of ductile materials is frequently initiated by multiple necking as illustrated in Fig.1, [1–5]. The following observations can be made:

(1) The number of necks and of fragments increases with the loading rate (Fig. 1a).
(2) Some necks are arrested before fracture (Fig. 4b).
(3) The overall fracture strain is increasing with the loading rate (increased global ductility).

These features are intimately related to inertia effects.

2.1. Perturbation analysis

The early stages of multiple necking can be well captured by a linear stability approach. Perturbation methods were developed by Fressengeas and Molinari [6] and Mercier and Molinari [7, 8] for the analysis of dynamic necking of viscoplastic materials. The quasistatic problem was examined by Hutchinson and Neale [9] and Hutchinson et al. [10]. Dynamic necking of rate independent plastic materials was studied by Shenoy and Freund [11] and Guduru and Freund [12] by extending the quasistatic bifurcation analysis of Hill and Hutchinson [13]. Simplified one dimensional perturbation analyses with the Bridgman correction to account for multiaxial effects in necked regions have been also used [14–17].

The effects of inertia are investigated here by following the development proposed by Mercier et al. [2]. Results were obtained for a uniform plate subject to dynamic plane strain stretching and for incompressible visco-plastic materials obeying the $J_2$-flow theory and with flow stress of the form

$$\sigma_y = g(\varepsilon_p, \dot{\varepsilon}_p, T) \quad \text{with} \quad \dot{\varepsilon}_p = \sqrt{(2/3)d_y d_y} \quad \text{and} \quad \varepsilon_p(t) = \int_0^t \dot{\varepsilon}_p(\tau)d\tau,$$

(1)

where $\dot{\varepsilon}_p$ is the effective plastic strain rate, $d_y$ is the plastic strain rate tensor, $\varepsilon_p(t)$ is the cumulated plastic strain at time $t$ and $T$ is the temperature. Strain hardening, strain rate hardening and thermal sensitivity are accounted for in the constitutive relation (1). Elastic deformations are neglected.

The plate occupies the domain defined by $-L_1 \leq X_1 \leq L_1$ and $-L_2 \leq X_2 \leq L_2$ in the initial reference
configuration and is stretched in the direction $X_1$ with the constant velocity $± V$ applied at the extremities $X_1 = ± L_1$, see Fig. 2. We denote by $l_1$ the half current length, $λ = l_1 / L_1$ the longitudinal stretch.

![Image](image-url)

Fig. 1. (a) Necking and fragmentation of thin rings of solutionized 6061 Aluminum subject to rapid radial expansion by the effect of an intense magnetic field, Altynova et al. [1]. The number of necks and the number of fragments increase with the energy input (i.e. with the stretching rate): (1) original, (2) 0.94 kJ, (3) 1.38 kJ, (4) 2.06 kJ, (5) 2.38 kJ, (6) quasistatic tensile test (single neck); (b) multiple necking of a tantalum hemisphere under rapid expansion (strain rates of the order of 10 000 s⁻¹) by the effect of a shock loading generated by an explosive charge, Mercier et al. [2].

The theoretical homogeneous background solution which exists in absence of any flow instability can be analytically determined [6] and involves lateral inertial effects. We denote by $σ^0$, $ε^0$, $ε^0_H$ and $T^0$ respectively the flow stress ($σ = (3/2)s_y s_y$, is the effective stress and $s_y$ the deviatoric stress), the cumulated plastic strain, the equivalent strain rate and the temperature of the background solution. These quantities are uniform. The evolution of $T^0$ results from adiabatic heating associated to plastic work.

![Diagram](diagram-url)

Fig. 2. Plate under plane strain deformation subject to the constant stretch-rate $λ = V / L_1$

Plastic flow stability is analysed at any time $t_*$ by perturbing the background solution with a small displacement field $\tilde{X}_1(X_1, X_2, t)$, $\tilde{X}_2(X_1, X_2, t)$ superimposed to the current position of material particles. $(X_1, X_2)$ and $(x_1, x_2)$ are respectively the Lagrangian and the Eulerian coordinates. The problem equations are linearized with respect to the corresponding disturbances of velocity, acceleration, strain-rates, cumulated plastic strain and stresses. To satisfy incompressibility of plastic flow a stream function, $I$, is introduced such that $\tilde{X}_1 = \lambda^2 \phi_2$, $\tilde{X}_2 = \lambda^2 \phi_1$, where $(.)_j$ is the partial derivative with respect to $X_j$.

The analysis investigates the exponential growth rate of modes of the form

$$\phi(X_1, X_2, t) = A \exp\left[\eta(t - t_0)\right] \sin(kX_1) \exp(i\xi X_2),$$

where $\eta$ characterizes the time evolution, $k$ and $\xi$ are Lagrangian longitudinal and transversal wavenumbers. The boundary conditions at extremities are satisfied if $\sin(kX_1) = 0$ at $X_1 = ± L_1$, i.e.

$$kL_1 = j\pi, \quad j \text{ positive integer}.$$
The relationship between the growth rate $\eta$ and the wave number $k$ is obtained by using the lateral boundary conditions and can be written in terms of dimensionless parameters as

$$\overline{\eta} = \eta\left(\overline{p}_I / \overline{k}^2, \overline{k}, m, n, \varepsilon_e^0, q\right)$$

with

$$\overline{\eta} = \eta / \eta_{L1} = \eta \frac{\lambda_{L1}}{V}, \quad \overline{k} = kL_{\lambda}^{-2}, \quad \overline{p}_I = \frac{\rho \lambda^2}{\sigma_e^0} L_2 \lambda^{-4},$$

where $m$, $n$ and $q$ represent respectively strain rate sensitivity, strain hardening and thermal softening parameters. Inertia effects are embedded in the expression of the normalized inertial pressure $\overline{p}_I$.

This linearized perturbation approach was recently used to analyze multiple necking in the dynamic expansion experiments of hemispherical metallic shells, Fig. 1b, Mercier et al. [2].

2.2. Stabilizing effect of inertia

Effects of inertia on the strain localization process can be illustrated by considering a rate dependent non-hardening material. The flow stress is taken as a power-law of the equivalent plastic strain rate

$$\sigma_e = \sigma_{e0} (\dot{\varepsilon}_e / \dot{\varepsilon}_0)^m.$$ (6)

The relationship (4) simplifies into $\overline{\eta} = \eta\overline{p}(\overline{p}_I \overline{k}^2, \overline{k}, m)$. Figure 3 shows results of the perturbation analysis for a plate of infinite length under plane strain stretching. The dependence of the normalized growth rate $\overline{\eta}$ is displayed in Fig. 3a with respect to the normalized wave number $\overline{k}$ for $\lambda = 1$ (initial state) and the stretch-rate $\dot{\lambda} = 10^4$ s$^{-1}$. Material parameters are representative of copper and are reported in Table 1 together with loading conditions. A dominant instability mode is emerging with maximum growth rate $\overline{\eta}_{max}$ and wave number $\overline{k}_{max}$. The initial neck spacing is given by

$$l_{neck} = \frac{2\pi}{\lambda} \approx 2\pi L_2 / \overline{k}_{max}.$$ (7)

The quasistatic theory, obtained by setting $\overline{p}_I = 0$, indicates that $\overline{k}_{max} = 0$, see Fig. 3a, i.e. the neck spacing is infinite. For a plate of finite length, $\overline{k}_{max}$ would correspond to the smallest value of the wave number compatible with the boundary conditions at $X_1 = \pm L_1$, i.e. $j=1$ in Eq. (3). Thus, in agreement with experimental results, a single neck is predicted by the quasistatic approach. Multiple necking is clearly an outcome of inertia effects. The effect of the stretch-rate is illustrated in Fig. 3b: $\overline{k}_{max}$ increases with $\dot{\lambda}$. Consequently, the neck spacing decreases at higher stretch-rates according to Eq. (7) and the number of necks increases, as observed in Fig. 1a.

Table 1. Material and loading parameters for the plate stretching problem

| Material parameters | $\rho = 8900$ kg.m$^{-3}$ | $\sigma_{e0} = 109$ MPa | $\dot{\varepsilon}_0 = 1$ s$^{-1}$ | $m = 0.05$ |
|---------------------|---------------------------|-------------------------|---------------------------|
| Loading conditions  | $\dot{\lambda} = 10^4$ s$^{-1}$ | $L_2 = 0.3$ mm |

Inertia stabilizes the strain localization process. By comparing the dynamic and quasistatic theories it is seen in Fig. 3a that small wave-number modes (large wavelength) are slowed down by inertia. In addition, Fig. 3b shows that the relative growth rate $\overline{\eta}_{max}$ is a decreasing function of the stretch-rate. Therefore, necking is retarded by inertia at high loading rates.

The damping of short wave-length modes seen in Fig. 3 is a consequence of stress multiaxiality as shown in [6, 7] by comparing the complete 2-D theory to a simplified one dimensional dynamic approach developed by Fressengeas and Molinari [15]. Finally, the neck spacing appears to be (via the selection of
a dominant instability mode) the outcome of the competition between material inertia that extinguishes long wavelength perturbations and stress multiaxiality effects that slow down short wavelength perturbations. However, these results are only related to the onset of instability in a sample free of defects. Material and geometrical defects and wave propagation effects are seen in the following to play an essential role at the late stage of strain localization.

Fig. 3 (a) Normalized growth rate $\bar{\sigma}_{\text{max}}$ in terms of the normalized wave number $\bar{k} = kL_2$ in the initial state ($\lambda = 1$) for $\dot{\lambda} = 10^4 \text{s}^{-1}$. Material parameters are given in Table 1. The dominant mode is characterized by the wave number $k_{\text{max}}$ and the normalized growth rate $\bar{\sigma}_{\text{max}}$. With respect to the quasistatic theory, inertia slows down the long wavelength perturbations (small wave number) but has negligible effect on small wavelengths; (b) The number of necks increases (larger $k_{\text{max}}$) with the stretch-rate $\dot{\lambda} = V/L_1$ as in Fig. 1a, while the growth rate $\bar{\sigma}_{\text{max}}$ decreases (neck retardation).

2.3. Neck retardation

It was qualitatively shown with the linearized perturbation analysis that, in addition to the usual retarding effects of strain hardening and strain rate hardening, strain localization could be strongly slowed down by inertia. This delay leads to the phenomenon of neck retardation which is considered as beneficial in terms of an overall increase of ductility. To quantify neck retardation one has to recourse to a fully non-linear analysis. Initial defects have an essential role in controlling the level of strain at which localized necking is triggered [9]. Inertia effects and neck retardation due to inertia were explored by finite element simulations for bars under simple tension [17–20], and for ring expansion tests [21–23].

Xue et al. [20] considered an infinite plate under plane strain constraint subject to the constant stretch-rate $\dot{\lambda}$. The flow stress is taken as rate-independent and given by the Hollomon law, $\sigma_c = \sigma_0 e^{N}$. They worked with periodic unit-cells to explore the effect of a geometrical defect (amplitude and wavelength), material parameters and inertia on strain localization and neck retardation. Finite element calculations were performed on unit-cells of the type shown in Fig. 2, with $\dot{\lambda} = V/L_1$. The initial thickness of the plate, $h$, has a sinusoidal imperfection with initial wavelength, $L_w = 2L_1$, and amplitude, $\eta_0 : h = h^0(1 - 0.5 \eta_0 \cos(2\pi X_1/L_w))$, with $h^0 = 2L_2$.

For several values of $\dot{\lambda}$, $\eta_0$ and $L_w$, Xue et al. [20] calculated the overall strain at localized necking, $\varepsilon_{\text{Neck}}$. They demonstrated the existence of a critical wavelength for which $\varepsilon_{\text{Neck}}$ is minimized for given
values of $\dot{\lambda}$ and $\eta^0$. This critical wavelength corroborates the emergence of a dominant instability mode suggested by the linearized stability analysis. The critical wavelength is a decreasing function of the applied stretch-rate, in qualitative agreement with the linearized stability analysis. Thus, the number of necks increases at high strain rates. The retarding effect of inertia on localized necking was also quantified by Xue et al. [20], see Fig. 4.

![Fig. 4 (a) Results of the cell-model, Xue et al. [20]. The necking strain $\varepsilon_{\text{Neck}}$ increases with the normalized stretch-rate (neck retardation). The material response is rate-insensitive and described by the Hollomon law. Several hardening exponents, $N$, are considered. The amplitude of the geometrical defect is $\eta^0 = 0.04$. The effect of the Young modulus, $E$, appears to be negligible;](image)

(b) Arrested necks in a fragment of aluminum ring which was dynamically expanded, Zhang and Ravi-Chandar [5].

2.4. Fragmentation

The process of fragmentation of ductile rings subject to rapid expansion has been simulated with finite element calculations [24–26]. Zhou et al. [16] have described the entire process of strain localization and fragmentation with a simplified one-dimensional framework using the Bridgman correction to account for stress multiaxiality within necked regions. The first stage is characterized by multiple necking with a characteristic neck pattern resulting from the selection of a dominant wavelength dictated by the interplay between inertia and material parameters. However, another selection process is appearing later. When strain localization proceeds, it is observed that some necks are arrested while others are evolving to complete fracture. The phenomenon of neck arrest is clearly seen in the experiments of Zhang and Ravi-Chandar [5], Fig. 4b, and in the numerical simulations of Zhou et al. [16]. Slow necks are arrested by unloading Mott waves [27] emanating from fast growing necks.

The model of periodic unit-cells previously discussed has a limitation since it assumes that the defects are periodically distributed along the sample (uniform wavelength and uniform imperfection amplitude). In this approach all necks develop equally and are equally spaced. If fracture is assumed to occur at the same failure strain, all necks would break simultaneously and none of them would be arrested. Thus, neck arrest is due to irregularities in the material properties and sample geometry. Consequently, statistical aspects are essential features of a fragmentation theory. Elegant and efficient theories of fragmentation based on statistical generation of fracture sites and occultation by unloading waves have been developed by Mott [27] and Grady [28]. However, in these approaches the material properties and defects are embedded in a “nucleation function” which governs the statistical generation of fracture sites. Actually, there is a need to link the fracture process to material and geometrical properties. Finite element
simulations presented above are steps towards establishing this link, but further advances are still needed.

A simple heuristic view of the process of neck arrest can be attempted. For convenience, we assume that a constant overall strain rate \( \dot{\varepsilon} = \dot{\lambda} / \lambda \) is applied instead of the constant stretch-rate \( \dot{\lambda} \) as before. Consider two neighbour necks denoted as “Neck1” and “Neck2”. We neglect in a first step any interaction between necks and we denote by \( \varepsilon_{\text{fail}}(1) \) (resp. \( \varepsilon_{\text{fail}}(2) \)) the value of the overall strain at which failure occurs within “Neck1” (resp. “Neck2”). Differences between \( \varepsilon_{\text{fail}}(1) \) and \( \varepsilon_{\text{fail}}(2) \) are due to fluctuations in material and geometrical defects. Failure strains can be calculated by using finite element computational cell-models accounting for the dynamic localization process and a failure criterion. Failure occurs within necks with a time delay \( \delta t_{\text{Travel}} \) which is roughly given by \( \delta t_{\text{Travel}} = \delta t_{\text{Fail}} / \dot{\varepsilon} \), where \( \delta \varepsilon_{\text{Fail}} = |\varepsilon_{\text{fail}}(1) - \varepsilon_{\text{fail}}(2)| \). Denoting by \( L_{\text{neck}} \) the neck spacing, the time for the Mott unloading wave to travel with speed \( c_{\text{Mott}} \) from one neck to the next one is \( \delta t_{\text{Travel}} = L_{\text{neck}} / c_{\text{Mott}} \). The slow neck is arrested by the unloading wave emanating from the fast neck if this wave arrives before completion of failure, i.e. if \( \delta t_{\text{Travel}} < \delta t_{\text{Fail}} \). This analysis is certainly oversimplified (we refer the reader to [28] for a complete analysis accounting for all interactions between fracture sites), but it clearly indicates that the phenomenon of neck arrest is more likely to occur at low loading rates and for large fluctuations of the defect’s amplitudes (leading to large \( \delta \varepsilon_{\text{Fail}} \)), conditions under which large values of \( \delta t_{\text{Fail}} \) are expected.

In the ideal case of a material and structure free of defects, we have \( \delta \varepsilon_{\text{Fail}} = 0 \) and no neck is arrested.

When the loading rate \( \dot{\varepsilon} \) is increased, \( \delta t_{\text{Fail}} = \delta \varepsilon_{\text{Fail}} / \dot{\varepsilon} \) is approaching zero, although \( \delta \varepsilon_{\text{Fail}} \) may be slightly growing with inertia effects. On the other hand it can be shown that \( L_{\text{neck}} \) is decreasing to a non-zero asymptotic limit at high strain rates, Rodriguez et al. [29]. It follows that at high values of \( \dot{\varepsilon} \) the characteristic unloading time \( \delta t_{\text{Travel}} = L_{\text{neck}} / c_{\text{Mott}} \) tends to a non-zero limit, while \( \delta t_{\text{Fail}} \) is approaching zero. Therefore, the proportion of arrested necks is a decreasing function of \( \dot{\varepsilon} \), as at high loading rates less time is left to the Mott unloading waves to communicate between necks. At very high strain rates, a situation is approached where all necks are fractured, and the fragment size is coming close to the neck spacing. Naturally the fragment size decreases with \( \dot{\varepsilon} \) since the neck spacing and the number of arrested necks shrink with higher values of \( \dot{\varepsilon} \).

To summarize, the fragmentation process is controlled by interplay between material properties, inertia and statistical defects. Inertia effects are more pronounced at higher strain rates (reduction of the neck spacing and fragment size, less communication between necks, increasing of the overall ductility). Then, the relative effect of statistical defects is getting less important and the problem turns out to be more deterministic.

It is worth mentioning that similar conclusions are reached when dealing with multidimensional dynamic fracture of ductile materials (Grady [28]) or when considering brittle materials (Denouald and Hild [30], Forquin and Hild [31]) although the failure mechanics are quite different (necking versus micro-cracking).

3. Adiabatic shear banding

Ductile failure by adiabatic shear banding is frequently observed in metals subject to high loading rates especially when compressive and shearing modes are involved. Adiabatic shear bands (ASB) are seen in impact and penetration problems. They are also involved in fast forming processes such as forging and high speed machining. ASB are narrow zones with thickness of the order of few micro-meters where shear deformation is highly localized. Here again, the process of strain localization is a consequence of plastic flow instability, generally attributed to thermal softening due to heating by plastic work and quasi-
adiabatic conditions. However, for some materials, other softening mechanisms such as dynamic recrystallization or phase transformation may be involved, see for instance Rittel et al. [32]. Reviews on ASB can be found in Bai and Dodd [33] and Wright [34].

The spontaneous occurrence of a family of ASB and their collective behavior can be experimentally captured by considering the radial collapse of cylinders driven by explosive loading, Nesterenko et al. [35] or electromagnetic pulses, Lovingier et al. [36] It is of interest to note that families of ASB with regular spacing are also observed in high speed machining of metals (chip segmentation) where shearing is the main deformation mode, Komanduri and Von Turkovich [37].

There is a strong similarity between the analysis of dynamic necking in expanding rings and shells and adiabatic shear banding in collapsing cylinders, with however some important differences. Geometrical softening is not playing any role in the case of ASB. Instead, shear localization is driven by thermal softening and short wavelength perturbations are damped by heat diffusion. Similarly to the problem of multiple necking, higher strain rates promote inertia effects that are conducive to multiple shear banding and to a decreasing of the shear band spacing and fragment size. The shear band spacing was characterized by using perturbation approaches by Wright and Ockendon [38] for viscoplastic response with no-strain hardening and by Molinari [39] for strain-hardening materials. Recently, the fragmentation process associated to adiabatic shearing was addressed by Zhou et al. [40] with the type of approach developed by Zhou et al. [16] to investigate fragmentation by multiple necking. It was shown that the early stage of the localization process is well described with the linear stability analysis. The entire process of fragmentation, including shear band generation and growth as well as shear band arrest by unloading waves was described with non-linear calculations. The spacing between mature (non arrested) shear bands was found to be closer to that predicted by the momentum diffusion theory of Grady and Kipp [41] than by the linearized stability approach.

4. Dynamic damage and fracture

Fracture of ductile materials is often due to the nucleation, growth and coalescence of microscopic voids. Under dynamic loading conditions, these mechanisms can be substantially affected by inertia. In the present section, the distinctive features of dynamic ductile damage are discussed and a multiscale modelling of voided visco-plastic solids, incorporating micro-inertia effects, is presented. The results suggest that considering microscale inertia is of primary importance in the modelling of spall fracture and dynamic ductile crack growth.

4.1. Dynamic damage by micro-voiding, spall experiments

Dynamic damage by micro-voiding can be investigated via spall fracture experiments, [42–44]. A target plate is impacted at high velocity by a flyer plate. Compressive shock waves generated during impact are reflected as rarefaction waves when they encounter the target and flyer free surfaces. When these reflected waves interact, a large tensile stress is brought forth along the spall plane, Fig. 5. The tensile pressure is scaled by the impact velocity and may reach several GPa. Under this intense pressure loading, micro-voids are nucleated along the spall plane and undergo a rapid expansion, Fig. 5. As material particles at the vicinity of micro-voids sustain large radial accelerations, micro-inertia effects are significant. Indeed, theoretical works dealing with the growth or collapse of a single cavity in an unbounded or bounded matrix have revealed that inertia strongly slows down the void evolution, [45–48].

For high impact velocities, the material can be fractured along the spall plane, Fig. 5. Spall fracture occurs in several situations such as armor penetration, collision of particles against satellite, laser induced shock. An enlarged view of micro-voids is shown in Fig. 6a. Three key features can be noted:
(1) Voids are growing by remaining almost spherical and without the occurrence of localized phenomena.

(2) Coalescence occurs through a direct impingement mechanism.

(3) Complete fracture occurs at high value of the porosity.

Coalescence by direct impingement was observed in spall experiments, [49–52], and in dynamic crack propagation [53].

The role of microscale inertia on void growth and void-shape evolution has been often neglected. Most of the constitutive models of dynamic ductile damage and spalling assume that void expansion is mainly governed by viscous effects [54–61]. We shall see in the following that dynamic ductile fracture is significantly influenced by micro-inertia effects.

Fig. 5. Development of micro-voiding along the spall plane parallel to the impacted surface [50]. The level of porosity is increasing with the impact velocity $V$. Fracture is observed at $V = 306$ m/s.

Fig. 6. (a) Spall experiments on tantalum, Gray III et al. [52]. Micrvoids have spherical shape and coalescence occurs by direct impingement, (b) periodic cell model used to investigate the effects of the loading rate and of stress triaxiality on void evolution and coalescence, Jacques et al. [73].

4.2. Void coalescence

Accounting for void coalescence in constitutive damage models is a necessary step to accurately describe the fracture of ductile solids, [62–64]. Under quasi-static loading, the most common coalescence mode is necking of intervoid ligaments [65–67]. However, at very high loading rates direct impingement of voids appears to be also a possible coalescence mechanism for ductile metals, Fig. 6a.

Computational cell models, Fig. 6b, are commonly used to investigate the response of porous solids, but almost all these studies were devoted to quasi-static loading conditions, [68–72]. Dynamic computations were recently carried out by Jacques et al. [73] to investigate void coalescence under high...
loading rates. An axisymmetric model of a cylindrical cell (initial half-height $H_0$; initial radius $R_0$; $H_0=R_0$) containing a single initially spherical void at its centre was considered, Fig. 6b. The initial porosity, $f_0$, is equal to $10^{-3}$. Since dynamic conditions are considered, the cell response is size-dependent. Two values of the initial void radius are considered: $a_0=5$ μm and $a_0=10$ μm. To have same initial porosity, the values of $R_0$ are respectively 43.7 μm and 87.4 μm.

The matrix material is described as an isotropic elastic-plastic (rate-independent) solid, obeying the $J_2$ flow theory. Material parameters representative of a medium-strength steel were adopted.

![Fig. 7. Effect of inertia on void coalescence, Jacques et al. [73]: (a) Evolution of the porosity versus macroscopic axial strain for several values of the stress triaxiality, $T$; dynamic simulations with $\dot{\Sigma}_{22} = 400$ MPa/$\mu$s; (b) Quasistatic simulation: contours of equivalent plastic strain at a macroscopic axial strain of 0.81 for $T=1$; c Dynamic simulations with $a_0=5$ μm and $\dot{\Sigma}_{22} = 400$ MPa/$\mu$s.](image)

![Fig. 8. The stabilizing effect of inertia is apparent when comparing, for the same macroscopic axial deformation $E_{22} = 0.22$, the void shape obtained (a) with the quasistatic simulation to (b) the dynamic case with $\dot{\Sigma}_{22} = 400$ MPa/$\mu$s. The initial void is spherical. The triaxiality ratio is $T = 3$. The lateral enlargement of the void and the deformation of the ligament are reduced under dynamic conditions, Jacques et al. [73]. These calculations support the conjecture of Thomason [74] that the loss of localization between voids under dynamic loading may be due to the fact that inertia causes a shielding effect on the inter-void ligaments.](image)

The macroscopic stress components along the radial and axial axes are respectively denoted by $\Sigma_{11}$ and $\Sigma_{22}$. The cell is initially at rest and the time evolution of the macroscopic stress is imposed according to: $\dot{\Sigma}_{11} = \dot{\Sigma}_{11}^0 t$, $\dot{\Sigma}_{22} = \dot{\Sigma}_{22}^0 t$ ( $\dot{\Sigma}_{11}$ and $\dot{\Sigma}_{22}$ are two positive constants with $\dot{\Sigma}_{22} > \dot{\Sigma}_{11}$).
Simulations have revealed that inertia may play an important role in the coalescence process. Fig. 7a represents the evolution of the cell porosity (ratio of the void volume to the cell volume) with respect to the macroscopic axial strain $E_{22}$ for several values of the stress triaxiality ratio $T$. Results are obtained for quasistatic and dynamic computations.

A notable effect of inertia on porosity evolution is seen for $T=1$. For this value of stress triaxiality the quasi-static simulation predicts a rapid increase of porosity at $E_{22}=0.75$. Acceleration of the void growth is due to necking of the inter-void ligament, see Fig. 7b, and corresponds to the onset of void coalescence. By contrast, under dynamic conditions the acceleration of the porosity growth is triggered at higher strain and porosity, and the transition is smoother. This delay is a manifestation of the stabilizing effect of inertia. Void shape and contours of plastic strain are shown in Fig. 7c (dynamic case) at the same strain $E_{22}=0.81$ as in Fig. 7b (quasistatic case). In the dynamic case the void has an ellipsoidal shape typical of the pre-coalescence stage. Thus, necking of the intervoid ligament is retarded by inertia. This situation is quite similar to the phenomenon of neck retardation that was discussed in Sect. 2 for plates and bars under high stretching rates, although the length scale is smaller in the present configuration.

The size effect associated to inertia is apparent in the results displayed in Fig. 7a for $1.5T=1$: neck retardation is more pronounced for the initial void radius $a_0=10\ \mu m$ as compared to $a_0=5\ \mu m$.

From Fig. 7a the occurrence of coalescence seems to be unaffected by inertia for $T=2$ and $T=3$. However, inertia effects are also important at this higher level of triaxiality but they manifest themselves in a different way. The evolution of the void shape appears to be significantly affected by inertia, Fig. 8.

For $T=3$, the void remains nearly spherical under dynamic conditions, Fig. 8b. Coalescence occurs by direct impingement in this case. Thus, material inertia appears to play a crucial role by refraining changes in the void morphology, consistent with several experimental studies [50–52] which showed essentially spherical voids in spall fracture of ductile metals, see Fig. 6a.

It should be noted that coalescence retardation induced by inertia effects prompts a significant increase of the material ductility at high loading rates [73].

### 4.3. Micro-mechanical modelling of dynamic damage

Considering the central contribution of micro-inertia to dynamic damage observed previously, it appears essential to include this feature in the modeling of the response of damaged materials. We expose here a multiscale modelling of the dynamic response of voided viscoplastic materials that includes microscale inertia effects, based on the work of Czarnota et al. [75, 76], Jacques et al. [77], Molinari and Mercier [78], Molinari and Wright [79]. It should be mentioned that different formalisms to model micro-inertia were proposed by Wang and Jiang [80, 81], Weinberg et al. [82] and Trumel et al. [83].

A representative volume element (RVE) is considered which contains a large number of voids with various sizes. In classical approaches of voided viscoplastic materials such as the GTN model, [84, 85], the non-uniformity of void size does not influence the overall material response. The influence of voids is essentially accounted for through the effect of the overall porosity on the overall viscoplastic potential. However, under dynamic conditions, void radii are crucial parameters to be considered since inertia effects are size dependent [86].

To each void in the RVE we associate a unit-cell (hollow-sphere model) composed of the void (with radius $a$) surrounded by a spherical shell of matrix material (with external radius $b$), Fig. 9. Different unit-cell configurations can be envisaged (uniform cell porosity, uniform cell size etc.) [76, 87].

Two simple averaging schemes are used to link the RVE and the unit-cell scales. The first scheme, named $D$-model, assumes that the overall strain rate $\dot{\mathbf{D}}$ is applied to the external boundary of each unit-cell. In the second approach ( $\Sigma$-model) the macroscopic stress $\Sigma$ is applied to each cell. At this stage
elastic deformation are neglected.

We denote by \( D_{\text{cell}} \) the homogeneous strain rate applied at the boundary of an individual unit-cell (\( D_{\text{cell}} = D \) for the D-model) and by \( \Sigma_{\text{cell}} \) the stress associated by the principle of virtual work (accounting for micro-inertia). \( \Sigma_{\text{cell}} \) is the sum of a quasistatic component representative of the overall response of the hollow-sphere under quasistatic loading and a dynamic component inherited from material accelerations at the microscale,

\[
\Sigma_{\text{cell}} = \Sigma_{\text{stat}} + \Sigma_{\text{dyn}}.
\]  

The quasistatic stress \( \Sigma_{\text{stat}} \) is given by any classical homogenization approach well suited to the hollow-sphere configuration. In the following, the GTN model will be adopted to estimate \( \Sigma_{\text{stat}} \) [84, 85].

A closed-form expression for the dynamic stress \( \Sigma_{\text{dyn}} \) was obtained by Molinari and Mercier [78] using a trial velocity field similar to that of Gurson [84]. For applications to dynamic fracture, the spherical component of the dynamic stress tensor is predominant [88,89]. Thus, the dynamic stress can be approximated as

\[
\Sigma_{\text{dyn}} = P_{\text{dyn}} I \quad \text{with} \quad P_{\text{dyn}} = \rho_0 a^2 \left[ D_{\text{m}} \left( \frac{1}{3} f_{\text{cell}}^3 - \frac{1}{2} f_{\text{cell}}^2 \right) + \left( D_{\text{m}} \right)^2 \left( \frac{3}{2} f_{\text{cell}} - 5 f_{\text{cell}}^2 - 3 f_{\text{cell}}^3 \right) \right].
\]

Here, \( D_{\text{m}} = \text{tr}(D)/3 \) and \( f_{\text{cell}} = a^3/b^3 \) is the porosity of the hollow sphere. It is interesting to point out that the dynamic pressure is function of the void radius \( a \).

For the D-model, the macroscopic stress \( \Sigma \) is obtained with averaging of \( \Sigma_{\text{cell}} \) over all unit-cells: \( \Sigma = \langle \Sigma_{\text{cell}} \rangle \) (brackets represent the volume average). For the \( \Sigma \)-model the macroscopic strain rate is given by \( D = \langle D_{\text{cell}} \rangle \).

So far the micromechanical analysis has been developed by neglecting elastic deformations. Elasticity is introduced at the macroscopic RVE’s level by considering the decomposition of the macroscopic strain rate into elastic and plastic parts. In the constitutive equations derived from the previous micromechanical analysis, the strain rate is replaced by the plastic strain rate. The elastic response is described using a hypoelastic relation.
4.4. Comparison with finite element cell computation

A first theoretical validation of the proposed averaging model is provided by comparison with numerical simulations on voided unit-cells carried out with the finite element code ABAQUS/Explicit [87]. The matrix material has an elastic-viscoplastic behaviour obeying the $J_2$ flow theory (details on constitutive behaviour are given in [87]). Adiabatic heating is taken into account.

Figure 10 presents a comparison of stress—strain responses derived from finite element cell computations with results obtained with the present constitutive framework (GTN augmented by microscale inertia effects) and the quasistatic GTN model. The time evolution of the prescribed axial strain rate is given in Fig. 10b ($D_0 = 10,000 \text{ s}^{-1}$, $D_0' = 4.75 \times 10^{11} \text{ s}^{-2}$, $t_0 = 4 \times 10^{-6} \text{ s}$). To investigate the influence of micro-inertia during transient stages, the strain rate is changed for $t > t_0$. From Fig. 10b, it appears that including micro-inertia in the cell constitutive response is of primary importance.

![Fig. 10](image-url)

4.5. Application to spall fracture

Spall fracture has been modeled by using several damage approaches based either on semi-empirical law for the damage rate, Kanel et al. [90], or on averaging techniques requiring the definition of a representative volume element (RVE), Seaman et al. [54], but neglecting micro-inertia effects.

The micro-mechanical approach of dynamic damage presented in Sect. 4.3 is used here to model spall experiments on high-purity tantalum performed by Roy [50]. Finite element calculations were carried out with the ABAQUS/Explicit software, according to the configuration shown in Fig. 9 [76–77]. At each Gauss point, the constitutive response of the voided material is represented by those of a RVE. Homogenization within the RVE is performed by using the $\Sigma$-model, but results obtained with the $D$-model are found to be close [76].

The modeling is completed by introducing a nucleation law describing the generation of voids with increasing pressure. The RVE is initially free of voids since the high purity tantalum is fully dense in the
initial state (however, the modelling could be extended to account for an initial distribution of voids). It is assumed that the number $N_c$ of voids which are nucleated per unit initial volume of the RVE is function of the macroscopic pressure $P$ applied to the RVE and given by the following nucleation law [79]

$$N_c(P) = N \int_{p_c}^{P} w(p_c; P_{0c}, \beta, \eta) dp_c \quad \text{for} \quad P \geq P_{0c}; \quad N_c(P) = 0 \quad \text{for} \quad P \leq P_{0c},$$

(10)

where $w(p_c)$ is the Weibull probability density function characterized by three parameters $P_{0c}$, $\beta$ and $\eta$. $N$ represents the maximum number of nucleation sites (per unit initial volume) where voids can be nucleated ($N_c(P)$ tends to $N$ for large values of the applied pressure $P$). The threshold pressure below which voids do not nucleate, $P_{0c}$, can be measured experimentally, [43, 50]. For high-purity tantalum, the value $P_{0c}=3$ GPa was identified by Roy [50].

![Fig.11. Prediction of the proposed dynamic damage model compared against experimental data from plate impact experiments of Roy [50] on high-purity tantalum initially void free, Jacques et al. [77]: (a) Time history of the particle velocity at the middle of the rear surface of the target, for several impact velocities; (b) Mean void radius versus impact velocity. We note that the model shows quite good correlation with respect to (a) macroscopic and (b) microscopic experimental measurements.](image)

The quasi-static GTN model is modified for very low values of cell porosity. Indeed, the GTN model predicts an infinite yield pressure at vanishing cell porosity which is unphysical. A nucleation pressure has been introduced in the modeling to avoid this singular response, Czarnota et al. [75].

The voids are assumed to grow by keeping a spherical shape in accordance to experimental and theoretical considerations exposed in Sects. 4.1–4.2. Thus, fracture occurs by direct impingement of voids. A material particle is considered to be fractured when the macroscopic porosity of the associated RVE reaches a critical value $f_c$. For high purity tantalum the critical porosity was identified as $f_c=0.35$ from metallographic observations, and the viscoplastic properties were characterized from Hopkinson bar experiments [50]. A nonlinear formulation of the elastic response based on the Mie-Grüneisen equation of state was used to describe the high velocity impact tests [77].

The parameters $N$, $\beta$ and $\eta$ related to the nucleation law Eq. (9) are characterized by an inverse identification method using three free-surface velocity profiles (at low, intermediate and high impact velocities) that were measured by laser interferometry at the rear surface of the target.

The nucleation law being characterized, the model is validated by predicting impact tests at other loading conditions than those taken for the identification of the parameters $N$, $\beta$ and $\eta$ (i.e. for different
flyer thicknesses and impact velocities). Moreover, the model is able to characterize the internal damage generated by micro-voiding and its dependence upon loading conditions, Fig. 11. The present multiscale model shows excellent capabilities in predicting the decreasing of the mean void radius for higher impact velocity, the statistical distribution of void radii and the spatial distribution of the porosity in the vicinity of the spall plane, Jacques et al.[77]. In this modeling, the role of microscale inertia is crucial as it controls the growth-rate of voids by refraining large voids with respect to small voids.

4.6. Dynamic crack propagation

Local approaches of ductile fracture are aimed to relate the fracture resistance of a specimen to the mechanical behavior of its constitutive material by analyzing the deformation in the vicinity of the crack tip. This approach requires a constitutive model that accounts for the progressive loss of stress-carrying capability of the material due to the development of damage. In that perspective, the GTN model of ductile damage [84, 85], and its extensions have been widely used to analyze crack propagation under quasi-static conditions [91–94], and dynamic conditions [95–98].

The local approach of fracture has several interesting features. For instance, the crack speed appears to be a natural outcome of the simulation and depends on the micro-mechanisms of failure included into the material model. However, the problem of spurious strain localization and pathological mesh sensitivity is an important issue of this approach. For dynamic problems, material rate dependence implicitly introduces a length scale into the governing equations and also provides a regularizing effect (Needleman [99]; Sluys and de Borst [100]). Nevertheless, this regularizing effect rapidly decreases for low loading rates. Furthermore, Needleman and Tvergaard [101] observed that the crack growth behavior predicted by simulations based on a viscoplastic version of the GTN model (with material parameters representative of some high strength steels) is, in general, clearly mesh sensitive.

According to several analyses, [98, 102, 103], the material in the vicinity of a crack propagating at high speed may sustain strain rates greater than $10^7$ s$^{-1}$. Besides, the region of high stress triaxiality that occurs ahead of a crack tip promotes ductile void expansion. For such loading conditions, micro-inertia can have an important effect as seen in previous sections. Glennie [104] proposed a simple analytical model for void growth in the vicinity of a blunted crack tip in which the effect of micro-inertia is taken into account. Even if this model is built on some strong assumptions (mainly the effect of damage due to void growth on the stress state in the vicinity of the crack tip is neglected), it suggests that inertial resistance to void expansion limits the speed at which cracks can propagate. Rosakis et al. [105] investigated the effect of micro-inertia by means of an elementary energetic analysis which was found to be qualitatively consistent with the increase of the dynamic fracture toughness with the crack speed observed in experiments [106].

Jacques et al. [89] have recently analyzed dynamic crack growth by using an extension of the GTN model including the effects of micro-inertia, along the lines exposed in Sect. 4.3. In a first approach, they considered that the evolution of the internal damage was due to the growth of pre-existing voids, the nucleation of new voids being neglected.

Fracture of an axisymmetric notched bar is investigated in Fig. 12. Failure of a material particle is assumed to occur when the overall porosity is equal to 0.5. Finite element calculations have been carried out with coarse and fine meshes, see Fig. 12b. Simulations of the dynamic crack growth have been performed with a viscoplastic version of the classical GTN model (in which micro-inertia effects are neglected) and with the dynamic damage model proposed by Jacques et al. [89], for an initial mean void radius of $a_i = 5 \ \mu m$ and the initial overall porosity $f_0 = 1.5 \times 10^{-4}$. It is worth noting that the dynamic damage model coincides at the limit with the quasistatic GTN model when $a_i$ is tending to zero (for a fixed value of the initial porosity). Figure 12b indicates clearly that the results obtained with the dynamic
damage model are mesh insensitive. The same crack path, crack speed and extension of the highly
damage zone (porosity > 0.3) are obtained by using either the coarse or the fine mesh. This is not the case
for the quasistatic GTN model. Strain localization is much less pronounced and damage is more diffuse
when microscale inertia is accounted for.

![Diagram](image)

Fig.12. Crack growth in an axisymmetric notched bar, Jacques et al. [89]: (a) Geometry and boundary conditions for the
axisymmetric notched specimen. A uniform surface traction, $T_s$, is applied on the top surface and the axial displacement is zero on
the bottom surface; (b) Effect of the mesh size on the crack path plotted on the initial (undeformed) configuration at $t = 40.125$ $\mu$s
for an applied surface traction of 700 MPa. Elements with porosity greater than 0.3 are painted in black. The region presented in this
figure is indicated by a dashed box in the part a of the figure. The GTN model shows mesh dependence. Instead, the crack path, the
crack speed and the extension of the highly damaged zone (porosity > 0.3) are identical for the coarse and fine meshes when
microscale inertia is accounted for.

It has been also shown by Jacques et al. [89] that the crack speed can be substantially slowed down and
fracture toughness enhanced by micro-inertia effects. These are first order effects which have been
neglected so far. The role of inertia in dynamic fracture was mainly considered to be due to macroscopic
inertia effects associated to plastic deformation in the crack tip region, Siegmund and Needleman [103],
Lam and Freund [107]. Indeed, the contribution of microscale inertia seems to be as essential as that of
macroscopic inertia to achieve a complete picture of dynamic fracture.

5. Conclusion

We have seen that inertia can deeply act upon the mechanisms of dynamic ductile failure of metallic
materials. The mark of inertia was recognized at the macro-scale and micro-scale levels and can bear
different aspects:

1. Long wavelength perturbations and large scale defects are damped by inertia. This feature contributes
to the decreasing of the characteristic spacing between localization, fracture and damage sites at
higher strain rates.

2. The growth rate of perturbations and imperfections is slowed down by inertia. Therefore, strain
localization is retarded at high strain rates. This results in an increase of the apparent overall ductility.

3. Defects and stress release waves have essential effects on the late development of the fragmentation
pattern.

4. Microscopic damage mechanisms (void growth and coalescence) can be substantially affected by
microscale inertia.

5. Microscale inertia has first order effects on dynamic crack growth by diffusing damage in a larger
zone (regularizing effects), slowing down the crack advance and increasing the fracture toughness.

References


[87] Jacques N, Mercier S, Molinari A. Multiscale modelling of voided ductile solids with micro-inertia and application to dynamic...


