Short Communication

Two efficient iterative algorithms for error prediction in peripheral milling of thin-walled workpieces considering the in-cutting chip

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ARTICLE INFO

Article history:
Received 6 October 2008
Received in revised form 3 June 2013
Accepted 5 June 2013
Available online 18 June 2013

Keywords:
Flexible iterative algorithm (FIAL)
Thin-walled workpiece
Surface form errors
Double iterative algorithm (DIAL)
Peripheral milling
In-cutting chip

ABSTRACT

Due to the deflection of tool and workpiece induced by cutting force, there is a high complexity associated with the prediction of surface form errors in peripheral milling of thin-walled workpieces. Based on the systematic study of in-cutting chip, this paper proposes a new efficient iterative algorithm named flexible iterative algorithm (FIAL), which is suitable for surface form errors prediction in peripheral milling of low rigid thin-walled workpiece. In FIAL, an iterative scheme for calculations of tool/workpiece (TW) deflections are developed by considering the former convergence cutting position, and in the scheme a new important variable $\Delta$ is proposed for the calculation of radial cutting depth which never been considered before. Based on FIAL and the analytical study of in-cutting chip, a double iterative algorithm (DIAL) is brought forwarded to calculate the positions and magnitude of the maximum surface form errors, which always take the peak point include in each iterative step. Comparisons of the form errors and cutting forces obtained numerically and experimentally confirm the validity of the proposed algorithms and simulation procedure. The experimental and analytical results have shown that FIAL is faster in the iteration convergent speed and more accurate than the rigid iterative algorithm in surface form errors prediction, and DIAL is proved to be valid in the maximum errors prediction.

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1. Introduction

Thin-walled workpieces such as jet engine impellers and monolithic spar-ribs are widely used in aerospace industries, which are easy to deform under the cutting forces for the poor rigidity. Well, how to control the surface form errors induced by deflection of tool/workpiece (TW) is the most striking problem that aerospace manufacturing technologies have to face up. To achieve high machining quality, the surface form errors must be controlled. Based on the reliable prediction of surface form errors by the finite element (FE) method, the milling process optimization and error compensation can achieve the aim of error control. Thus, more attention has been paid to the surface form error prediction.

Generally, due to the variable TW deflections and changing tool immersion angle there is a high complexity associated with modeling of cutting forces in peripheral milling of thin-wall workpieces [1], and periodically varying milling forces excite significant deflections of the flexible plate structures both statically and dynamically [2]. Kline et al. [1] used a rigid model without an iterative algorithm to simulate the deflections of a rectangular plate and the end milling tool. Sutherland and Devor [3] considered the effects of static deflections of the tool in estimating the instantaneous uncut chip thickness, and included the iterative algorithm in the cutting force prediction, the predicted errors decreased correspondingly. Considering the cutter deflections in peripheral milling operation, Desai and Rao [4] present a methodology to classify surface error profiles and to relate the same with cutting conditions in terms of axial and radial engagement between cutter and workpiece.

To increase the precision of prediction models, flexible models must be taken. So, Budak and Altintas [5], Tsai and Liao [6] considered the TW deflections, and used iteration to predict the cutting force distribution and the surface form errors in the peripheral milling of a flexible plate. During the iterative process, Wan and Zhang [7,8] studied two sub-iteration algorithms for the determination of chip thickness and radial cutting depth to avoid numerical oscillations that may occur. And Wan et al. [9] presented three methods for controlling the force-induced surface
dimensional errors in peripheral milling of thin-walled structures. Emphasis was put on how to select the feed per tooth and cutting depth simultaneously for tolerance specification and maximization of the feed per tooth simultaneously. Chen et al. [10] proposed a simulation method for predicting machining deformation considering multilayer machining of a thin-walled part with single-tooth end milling. To modify the off-line tool path on the basis of TW deflections, Ratchev [11] used an iterative method to implement the NC tool path optimization strategy. Bera et al. [12] presented a methodology to compensate TW induced surface errors in machining of thin walled geometries by modifying tool paths, in which a cutting force model accounting for change in process geometry due to static deflections of TW was adopted.

The references show the importance of iterative algorithm in the error prediction and compensation process. It is the key difference between the rigid model and the flexible model in the peripheral milling of thin-walled workpieces. On the assumption of no deflection, the iterative algorithms abovementioned that we called rigid iterative algorithm (RIAL) here, always take the nominal position (equaled to the effect without cutting forces act on the workpiece) as the initial iterative position on each iterative step along the whole workpiece. As a result, the simulation time for the iterative process is too long and the errors between predicted and tested are relatively large. At the same time, the maximum surface form error must be determined by calculating the surface form errors all over the workpiece for each feed step and angular increment. This may cost long time to obtain the maximum surface form error at a certain feed step by comparing the simulated errors.

In this work, based on the analytical study of the in-cutting chip, two efficient flexible iterative algorithms with a systematic simulation procedure are presented for prediction of surface form errors in peripheral milling of thin-walled workpieces, which aims to reduce both the simulation time and the algorithm errors. Firstly, a flexible TW system is modeled and the FE based simulation method is used to predict the deflections of the TW. Secondly, the cutting forces for the flexible TW system are settled. Thirdly, FIAL and DIAL are described in detail for the prediction of the actual cutting depth, the surface form errors and the maximum surface form error. Finally, the validation of the algorithms is demonstrated by comparing the numerical results with the experiment results.

2. In-cutting chip in flexible TW system

As shown in Fig. 1, down-milling a thin-walled workpiece with peripheral milling tool is studied, which includes a \( L \times H \times t_0 \) thin-wall (in Fig. 1(a)). The in-cutting chip (which means the chip to be removed) is taken as the key discussing object as shown in Fig. 1(b). In-cutting chip, (c) deformed in-cutting chip and (d) surface form error.

![Fig. 1](image_url)

For surface form error prediction, the cutting forces and the deflections of TW were large enough influence the material removing as planed (fewer material removed in down-milling see Fig. 1(c)) and produce surface form errors in result (see Fig. 1(d)).

As described in the previous works [6–9,13], the distribution of surface form error can be obtained by projecting the deflections induced by the cutting forces along the normal of the machined surface. By definition, the surface form error is the normal deviation of the actual machined surface from the desired machined surface at the direction of \( Y_{w_c} \). For instance, at point \( p \) in Fig. 1(d), the corresponding surface form error is \( e_\theta(p) \). The calculation of \( e_\theta(p) \) for a certain feed position \( I \) can be given by

\[
e_\theta(p) = \delta Y_c(p,j,k) + \delta Y_w(p,j,k)
\]

(1)

where \( \delta Y_c(p,j,k) \) and \( \delta Y_w(p,j,k) \) are the normal projections of the cutting tool deflection and the workpiece deflection corresponding to point \( p \) respectively. The values of \( \delta Y_c(p,j,k) \) is estimated by assuming the cutting tool as a cantilevered beam [2,14], \( \delta Y_w(p,j,k) \) is calculated using the finite element method [7–9], and of course the cutting forces is the key factor in the calculation process. Surface form errors at a certain feed position can be obtained by repeating this computing process. By splitting the milling path into a sequence of discrete feed positions, the surface form errors all over the workpiece can be obtained.

Since the flexible TW system is adopted here, the iteration algorithm must be used to determine the chip load, the cutting forces and the surface form errors, to meet the static equilibrium condition. After the iteration, we can get the actual cutting chip as the shaded area in Fig. 1(c), and the actual cutting depth \( d' \), the actual immersion angle \( \alpha_{\text{sim}} \). We have

\[
\alpha_{\text{sim}} = \cos^{-1}\left(1 - \frac{d'}{R}\right) = \cos^{-1}\left(1 - \frac{d - \delta Y_c - \delta Y_w}{R}\right)
\]

(2)

For \( \alpha_{\text{sim}} \), \( \delta Y_c \), \( \delta Y_w \), \( d \), \( R \) are the cutting force modeling procedure.

3. Cutting force model for a flexible TW system

For surface form error prediction, the cutting forces and the force induced deflections should be predicted firstly. In this study, the cutting force model proposed by the authors [1,15] is adopted to estimate the cutting forces. For the sake of completeness, this paper introduces briefly the cutting force modeling procedure.
The helical fluted peripheral milling tool is equivalently divided into a finite number of axial segments. The total X-c, Y-c and Z-c cutting forces acting on the cutter at a particular instant are acquired by numerically summing the force components acting on each individual disk element. For the convenience of study, take \( k, i \) designates the cutting tool node which is the intersection between \( i \)th horizontal mesh line and the \( j \)th cutting flute, while notation \( [i, j, k] \) means the tool element which is the cutting flute segment between cutting tool node \( (k, i) \) and \( (k, i+1) \) with \( j \)th angular increment, see Fig. 1(a) and (b). During the milling process, the cutting forces can be expressed as \[1,15],

\[
F_x = \sum_{i=1}^{N_i} \sum_{k=1}^{N_k} \left\{ -K_k \alpha_i d_i(i,j,k) f_z \sin^2[\varphi(i,j,k)] \\
+K_i d_i(i,j,k) f_z \cos[\varphi(i,j,k)] \sin[\varphi(i,j,k)] \right\} 
\]

(3)

\[
F_y = \sum_{i=1}^{N_i} \sum_{k=1}^{N_k} \left\{ K_k \alpha_i d_i(i,j,k) f_z \sin[\varphi(i,j,k)] \cos[\varphi(i,j,k)] \\
+K_i d_i(i,j,k) f_z \sin^2[\varphi(i,j,k)] \right\} 
\]

(4)

where \( K_k \) and \( K_i \) are the cutting force coefficients determined experimentally, while \( \varphi(i,j,k) \) is the corresponding rotation angle of the \( k \)th flute for the \( i \)th axial segment and \( j \)th angular position which can be given as

\[
\varphi(i,j,k) = -[\theta(j) + 2\pi(k-1)/N_i] + d_z \tan(\alpha_{th})/R
\]

(5)

where \( \theta(j) \) is the rotation angle of the \( j \)th angular position, and we define \( \theta(0) = 0 \) when the tool nose meet the point \( C \) in Fig. 1(b); \( N_i \) is the number of flutes on the cutting tool; \( R \) is the nominal radius of the helical fluted cutting tool; \( \alpha_{th} \) is the helix angle of the peripheral milling tool; \( d_z \) is the axial length of each segment, which can be given as

\[
d_z(i,j,k) = \frac{\alpha_{th} \pi R}{\tan(\alpha_{th})} = \frac{R \cos^{-1} \left(1 - \frac{d_y(i,j,k)}{R}\right)}{\tan(\alpha_{th})}
\]

(6)

From Eqs. (5) and (6), can we see the cutting forces (Eqs. (3) and (4)) are directly influenced by the oscillations of the actual radial cutting depth. For a certain cutting tool element \([i, j, k]\), the actual radial cutting depth is iteratively corrected by

\[
d_z^{\text{corr}}(i,j,k) = d_z - \delta Y_r(i,j,k) \]

(7)

where \( d_z^{\text{corr}}(i,j,k) \) is the actual radial cutting depth due to deflections after \( t \)th iteration, \( d_z \) is the ideal nominal radial cutting depth without deflections, \( \delta Y_r(i,j,k) \) is the cutting tool deflection of the \( i \)th axial segment at the \( j \)th angular position, \( \delta Y_w(l,j,1) \) is the workpiece deflection at position \( Z_w(l) \) corresponds to the \( i \)th tool axial segment.

4. The flexible iterative algorithm (FIAL)

4.1. FIAL for the process of iteration

During solving of Eqs. (3), (4) and (7), following iterative process about \( \delta Y_r \) and \( \delta Y_w \), \( d_z^{\text{corr}} \) and cutting forces can be found, as shown in Table 1.

In order to save the simulation time and improve the simulation precision, a new iterative algorithm called FIAL here is proposed to sole the iterative problem.

Fig. 2(a) describes an unfolded in-cutting chip which is the normal projection of in-cutting chip in \( Y_w \) direction, and some cutting flutes in contacting with workpiece. In Fig. 2(a) the oblique lines represent TW contact zone. Take \( E_{1i}F_{1k} \) as the initial position of the iteration during the cutting process, cutting flutes will vibrate up and down in the iterative process. After \( k \)th iteration, cutting flutes reach the convergence position: \( E_{1k}F_{1k} \), and the point \( E_{1k} \) will be the corrected surface generation point. After the first iteration convergence, instead of taking \( E_{20}F_{20} \) as initial iteration position for the next cutting increment (the method which used in former literatures), the FIAL taking \( E_{20}F_{20} \) as the next initial iteration position. Based on the first convergence position \( E_{1k}F_{1k} \), we can see that \( BF \) and \( E_{1k}E_{20} \) are the cutting entrance boundary line of the thin wall and surface generation line after first iteration convergence respectively. So considering the continuity of cutting process, \( E_{20}F_{20} \) is obviously the better choice as start position for the second iteration step than \( E_{20}F_{20} \).

### Table 1

<table>
<thead>
<tr>
<th>Iteration no.</th>
<th>( \Delta Y_r )</th>
<th>Actual cutting depth ( d_z^{\text{corr}} )</th>
<th>( F_x ) and ( F_y )</th>
<th>Surface generation point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>↑</td>
<td>↓</td>
<td>( E_{10} )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>↑</td>
<td>↓</td>
<td>( E_{11} )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>↑</td>
<td>↑</td>
<td>( E_{12} )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>↓</td>
<td>↑</td>
<td>( E_{13} )</td>
<td></td>
</tr>
</tbody>
</table>

* Denote ↑ means increase; ↓ means decrease (down-milling).

Fig. 2. The FIAL: (a) the deflections iteration process of TW and (b) calculation of actual radial cutting depth in FIAL.
Moreover, the method can obviously improve the algorithm convergence speed because the initial iterative position is closer to the convergence one. In this paper, we call the algorithm “flexible iterative algorithm” (FIAL) as contrast with the “rigid iterative algorithm” (RIAL). It can be inferred that the cutting contact length of $E_{1k}F_{vk}$ and $E_{1k}F_{1k}$ would keep same after the cutting tool gets across from peak point $F$ as shown in Fig. 2(a). The point $F$ is always the actual peak point of cut, and after it there is no iterative step needed until the end of the cutting. In fact, it is because there is no contact between cutting tool and workpiece up the line $\Gamma F$.

### 4.2. Improvement of radial cutting depth predict by FIAL

In actual cutting, the deflections of TW may induce the workpiece to be inclined, just like the line $BF$ as shown in Fig. 2(b). Based on FIAL, considering the inclined cutting entrance line, the nominal radial cutting depths must be revised for each increment, which is important for predicting cutting forces at the next iterative step and increment step.

As seen in Fig. 2(b), $E_{k}$ and $E_{k}$ denote surface generation points correspondent to the $k$th iteration, while $E_{k}F_{vk}$ and $E_{k}F_{1k}$ are the cutting contact edges between cutting tool and workpiece. $\delta Y_{c}$ and $\delta Y_{w}$ are the deflection of cutting tool at the bottom of workpiece and actual radial cutting depth correspondingly; while $\delta Y_{c}$, $\delta Y_{w}$ are the deflections of workpiece and tool at the free end of toolpoint, $d_{rr}$ is the actual radial cutting depth at $v$th iteration. The actual radial cutting depth can be given as

$$d_{rr} = d_{v} - \delta Y_{c} - \delta Y_{w} - \Delta$$

(7)

$$d_{rr} = d_{v} - \delta Y_{c}$$

(8)

In Eqs. (7) and (8), an important variable $\Delta$ is proposed, it is never been mentioned before. For this reason, the RIAL used in the literatures which remains cutting entrance line unchanged, will surely leading to the bigger prediction results of $d_{rr}$ and may consume more time to reach equilibrium condition.

From Fig. 2(b), considering the law of sine, there is

$$\frac{B_{1}F_{vk}}{\sin(90^\circ - \alpha_{nx})} = \frac{B_{1}E_{k}}{\sin(\alpha + \alpha_{nx})} = \frac{\Delta}{\sin \alpha} \frac{\lambda}{\cos(\alpha + \alpha_{nx})} = \frac{d_{v} - \delta Y_{c} - \delta Y_{w}}{\sin(\alpha + \alpha_{nx})}$$

and then

$$\Delta = \frac{(d_{v} - \delta Y_{c} - \delta Y_{w}) \cos(\alpha_{nx})}{\sin(\alpha + \alpha_{nx})} \sin \lambda$$

(9)

where $\lambda = \arctan(\delta Y_{c}(F)/\delta Y_{w})/(d_{v} - H_{c}^{*})$, $H_{c}^{*}$ is the distance from the $v$th cutting position to cutting bottom point $C$ along $Z_{0}$ direction, $\delta Y_{c}(F)$ is the deflection of point $F$ correspond to $v$th actual cutting position. $\delta Y_{c}$ and $\delta Y_{w}$ can be calculated based on FE model at point $B_{k}$ at the $k$th increment segment.

As for $\Delta$ in Fig. 2(b), the following relationship can be found

$$\frac{B_{1}F_{1k}}{\sin(90^\circ - \alpha_{nx})} = \frac{B_{1}E_{k}}{\sin(\alpha + \alpha_{nx})} = \frac{\Delta}{\sin \alpha} \frac{\lambda}{\cos(\alpha + \alpha_{nx})} = \frac{d_{v} - \delta Y_{c}}{\sin(\alpha + \alpha_{nx})}$$

then we get

$$\Delta = \frac{\sin(\alpha)(d_{v} - \delta Y_{c})}{\cos(\alpha_{nx})} \cos(\alpha_{nx})$$

(10)

where

$$\lambda = \arctan(\delta Y_{c}(F)/d_{v})$$

### 5. The double iterative algorithm (DIAL)

To control the surface form errors, keeping the maximum values under the permissible is the ultimate purpose of the surface form errors prediction. DIAL is the method that aims to calculate the position and the magnitude of maximum surface form errors directly. By using DIAL, the surface form errors can be controlled without calculating the whole surface form errors, and the speed of simulating will be greatly increased.

#### 5.1. DIAL and the maximum surface form error

As shown in Fig. 1(b) and (c) for example, for the form errors, there are two possible extreme point: (1) $\delta Y_{w}^{(\max)}$ on point $E$ and (2) $\delta Y_{c}^{(\max)}$ on point C. $\delta Y_{w}^{(\max)}$ is mainly caused by the deflection of thin-wall, while $\delta Y_{c}^{(\max)}$ is mainly caused by the deflection of cutting tool. In this paper, the stiffness of cutting tool is much stronger than thin-walled workpiece. So, $\delta Y_{w}^{(\max)}$ is the maximum form error we need.

In a flexible cutting system, for the deflection of TW, $H_{en}$ is no longer a constant, the correspond point $E$ would change too. The position of point $E$ should be resolved by "double iterative algorithm" (DIAL) which includes some similar iterative process as shown in Fig. 2(a), the difference from FIAL is that DIAL only concerns about the top cutting edge $\Gamma$ as shown in Fig. 1(b). Moreover, the DIAL always assures the initial position including the actual peak point $F$ in every iterative step. As shown in Fig. 3, $F_{10}$, $F_{20}$ must be same with point $F$.

The DIAL and the method of determining the position and the magnitude of maximum surface form errors is studied as shown in Fig. 3, and the following steps are adopted to get the maximum form error using DIAL.

1. The first iterative process: Take $E_{10}F_{10}$ as the first initial iteration position, using FIAL to get the convergence position $E_{1k}F_{vk}$, get the deflection of TW, $d_{v}^{*}$ and $H_{en}^{*}$ using Eq. (11).

$$H_{en} = d_{v}^{*} - R \cos^{-1}(1 - d_{v}^{*}/R) \tan \alpha_{nx}$$

(11)

2. Find the second initial iteration position $E_{20}F_{20}$ using two straight dashed lines $E_{1k}F_{20}$ and $F_{1k}F_{20}$, $F_{20}$ is the new position of peak point $F$. 

![Fig. 3. Calculation of maximum deflection.](image-url)
(3) The second iterative process: Take $E_{2011}$ as the initial iteration position, using FIAL to get the convergence position $E_{2k}F_{k}$. 
(4) Repeat the method of step (1) to step (3) to get the $v$th convergence position $E_{vk}F_{vk}$, get $d_{v}^{*}$ and $H_{vn}^{*}$. 
(5) The iteration would be stopped when 
$$\left|\frac{d_{v}^{*} - d_{v}^{*-1}}{d_{v}^{*-1}}\right| \leq \varepsilon$$

where $\varepsilon$ is a prescribed precision. 
Once this qualification is achieved, pick out the maximum deflection $\delta_{E_{nk}}^{*}(\text{max})$ through comparing the results in DIAL process, and get corresponding surface generation point $E_{nk}$ and the corresponding $d_{v}^{*}$.

5.2. Surface form errors control

Based on the methods abovementioned, it is not necessary to calculate the errors all over the machined surface, but surface form errors are controlled by limiting the maximum surface form errors within the specified tolerance 
$$\delta_{E_{nk}}(\text{max}) < \max[|\epsilon_{e}(p)|] = \tau_{l} \quad (15)$$

where $\tau_{l}$ denotes the permissible surface form error in a certain feed step $l$.

6. Experimental verification

Tests with two rectangular plates as shown in Fig. 1(a) are selected to investigate the milling processes and the maximum form errors, in order to verify the validity of FIAL and DIAL both in prediction accuracy and efficiency. $L \times H = 105 \times 24 \text{ mm}^2$ for test 1 and $45 \times 38.1 \text{ mm}^2$ for test 2, the materials for tests 1 is Al7050-T7451, and Al7075-T6 for test 2, which are extensively used for low rigidity workpieces, e.g., jet engine compressor blades in aero-nautical and aerospace industries, vertical type machining center JOHN FORD VMC-850 is used in test 1. Workpieces are clamped on a table-type Kistler dynamometer Kistler 9255B in order to measure the cutting forces during milling. A CMM of Global Status $121510$ is used to measure the surface form errors. For test 1, the helical fluted peripheral milling tool is two-fluted Y330 with a diameter of 12 mm, helix angle is 30°. Young's moduli of the cutting tool and workpiece are 207 GPa and 70 GPa respectively. For test 2, the helical fluted peripheral milling tool is one-fluted Co-HSS with a diameter of 20 mm, helix angle is 30°. Young's moduli of the cutting tool and workpiece are 207 GPa and 70 GPa respectively. The cutting speeds for test 1 and test 2 is 2000 n/min and 244 n/min respectively. Other parameters are listed in Table 2 in detailed.

6.1. To verify FIAL

In Fig. 4(a), measured and simulated surface form errors for test 1 at feed position of $l=55 \text{ mm}$ are shown. At the beginning of milling, as the rigidity is relatively strong, the predicted and experimental results are similar to each other for both of the FIAL and RIAL models. But the iterative numbers for RIAL is larger than that of FIAL. At the fixed end of the workpiece, the protrude part indicate that the surface form errors mainly composed of the cutting tool deflection, while the maximum surface form error still occurs at 7.5 mm from the free end of the workpiece mainly composed of the deflection of workpiece. Note that iterative number are almost the same for both models in the region from the maximum surface form error position to the free end (region FEG as shown in Fig.1(b)), as the reason of the cutting tool flute loose contacting with workpiece up the line FG. As the workpiece thickness becomes smaller(test 1) at $l=100 \text{ mm}$, the maximum prediction results for the RIAL become worse both for the magnitude and the position as shown in Fig. 4(b), but the results for the FIAL model are still agree well with the experimental results. Meanwhile, it is worthy to note that surface form errors predicted by the RIAL are slightly smaller than those of FIAL. But Fig. 4(c) shows that the cutting forces simulated in RIAL are larger than FIAL. It is wondering that the larger forces produced large deflections combined with the surface form errors. Why? Well, in

### Table 2

<table>
<thead>
<tr>
<th>Test no.</th>
<th>$K_{f}$</th>
<th>$K_{b}$</th>
<th>$f_{c}$ (mm/tooth)</th>
<th>$a_{c}$ (mm)</th>
<th>$t_{0} - t_{a}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1458.062</td>
<td>0.708</td>
<td>0.05</td>
<td>2.0</td>
<td>5.2</td>
</tr>
<tr>
<td>2</td>
<td>5212.0</td>
<td>1.090</td>
<td>0.01</td>
<td>1.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

$^4 t_{0}$ and $t_{a}$ are the thickness of workpiece before and after cutting, respectively.
fact, the abnormality is caused by the ignorance of $\Delta$ or $\Delta'$ (as shown in Fig. 2(b)) in the calculating of cutting forces and deflections.

Moreover as shown in Fig. 4(c), the simulated cutting force for the FIAL model are close to the experimental results, but the RIAL model gives worse results. So, the accuracy of the present FIAL to determine the cutting forces for flexible systems is verified to be suitable for the thin-walled workpieces.

6.2. To verify DIAL

The position and the magnitude of the maximum form errors for $l=55$ mm and $l=100$ mm with test 1 are list in Tables 3 and 4. For a better rigidity situation (at the feed position $l=55$ mm), the position and magnitude of maximum surface form error predicted by the three models are all close to each other. While by rigid model, the predicted results are larger than the measured value. And as RIAL is used, we get the bigger distance value and smaller magnitude, but more accurate than the rigid model. Furthermore, it can be seen that DIAL model can always predict the maximum surface form errors in high precision no matter the rigidity of the workpiece is good or not.

An overall presentation of the surface form errors for test 2 along the feed direction are illustrated in Fig. 5(a) using FIAL. While Fig. 5(b) shows the simulation results of maximum surface form error using DIAL. The results show that the good agreement between the FIAL and the DIAL in maximum surface form error prediction.

It is shown in Fig. 5(a) and (b), as the rigidity of the thin wall reduces, the maximum surface form error intends to increase, and the distance from the position of the maximum errors to free end will be decreased. At the same time, because the number of nodes to be calculated is reduced obviously, the simulate efficiency has been highly improved. For test 2, 22 elements in axial cutting depth direction, the simulation time using ABAQUS 6.10 run on an IBM server was checked out. 766 min are required by using rigid model, while by FIAL, the time needed is 611 min, and only 307 min are required with DIAL model. So 50% of time is saved to obtain the maximum surface form error.

7. Conclusions

Through the analytical studying of in-cutting chip, a systematic simulation procedure together with two efficient flexible iterative algorithms for simulation of peripheral milling of thin-walled workpiece is developed. The proposed algorithms are able to predict the form errors before actual cutting. To ensure the reliability and the computing accuracy of the developed method, cutting force modeling in flexible TW, identification of engaged cutting tool with the workpiece, iterative corrections of radial cutting depths with correction factors, and the calculation of the actual radial cutting depth are settled. Some significant improvements have been conducted about the flexible iterative algorithm (FIAL) for form errors prediction and double iterative algorithm (DIAL) for the maximum surface form error prediction. The form errors in peripheral milling of thin-walled workpieces are studied numerically and experimentally to show the validity of the proposed procedure and algorithms. Numerical simulation results show that cutting forces and surface form errors evaluated by the developed method march well with the experimental data, and about 50% of time is saved in the prediction of maximum surface form errors.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant no. 50435020), and the Key Technology R&D Program of China (Grant no. 2011BAF13B03).

References


