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6th grade students' use of different strategies in solving ratio and proportion problems

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Abstract

The general purpose of this study was to determine the strategies used by sixth grade students in solving ratio and proportion problems. The research was carried out at three different elementary schools of Konya province in Turkey during 2009-2010 educational years. The study was conducted with 278 elementary school 6th grade students. In sample choice, random sampling method was used. General survey model was employed in order to determine students' solution strategies. In determining these strategies, an open ended test consisting of 8 items was developed by the researchers. Items were developed by following the objectives of renewed elementary mathematics curriculum. Students' answers were evaluated with rubrics which were developed for each item according to the strategies used. Data were analyzed by using descriptive analyses. The results revealed that students used six different solution strategies in solving ratio and proportion problems. In addition, they most frequently used cross multiplication algorithm strategy during the solution of these problems.

Keywords: Ratio and proportion; proportional reasoning; solution strategies; elementary education

1. Introduction

1.1. Significance of proportional reasoning

Reasoning has been defined in several ways by researchers. According to Kayhan (2005) reasoning can be defined as the process of considering all the factors and coming to a rational conclusion. As for Çüçen (1997), reasoning is defined as deducing one from another as a result of the relationship between two or more propositions. The Principles and Standards for School Mathematics (NCTM, 2000) states that reasoning is an important part of mathematics and it suggests 6th-8th grade students to sharpen and extend their their evaluations of their assertions and conjectures and using inductive and deductive reasoning to formulate mathematical arguments. Likewise, Umay (2003) reports that reasoning constitutes a basis for all the procedures and operations in mathematics. Therefore, related studies emphasize the necessity of reasoning in order for students to understand mathematics.

Mathematical reasoning both proceeds and takes its form through mathematical knowledge system. Mathematical reasoning is an essential component of learning and teaching process (Duatepe, Akkuş-Çıkla & Kayhan, 2005). Among reasoning types, proportional reasoning has an important position (Umay & Kaf, 2005). According to Lamon (2006) proportional reasoning refers to detecting, expressing, analyzing, explaining, and providing evidence in support of assertions about, proportional relationships and in colloquial terms, proportional reasoning is reasoning up and down in situations in which there exists an invariant (constant) relationship between two quantities that are linked and varying together. According to the NCTM Curriculum and Evaluation Standards (1989), ''the ability to

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reason proportionally develops in students throughout grades 5–8. It is of such great importance that it merits whatever time and effort that must be expended to assure its careful development'' (p.82). Also, some other researches related with proportional reasoning indicate that students' ability to reason proportionally is important for their own mathematical development. This kind of reasoning is fundamental to solve some daily life problems and is also essential for learning advanced mathematical topics as well as physics and chemistry (Abrantes, Serrazina & Oliveira, 1999; Post, Behr & Lesh, 1988).

The concepts of ratio and proportion are widely used in mathematics, science and everyday life (Karplus, Pulos & Stage, 1983). Reasoning with these concepts are widely regarded as a critical bridge between the numerical, concrete mathematics of arithmetic and the abstraction that follows in algebra and higher mathematics (e.g., Fuson & Abrahamson, 2005; Lamon, 2007; Post, Behr & Lesh, 1988). Therefore, ratio and proportion topics include critical reasoning skills and are related with many mathematical concepts (Akkuş & Duatepe-Paksu, 2006). Post, Behr and Lesh (1988) assert that reasoning with ratio and proportion is a pivotal concept and that it serves as the capstone of children's elementary school arithmetic and the cornerstone of all that is to follow.

1.2. Different strategies used for the solution of ratio and proportion problems

Since ratio and proportion are important mathematics topics, different mathematical problems intended to assess students' proportional reasoning ability were developed. When student responses to these problems are examined it showed that students were able to solve these problems in different ways. For this reason, this study aimed to determine the strategies used by 6^{th} grade students in solving ratio and proportion problems.

In mathematics education literature, different solution strategies are defined for ratio and proportion problems. In addition, these strategies are classified into two as correct solution strategies or erroneous solution strategies. This study, examined correct proportional reasoning strategies for the problems included in the instrument.

Six different correct strategies regarding the solution of ratio and proportion problems have been defined so far. These are as follows: cross product algorithm, unit rate strategy, factor of change strategy (Bart, Post, Behr & Lesh, 1994; Cramer & Post, 1993; Cramer, Post & Currier, 1993), equivalent fractions, equivalence class (Bart et al., 1994) and build-up strategy (Ben-Chaim, Fey, Fitzgerald, Benedetto & Miller, 1998; Parker, 1999). Each strategy will be described by using it to solve the following problem: In a bookstore, if 3 books of the same kind cost 30 dollars then find the total price of 9 books.

The cross product algorithm strategy involves setting up a proportion, forming a cross product and solving the resulting equation by division. In other words, if $a/b = c/x$, then $x = cb/a$ (Bart et al., 1994; Cramer & Post, 1993; Cramer, Post & Currier, 1993). For this problem, the proportion can be expressed as 3 books / 30 dollars = 9 books / x dollars. Thus, x = 30 dollars x 9 books / 3 books = 90 dollars.

The unit rate strategy involves computing one unit of a quantity and then multiplying the result with another quantity to generate the desired answer (Bart et al., 1994; Cramer & Post, 1993; Cramer, Post & Currier, 1993). For this problem, each book costs 10 dollars, so 9 books will cost 90 dollars, because 9 books x 10 dollar / book = 90 dollars.

The factor of change strategy involves comparing one quantity with another, determining the factor of change existing between two quantities and multiplying the factor with the value of given quantity (Bart et al., 1994; Cramer & Post, 1993; Cramer, Post & Currier, 1993). For this problem, 9 books are 3 times as many as 3 books. So the answer is 3 books times 30 dollars or 90 dollars.

The equivalent fractions strategy involves treating rate pairs as fractions by disregarding the labels and applying the multiplication rule for generating equivalent fractions. That is, the ratio with values given is multiplied by a particular fraction of the form n/n equal to one, so that the product ratio has a term equal to the desired answer (Bart et al., 1994). For this problem, the equivalent is $3/30 = 9/x$. Then a ratio which has 9 books as a numerator and is equal to the ratio 3 books / 30 dollars is sought for. The ratio 3 books / 30 dollars is multiplied by 3/3 to produce 9 books / 90 dollars. The desired answer is then 90 dollars, since the numerator is 9 books.

The equivalence class strategy involves determining the given rate pair as a fraction. Then a class of equivalent fractions are generated until the desired rate pair is identified (Bart et al., 1994). For this problem, the given rate pair

is 3 books / 30 dollars. Then the class of equivalent fractions can be written as 3/30=6/60=9/90. The desired rate pair is then 9 books / 90 dollars, as it corresponds to the fraction 9/90. The answer is thus, 90 dollars.

The build-up strategy involves beginning with the rate pair given in the problem and generating equivalent pairs until the desired pair is generated. This strategy can also be named as "making a list" and "looking for a pattern" strategy (Ben-Chaim et al., 1999; Parker, 1999). This problem can be used in the following way by using the build-up strategy; 3 books : 30 dollars, 6 books : 60 dollars , 9 books = 6 books + 3 books : 60 dollars + 30 dollars, then 9 books : 90 dollars

1.3. Statement of the problem

This study aimed to determine the strategies used by sixth grade students in solving ratio and proportion problems. Depending on this aim, the research problem was determined as "What are the strategies used by 6th grade students in solving ratio and proportion problems?"

2. Methods

2.1. Research Design

Since survey studies collect data from a group of people in order to describe some aspects or characteristics (such as abilities, opinions, attitudes, beliefs or knowledge) of the population of which that group is a part (Fraenkel &Wallen, 2005), this research was carried out by using survey method.

2.2. Sample

The sample of the study comprised of three different elementary schools of Konya province in Turkey during 2009 – 2010 educational years. 138 students (80 boys and 58 girls) from Mareşal Mustafa Kemal Elementary School (state school), 53 students (26 boys and 27 girls) from Vali Necati Çetinkaya Elementary School (state school) and 87 students (51 boys and 36 girls) from Meram Abdullah Aymaz Elementary School (private school) were chosen randomly. In total, 278 students (157 boys and 121 girls) took part in the study.

2.3. Instrument

Ratio and Proportion Test (RPT) was developed by researchers in order to determine the solution strategies used by sixth grade students. RPT consisted of 8 open ended items and these items were developed in parallel with the objectives of renewed elementary mathematics curriculum (MEB, 2008).

2.4. Data Analysis

Quantitative techniques were used in the analysis of data generated by RPT. Descriptive statistics were used in the analysis of the quantitative data. Quantitative analysis primarily consisted of frequencies and percentages. They were calculated using the SPSS 15.0 statistics programme and presented in a table.

Student responses for each item were scored via rubrics developed according to the strategies used. The highest score students could get from the RPT was 80 points as each item was scored between 0 and 10 points.

To check the internal consistency of the instrument, Cronbach alpha coefficient was calculated and was found to be 0,78. Criteria for reliability vary with the author and purpose of the test, but according to widely accepted guidelines (Doran, 1980), this value was found to be sufficient for group measurement.

"Content validity refers to the adequacy with which a measure or scale has sampled from the intended universe or domain of content." (Pallant, 2007, p.7). Relevance, balance and specificity are part of the content validity (Payne, 1968). A panel of experts examined the RPT for the qualities of relevance, balance, and specificity to establish content validity.

3. Findings

Analysis of students' correct responses showed six distinct solution strategies: cross product algorithm, factor of change, equivalent fractions, unit rate, build-up and equivalence class. The table given below depicts the distribution of strategies used for each item and for the overall test in general.

Table 1. The distribution of strategies used in solving ratio and proportion test

Strategy	Item 1	Item 2	Item 3	Item4	Item 5	Item 6	Item 7	Item 8	Total (%)
Cross product algorithm	106 38,13%	22 7,91%	24 8,63%	144 51,80%	111 39,93%	48 17,27%	57 20,50%	114 41,01%	626 28,15%
Factor of change	-	41 14,75%	135 48,56%	7 2,52%	37 13,31%	11 3,96%	66 23,74%	19 6,83%	316 14,21%
Equivalent fractions	119 42,81%	-	-	6 2,16%	42 15,11%	39 14,03%	27 9,71%	5 1,80%	238 10,70%
Unit rate	-	-	-	17 6,12%	-	99 35,61%	1 0,36%	23 8,27%	140 6,29%
Build-up	-	-	2 0,72%	21 7,55%	2 0,72%	-	-	14 5,04%	39 1,75%
Equivalence class	-	5 1,80%	-	1 0,36%	-	-	1 0,36%	6 2,16%	13 0,58%
Total correct strategies	**225 80,94%**	**68 24,46%**	**161 57,91%**	**196 70,50%**	**192 69,06%**	**197 70,86%**	**152 54,68%**	**181 65,11%**	**1372 62,0%**
Total incorrect strategies	**53 19,06%**	**210 75,54%**	**117 42,09%**	**82 29,50%**	**86 30,94%**	**81 29,14%**	**126 45,32%**	**97 34,89%**	**852 38,31%**

Item 1 was administered to students without any real world context. That is, the first item was a missing value problem aiming to determine students' procedural knowledge via the following equation: $a/b=x/d$. Equivalent fractions strategy (42,18%) and cross product algorithm (38,13%) were used in the solution of Item 1. This shows that the use of different strategies was rather limited. It is thought that the use of different strategies could be enhanced if the problem is presented in a real world context.

In Item 2, the problem was presented through a table and the students were expected to realize the pattern between numbers in order to reach a desired answer. Factor of change strategy (14,75%), cross product algorithm (7,91%), and equivalence class strategy (1,80%) were used in the solution of Item 2. These values show that students mostly used factor of change strategy in the solution of Item 2.

Item 3 aimed to determine whether students could relate ratio and scale concepts with each other. Students were intended to obtain the values via a picture. Factor of change strategy (48,56%), cross product algorithm (8,63%) and build-up strategy (0,72%) were used in the solution of Item 3. These values show that while students mostly used factor of change strategy in the solution of this item, they almost not used the build-up strategy.

Item 4 is a real-life problem aiming to determine students' capability in converting units of weight from one to another. The results obtained for this item reveals that students used all of the solution strategies existing in the mathematics education literature. In addition, students mostly used cross product algorithm (51,8%) in solving this item. But, on the other hand, only one student used equivalence class strategy (0,36%).

In Item 5, only one value is given and the students were asked to find the other two values with the help of a picture and finally reach the desired answer. Cross product algorithm (39,93%), equivalent fractions strategy (15,11%), factor of change strategy (13,31%) and build-up strategy (0,72%) were used in the solution of Item 5. The results show that students mostly used cross multiplication algorithm and almost not used the build-up strategy.

Item 6 aimed to determine whether students could relate fraction and ratio concepts with each other. The results presented for this item shows that students used unit rate strategy (35,61%), cross product algorithm (17,27%), factor of change strategy (3,96%) and equivalent fractions strategy (14,03%). When compared with other items, students highly used unit rate strategy in the solution of Item 6. This finding pointed out that students may mostly use unit rate strategy when the context of a problem is related with fractions.

Item 7 aimed to determine whether students were able to realize the relationship between similarity and ratio concepts. Students used factor of change strategy (23,74%), cross product algorithm (20,50%) equivalent fractions strategy (9,71%), unit rate strategy (0,36%) and equivalence class strategy (0,36%) in the solution of this item. The results show that students used all the strategies except for build-up strategy. Moreover, students mostly used cross multiplication algorithm and factor of change strategy nearly to the same degree. However, only one student used equivalence class strategy during the solution of this item.

Item 8 was a missing value problem presented in a map scale context. In the solution of this item, students used all the strategies existing in the literature, however, they mostly used cross product algorithm (41,01%) compared to other strategies.

When student responses for all items are taken into consideration generally, students mostly used cross product algorithm (40,37%) during the solution of the test. On the other hand, equivalence class (0,58%) was the least common strategy used by the students. When strategy uses are examined in general, it was found out that students used cross product algorithm, factor of change strategy, equivalent fractions and unit rate strategy more frequently than build-up and equivalence class strategies.

4. Conclusions and Recommendations

The research findings revealed that sixth grade students most frequently preferred to use cross product algorithm in solving ratio and proportion problems. The present data are congruent with the results of Akkuş-Çıkla and Duatepe (2002) and Kayhan (2005). However, being a proportional reasoner means more than applying the cross product algorithm since this strategy has no physical referent and therefore lacks meaning for students (Cramer, Post & Currier, 1993). Cramer and Post (1993) stated that textbooks were leading students to use cross product algorithm more frequently and thus the books were emphasizing the development of procedural skills rather than conceptual understandings. In this sense, students should be more encouraged to use multiple strategies including factor of change, unit rate, equivalent fractions, equivalence class and build-up strategy.

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