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Numerical analysis of hydromagnetic micropolar fluid along a stretching sheet embedded in porous medium with non-uniform heat source and chemical reaction

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A B S T R A C T

This paper presents the effects of non-uniform heat source and chemical reaction on the convected flow, heat and mass transfer of a micropolar fluid along a stretching sheet embedded in a porous medium in the presence of a volumetric non-uniform heat source. The generalization of the earlier studies centers round three aspects:

(i) The flow is made to pass through a porous medium characterized by a non-Darcian model affecting the momentum equation.
(ii) The presence of non-uniform heat source modifying the energy equation.
(iii) Consideration of chemically reactive species characterized by an order of chemical reaction modifying the equation of species concentration.

The governing equations of the flow have been transformed into ordinary differential equations by using similarity transformation technique and solved using the Runge-Kutta method associated with shooting technique. The numerical solutions are achieved showing the effects of pertinent parameters. For verification of the present findings the results of this study have been compared with the earlier works in particular cases; Darcian and non-Darcian fluids are discussed separately. It is worth reporting that effect of porosity of the medium combined with inertia gives rise to a transverse compression producing thinner boundary layer the solution by finite element method (FEM) and Runge–Kutta method, do agree within a reasonable error limit.

1. Introduction

In the last few decades, the theory of micropolar fluid has attracted significant attention among the engineering community due to the limitation associated with Newtonian fluids. The Newtonian fluid cannot characterize the flow in presence of suspended particles. On the other hand micropolar fluids could be able to model the fluid in presence of dust particle. The presence of micropolar fluid can be found in dumb-bell molecules, polymer fluids, fluids suspensions and animal blood. Also, micropolar fluids have significant practical applications to boundary layer and heat and mass transfer area. A large number of research papers have been available in the literature on the thermal boundary layer flows [1,2].

Pioneering work in micropolar fluids field has been carried out by Eringer [3], and explained that the micropolar fluid defined by the inertial characteristics of the substructure particles which go through rotation. Kim and Lee [4] studied the problem of the micropolar fluid over a semi-infinite vertical moving porous plate in the presence of magnetic field. In their study, they considered electrically conducting oscillatory two dimensional laminar viscous incompressible flows. Sakiadis [5] studied the behavior of laminar boundary layer on a moving flat surface. The obtained results showed very good agreement between two methods of solution. Grubka and Bobba [6] investigated the influence of surface temperature change on continuous and linearly starching surface. The formulated mathematical problem was solved by using a series solution to the energy equation based on Kummer's functions. It was found that the magnitude of the temperature

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Nomenclature

A^*, B^*	coefficients of space and temperature dependent heat source/sink
b	stretching parameter
B_o	magnetic field of constant strength
c_f	drag coefficient
C_f	local skin friction coefficient
c_p	specific heat
C	concentration of the fluid
C_w	stretching sheet concentration
C_∞	ambient concentration
Ec	Eckert number
F	inertia-coefficient
g	microrotation profile
j	reference length
K	material parameter
Kp	porous parameter
k_p^*	permeability of the porous medium
k_f	thermal conductivity
M	magnetic parameter
Nu_x	local Nusselt number
Pr	generalized Prandtl number
q'''	non-uniform heat source/sink
Re_x	local Reynolds number
s	surface condition parameter
Sc	Schmidt number
Kc	chemical reaction parameter
T	temperature of the fluid
T_w	stretching sheet temperature
T_∞	ambient temperature
u_w	characteristic velocity
u	velocity along the x-direction
v	velocity along the y-direction
x, y	coordinates
Greek symbols	
α^*	space dependent heat source/sink parameter
β^*	temperature dependent heat source/sink parameter
$\theta(\eta)$, $\phi(\eta)$	non-dimensional temperature and concentration parameter
η	similarity variable
υ	kinematic viscosity
α	thermal diffusivity
τ	stress tensor
σ	electrical conductivity of the fluid
ρ	density of the fluid
τ_{xy}	shear stress component of the stress tensor

parameter affects the direction and quantity of the heat flow. Minkowycz et al. [7] studied the effect of surface mass transfer in a porous medium using non similarity solution.

In recent years, several studies have been conducted on convective transport phenomena in a porous medium [8] due to its large practical applications such as geothermal reservoir, petroleum industry, thermal insulation engineering, soil mechanics, blood flow and artificial analysis. Instead of porous medium the flow characteristic in presence of stretching sheet has been studied by many flow conditions. Pioneering work in this area has been performed by Crane [9], who studied 2D boundary layer flow of an incompressible and viscous fluid. The velocity of the system was varied linearly from a fixed point on the sheet. Later, McCormack and Crane [10] discussed the physical characteristics of boundary layer flow over an elastic stretching sheet, where the sheet was moving along its own plane direction. Ali [11] analyzed general effect of power law surface and temperature variation on heat transfer characteristics. Ali and Magyari [12] studied the unsteady fluid and heat flow induced by a submerged stretching surface while its steady motion is slowed down gradually by using similarity analysis.

Mukhopadhyay and Layek [13] analyzed the effect of various fluid viscosity and thermal radiation on incompressible fluid over a vertical stretching porous plate. Pal [14] studied the two dimensional incompressible stagnation point flow over a stretching sheet by considering the effect of bouncy forces and thermal radiation. Abo-Eldahab and El-Aziz [15] studied electrically conducted mixed convection boundary layer flow over a continuous inclined stretching surface. Abel et al. [16] analyzed the effects of non-uniform heat source on viscoelastic fluid flow and heat transfer over a stretching sheet, while Bataller [17] reported the effect of both viscous dissipation and thermal radiation for the same fluid flow. Pal and Mondal [18] studied the effect of variable viscosity, Ohmic dissipation and non-uniform heat and source/sink on non-Dracy magnetohydrodynamics flow through a stretching sheet embedded in a porous medium. Mabood et al. [19,20] numerically studied the magnetohydrodynamics (MHD) slip flow and heat transfer problems on stretching sheet. The effect of heat and mass transfer of an electrically conducting nanofluid over a stretching sheet has been studied by Mabood [21]. Sandeep and Sulochana [22] studied the influence of heat and mass transfer of non-uniform source/sink on micropolar fluid over a stretching/shrinking sheet. The proposed model was solved by shooting technique using MATLAB package. Pal and Biswas [23] investigate the oscillatory MHD micropolar fluid in a porous medium with chemical reaction. Recently, Mabood et al. [24] investigated the effect of heat sourec/sink and Soret on MHD convective flow considering micropolar fluid with radiation.

Bhukta et al. [25] have analyzed heat and mass transfer on MHD flow of a viscoelastic fluid through porous media over a shrinking sheet. Recently, the flow characteristic of Jeffrey fluid in the boundary layer has been investigated over stretching sheet [26,27] by using Homotopy Analysis Method (HAM). The heat and mass transfer rate of boundary layer flow of different fluid could be controlled by using magnetic field. Many researchers have been studied the influence of transverse magnetic field on Newtonian and non-Newtonian fluids over a stretching sheet owing to some specific characteristic of magnetic field. Anika et al. [28,29] numerically investigate the effect of hall and ion-slip current on boundary layer flow in presence of transverse magnetic field. Rashidi et al. [30,31] studied the unsteady MHD Newtonian flow over a rotating stretching disk using HAM, Artificial Neural Network (ANN) and Particle Swarm Optimization (PSO) algorithm. Bhattacharyya [32] analyzed the effect of first order chemical reaction in boundary layer stagnant point flow over a stretching/shrinking sheet by using shooting technique.

Kumar [33] has studied the dynamics of hydromagnetic micropolar flow along a stretching sheet considering both effect of heat and mass transfer. His work was confined to the flow within the regions i.e. velocity, thermal and concentration boundary layers. It has considered that the mass transfer of species without chemical reaction. Further, the flow without porous matrix has been considered in the study of Kumar [33]. Moreover, though he has considered the viscous dissipation and Julian dissipation effect in energy equation but not considered any volumetric heat source either constant or variable strength which are of usual occurrence.

As the literature is beset with stabilizing/destabilizing effect of heat source on the flows in boundary layer, the necessity of inclusion of heat source is warranted. Add to it, the mass transfer of non-reactive species of industrial fluid is a far reaching cry, and hence the justification of accounting for the chemical reaction arises. Therefore, the present study deals with a flow model of micropolar fluid with heat source and chemical reaction. A nonlinear non-Darcy model has been applied to account for the effect of porous matrix on the flow field. Secondly, a non-uniform volume heat source has been added to the thermal boundary layer. Finally, a first order chemical reaction term has been added to concentration boundary layer to account for the effect of chemical reacting species.

In the analysis of flow through porous medium, Darcy's law usually has been assumed to be the fundamental equation [34,35]. The principle of Darcy's law is the velocity components are directly proportional to the gradient of pressure while the convective acceleration term of fluid does not exist. Hence, this law is only valid for low speed flows. The force of the fluid which is also proportional to the velocity components may be deviated from the Darcy drag force [36]. Brinkman have proposed generalization considering the convective force. To study the flow embedded by highly porous medium (such as fibers) the generalized Darcy law should be used. Hence, in the present study, generalized Darcy law has been used to account for the porosity of the medium. Moreover, as we have considered modified Darcy law due to high velocity, it is justified to account for the dissipative terms such as viscous dissipations and Julian dissipation characterized by Eckert number (Ec) which does not depend upon purely physical property but it grows in proportion to the square of the velocity. Therefore, the novelties of the present problem are laid down as follows:

(i) The momentum transport equation has been modified by inclusion of two terms i.e. $-\frac{\upsilon}{kp^*}u$ (Darcian) and $-\frac{c_f}{\sqrt{k_p^*}}u^2$ (Non-Darcian) which account for the effect of permeability of the medium on the flow phenomena.

(ii) The heat energy equation has been generalized by considering the non-uniform heat source/sink, not taken care of by Kumar [33] which affect the heat transport phenomena.

(iii) The mass transfer phenomenon due to diffusion of chemically reactive foreign species has been accounts for by considering the chemical reaction term of first order. This happens practically in engineering applications when the bounding surface of the plate is coated with evaporating material and gets heated, the foreign gasses are injected to the main flow. This causes a reduction in wall shear stress, the mass transfer conductance or the rate of heat transfer. Though Kumar [33] has considered the mass transfer aspect but restricted his discussion to non reactive species vis-à-vis disregarding the chemical reaction aspect.

Motivated by these applications the present study explores the effects of permeability of the porous medium embedding the plate, variable heat source/sink, chemical reaction modifying momentum, energy and solutal concentration equations, which primarily constitutive the flow model of any fluid.

2. Mathematical formulation

Consider an electrically conducting steady two dimensional incompressible micropolar fluid flow on a moving sheet (Fig. 1). A magnetic field of intensity B_0 is perpendicularly applied to the stretching sheet. It is assumed that the effect of both magnetic and electric field is very less while magnetic Reynolds number of the flow is small. Let the x axis represent the position of the sheet and y-axis perpendicular to it. The corresponding velocity components are u and υ along x-axis and y-axis respectively and N is micro-rotation component. Also T_w is the wall temperature of the sheet, T_∞ is temperature of the fluid far away from the sheet and T is temperature of the fluid in boundary layer. The governing equations of micropolar fluid following Kumar [33] with the appropriate boundary conditions are given by:

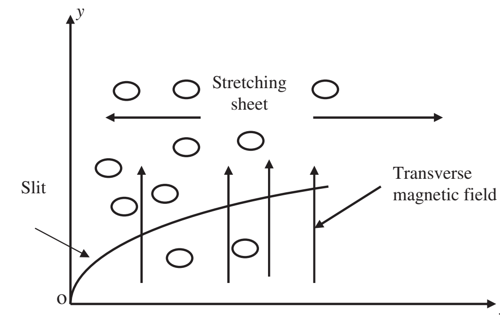

Fig. 1. A schematic representation of flow geometry.

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\left(\upsilon+\frac{k_v}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k_v}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma B_0^2}{\rho}u-\frac{\upsilon}{k_p^*}u-\frac{c_f}{\sqrt{k_p^*}}u^2 \tag{2}$$

$$u\frac{\partial N}{\partial x}+\upsilon\frac{\partial N}{\partial y}=\frac{\gamma}{j\rho}\frac{\partial^2 N}{\partial y^2}-\frac{k_v}{j\rho}\left(2N+\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\frac{k_f}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\left(\frac{\mu+k_v}{\rho c_p}\right)\left(\frac{\partial u}{\partial y}\right)^2+\frac{\rho B_0^2}{\rho c_p}u^2+\frac{q'''}{\rho c_p} \tag{4}$$

$$u\frac{\partial C}{\partial x}+\upsilon\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-Kc(C-C_\infty) \tag{5}$$

and the boundary conditions are

$$\left.\begin{array}{ll} u=u_w=bx,\ \upsilon=0,\ N=-s\frac{\partial u}{\partial y},\ T=T_w,\ C=C_w, & \text{at } y=0 \\ u=0,\ N=0,\ T=T_\infty,\ C=C_\infty, & \text{as } y\to\infty \end{array}\right\} \tag{6}$$

The fourth and fifth terms on the right hand side of Eq. (2) contribute due to non-Darcian flow over porous media. The second term in Eq. (5) represents the first order chemical reaction. Following the work of Rees and Pop [37], it is assumed that $\gamma=(\mu+\frac{k_v}{2})j$, where $j=\frac{\upsilon}{b}$ as a reference length. The non-uniform heat source/sink q''' is considered following [38]

$$q'''=\frac{\rho k u_w(\lambda)}{\chi K}\left[A^*(T_w-T_\infty)f'+(T-T_\infty)B^*\right] \tag{7}$$

Here it is worth to note that the internal heat generation/absorption corresponds to $A^*>0, B^*>0, A^*<0, B^*<0$ respectively. Following Kumar [33] the similarity transformations and dimensional variables used to transform Eqs. (2)–(5) into a set of ordinary differential equations which are given by:

$$\left.\begin{array}{l} \eta=\left(\sqrt{\frac{b}{\upsilon}}\right)y,\ u=bxf'(\eta),\ \upsilon=-\sqrt{b\upsilon}f(\eta) \\ N=\sqrt{\frac{b^3}{\upsilon}}xg(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \end{array}\right\} \tag{8}$$

In view of Eq. (6), the Eqs. (2)–(5) and the boundary conditions are reduced to

$$(1+K)f'''+ff''-(1+F)f'^2+kg'-(M+Kp)f'=0 \tag{9}$$

$$\left(1+\frac{K}{2}\right)g''+fg'-gf'-K(2g+f'')=0 \tag{10}$$

$$\theta'' + P_r f\theta' + (1+K)P_r Ecf''^2 + P_r EcMf'^2 + P_r(\alpha f' + \beta\theta) = 0 \quad (11)$$

$$\phi'' + Scf\phi' - ScKc\phi = 0 \quad (12)$$

and the boundary conditions are

$$\left.\begin{array}{l} f(0)=0, f'(0)=1, g(0)=-sf''(0), \theta(0)=1, \phi(0)=1, \\ f'(\infty)=0, g(\infty)=0, \theta(\infty)=0, \phi(\infty)=0 \end{array}\right\} \quad (13)$$

where the suffix prime denotes the order of differentiation with respect to η and the dimensionless parameter are

$$K = \frac{k_v}{\mu}, M = \frac{\sigma B_0^2}{\rho b}, P_r = \frac{\rho v c_p}{k_f}, Ec = \frac{u_w^2}{c_p(T_w - T_\infty)}, Sc = \frac{v}{D},$$
$$F = \frac{c_f x}{\sqrt{k_p^*}}, \frac{1}{Kp} = \frac{v}{bk_p^*}$$

The shear stress can be written as:

$$\tau_w = \left[(\mu + k_v)\left(\frac{\partial u}{\partial y}\right) + k_v N\right]_{y=0} = (\mu + k_v)bx\sqrt{\frac{b}{v}}f''(0) \quad (14)$$

The local skin friction coefficient C_f can be defined as

$$C_f = \frac{\tau_w}{(\rho u_w^2)} = \frac{(1+K)f''(0)}{\sqrt{\text{Re}_w}} \quad (15)$$

where $\text{Re}_w = \frac{u_w x}{v}$ is the local Reynolds number and $u_w = bx$ as a characteristic velocity.

The couple stress at the surface is defined by

$$M_w = \left(\gamma\frac{\partial N}{\partial y}\right)_{y=0} = \mu u_w\left(1 + \frac{K}{2}\right)g'(0) \quad (16)$$

The local surface heat flux $q_w(x)$, the local Nusselt number Nu_x, the local mass flux j_w and Sherwood number Sh_x are given as follows

$$q_w(x) = -k_f(T_w - T_\infty)\sqrt{\frac{b}{v}}\theta'(0) \quad (17)$$

$$Nu_x = \frac{xh(x)}{k_f} = -\sqrt{\frac{b}{v}}x\theta'(0) \Rightarrow \frac{Nu_x}{\sqrt{\text{Re}_w}} = -\theta'(0) \quad (18)$$

$$j_w = -D\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (19)$$

$$Sh_x = \frac{j_w x}{D(C_w - C_\infty)} = -\sqrt{\frac{b}{v}}x\phi'(0) \Rightarrow \frac{Sh_x}{\sqrt{\text{Re}_w}} = -\phi'(0) \quad (20)$$

3. Method of solution

In the present study, the robust Runge–Kutta method associated with shooting technique has been used to solved the couples non-linear Eqs. (9)–(12) subject to boundary conditions Eq. (13). The numerical solutions are obtained to exhibit the effects of pertinent physical parameters on the velocity, temperature and concentration distributions through Figs. 2–13 and Tables 1–5 represent the numerical computation of local skin friction, couple stress, Nusselt number and Sherwood number.

4. Results and discussions

To facilitate the working of flow problems in micropolar fluids which are essentially viscous fluids Eringen [3] presented subsequently the theory of subclass fluids which exhibit the microrotational effects and microrotational inertia and can sustain couple stress and body couples only. Physically they may represent adequately the fluids consisting of bar-like elements, and those which are made up of dumbbell shape molecules. The following discussion reveals the variation of different physical parameters which govern the flow, heat and mass transfer phenomena.

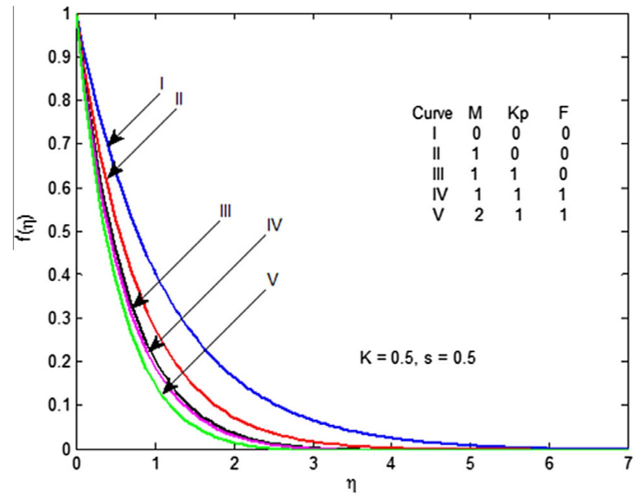

Fig. 2. Variation of M, Kp and F on velocity profile.

Table 1
Comparison of $\text{Re}_x^{1/2}C_f$ for different values of K with all other parameters are zero.

K	Qasim et al. [40]	Kumar [33]	$-f''(0)$
0	−1.000000	−1.000008	−1.000172
1	−1.367872	−1.367996	−1.367902
2	−1.621225	−1.621575	−1.621938
3	–	−1.827392	−1.827963
4	−2.004133	−2.005420	−2.007341

Table 2
Skin friction $-f''(0)$, with $s = 0.5$.

K	M	Kp	F	Kumar [33]	$-f''(0)$
0.5	0	0	0	0.8802	0.901878
0.5	1	0	0	1.2099	1.250358
0.5	1	1	0	–	1.510062
0.5	1	1	1	–	1.668701
0.5	2	1	1	–	1.874187
0	0.5	0	0	1.189	1.22559
1	0.5	0	0	0.9676	0.995088
1	0.5	1	0	–	1.265126
1	0.5	1	1	–	1.41095
2	0.5	1	1	–	1.219616

Table 3
Nusselt number, with $s = 0.5$, $Ec = 2$.

K	M	Kp	F	Kumar [33]	$\theta'(0)$ ($\alpha = \beta = 0$)	$\theta'(0)$ ($\alpha = \beta = 0.1$)
0.5	0	0	0	0.2732	0.318037	0.478903
0.5	1	0	0	1.1364	1.255576	1.432432
0.5	1	1	0	–	1.511936	1.687362
0.5	1	1	1	–	1.61693	1.791288
0.5	2	1	1	–	2.193974	2.375988
0	0.5	0	0	0.4979	0.56747	0.725812
1	0.5	0	0	0.9634	1.054947	1.23588
1	0.5	1	0	–	1.446563	1.62857
1	0.5	1	1	–	1.603956	1.78555
2	0.5	1	1	–	2.126203	2.121851

In Fig. 2, the curve II ($F = 0, Kp = 0$ and $M = 1$) flow without porous medium represents the case of Kumar [33]. It was observed that the present results make a good agreement with the previous results of Kumar [33]. Thus, the consistency of the solutions by

Table 4
Sherwood number, with $s = 0.5,\ Sc = 0.5, Kc = 0$.

K	M	Kp	F	Kumar [33]	$-\phi'(0)$
0.5	0	0	0	0.4289	0.428853
0.5	1	0	0	0.3977	0.397552
0.5	1	1	0	–	0.37855
0.5	1	1	1	–	0.372782
0.5	2	1	1	–	0.361366
0	0.5	0	0	0.4009	0.400998
1	0.5	1	0	0.4194	0.419252
1	0.5	1	0	–	0.395419
1	0.5	1	1	–	0.38802
2	0.5	1	1	–	0.423491

Table 5
Sherwood number.

$M = Kp = F = K = 1$, $s = 0.5$		
Sc	Kc	$-\phi'(0)$
0.25	0	0.312413
0.25	1	0.568529
0.5	0	0.38006
0.5	1	0.80043

Finite element method and Runge–Kutta method associated with shooting technique is established. From curves III, IV and V, it is evident that all the parameters M, Kp and F reduce the velocity distribution. This reduction of velocity due to Lorentz force, a resistive force of electromagnetic origin, M qualitatively agrees with desired result, causes a reduction along the main direction of flow. On careful observation it is remarked that interplay of electromagnetic force and the shear drag, the velocity decreases. From the above discussion it may be inferred that the additional body force due to porous medium characterized by permeability parameter and non-Darcian term F has a decelerating effect on the velocity with a significant contribution due to permeability parameter Kp. Due the fact of retard action separation will never occur and the heat transfer is never terminated, Cramer and Pai [39] as it is verified by figures for velocity and temperature distributions that the curves are asymptotic in nature.

The specialty of Fig. 3 is to bring out the effect of an important parameter $K(= k_v/\mu)$, the material parameter which is the ratio of two viscosities of the fluid under consideration i.e. dynamic viscosity and vortex viscosity. For $K = 1$ both the viscosities are of same order of magnitude. Comparing the curves I and II, it is observed that the velocity increases with an increase in material parameter K. This suggests that the vortex viscosity accelerates the fluid particles whereas the permeability of the medium and local inertia coefficient de-accelerates the flow motion. The cause of reduction of velocity is due to retarding effect of porous matrix and local inertia. The striking feature of the Fig. 4 is the back flow which is cause for high value of surface condition parameter i.e. microrotation ($s = 2$) in conjunction with retarding effect caused due to the presence of porous matrix and local inertia.

Fig. 5 exhibits the microrotational effect which characterize the micropolar fluid. It is note that the parameters M, Kp and F enhance the microrotation of the fluid under study subject to constant material properties ($s = 0.5, K = 0.5$). Thus, it concluded that the additional forces due to the presence of magnetic field, porous matrix and inertial effect enhance the microrotation. Fig. 6 illustrates the variation of material parameter and inertia coefficient in both the absence of porous matrix/presence of porous matrix. It is observed that increase in material parameter reduces the microrotation profile in both the presence/absence of porous matrix. Further inertia coefficient enhances it significantly.

Fig. 7 exhibits some interesting characteristics of microrotation component in response to surface condition parameter s. It is interesting to note that $s = 0, Kp = 0$ and $F = 0$ gives rise to a constant microrotation. For high value of $s (s = 2)$, microrotation attains higher value and asymptotically goes to zero. Now, the variation of temperature in the thermal boundary layer is depicted through the Figs. 8–11. The common characteristics of the profiles show that a sharp rise of temperature is indicated near the boundary then temperature falls gradually to attend the ambient temperature within a few layers near the plate. A sharp rise in temperature is attributed to the fact that the energy loss due to viscous dissipation and Ohmic dissipation becomes affective after a few layers near the stretching surface when shearing effect causes to affect the temperature distribution within thermal boundary layer. It is to note that all the parameters K, α, β and Ec enhance the temperature in the thermal boundary layers. Also from Fig. 8 it is remarked that an increase in permeability parameter and magnetic parameter increases the temperature significantly. Thus, it may be inferred that due to resistance offered by the porous matrix and electromagnetic force, temperature increases. It is observed that the higher shear rate i.e. higher value of $s, (s = 2)$ in the presence of source/temperature dependent parameter α and β contribute significantly to increase the temperature in all the layers. It is also

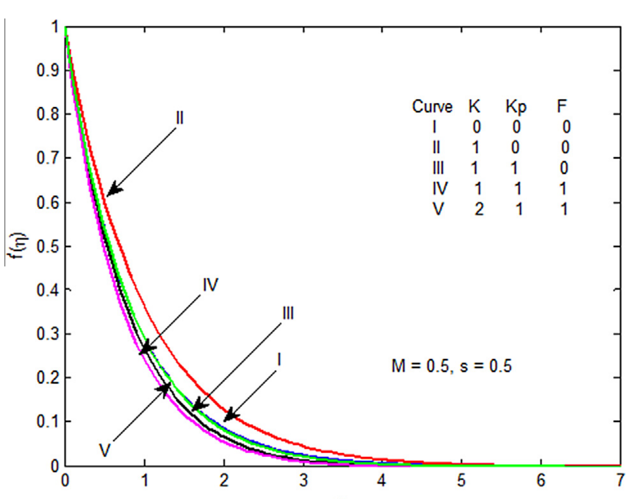

Fig. 3. Variation of K, Kp and F on velocity profile.

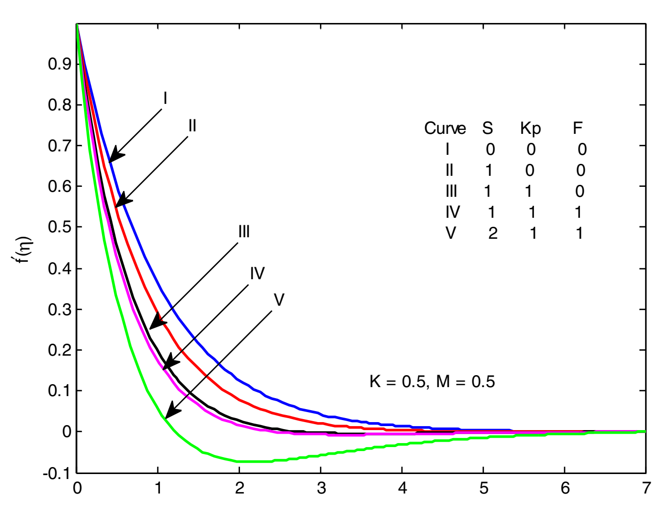

Fig. 4. Variation of s, Kp and F on velocity profile.

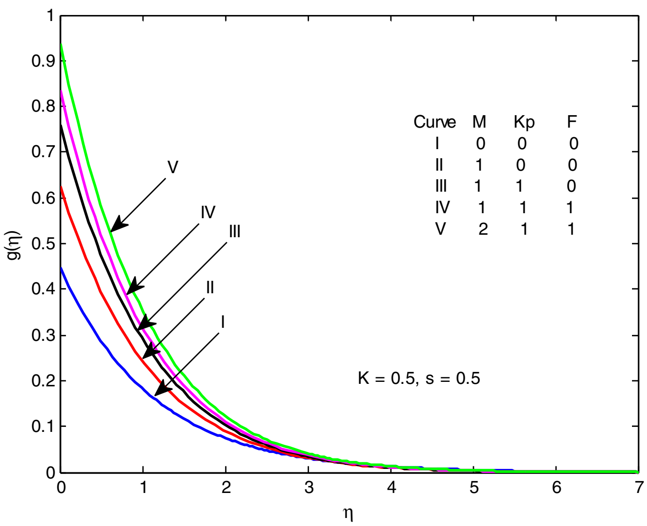

Fig. 5. Variation of *M*, *Kp* and *F* on microrotation profile.

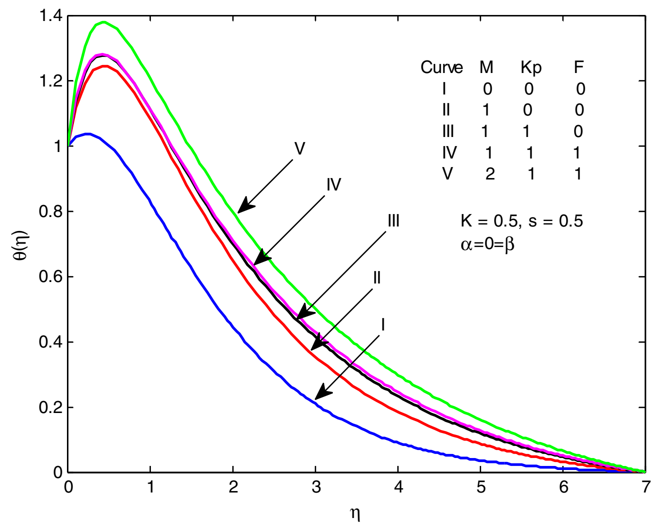

Fig. 8. Variation of *M*, *Kp* and *F* on temperature profile.

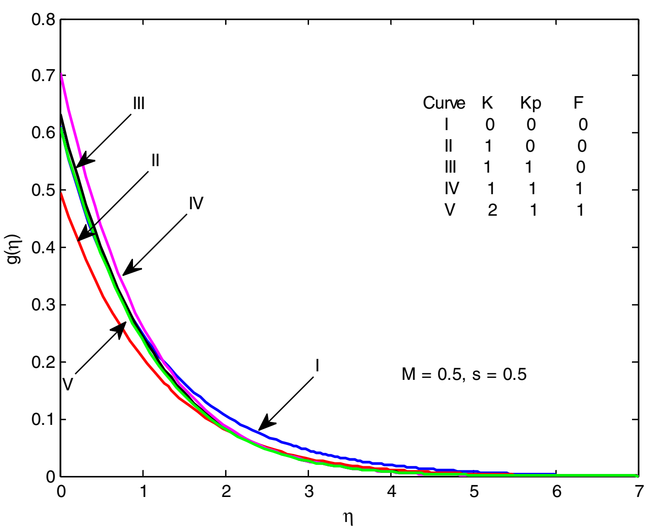

Fig. 6. Variation of *K*, *Kp* and *F* on microrotation profile.

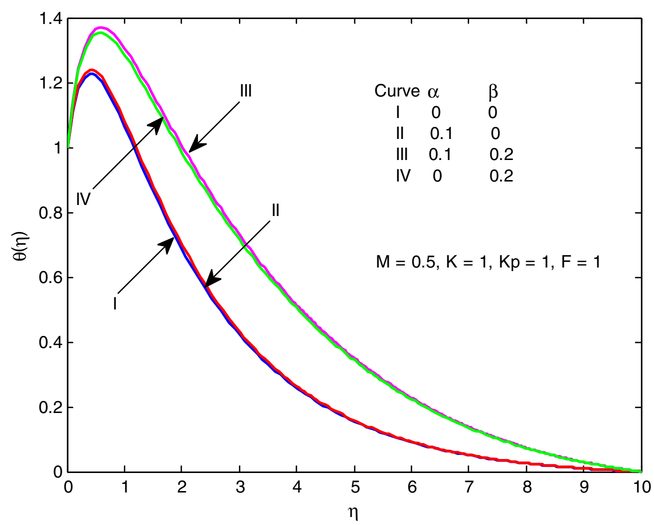

Fig. 9. Variation of on α and β on temperature profile.

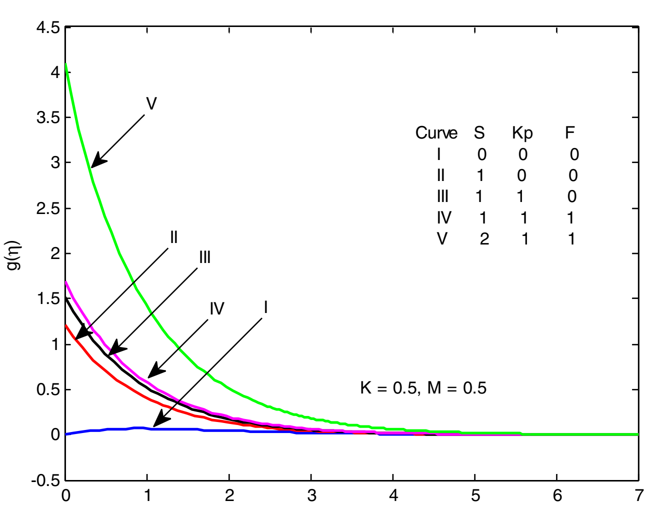

Fig. 7. Variation of *S*, *Kp* and *F* on microrotation profile.

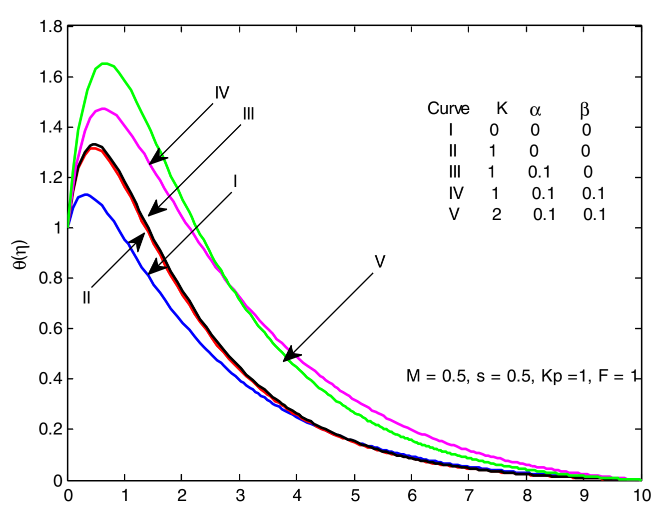

Fig. 10. Variation of *K*, α and β on temperature profile.

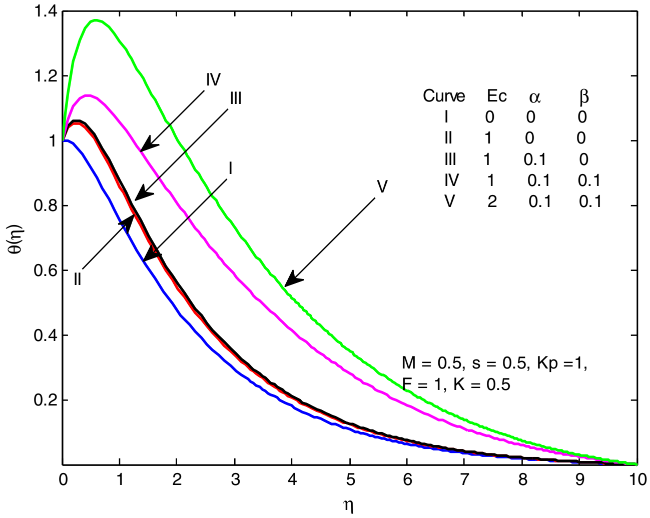

Fig. 11. Variation of *Ec*, α and β on temperature profile.

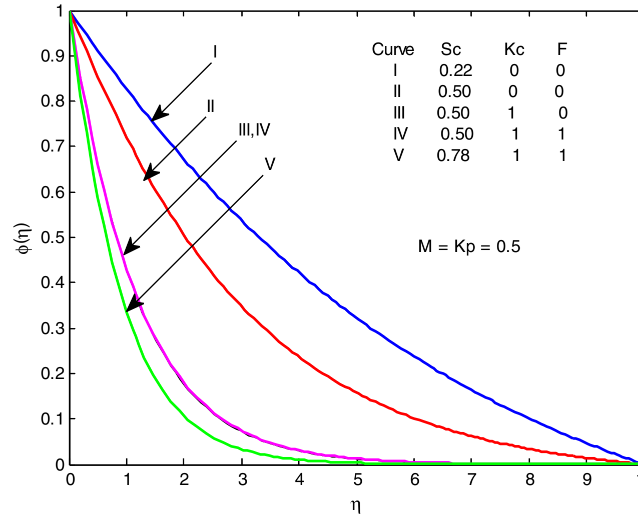

Fig. 13. Variation of *Sc*, Kc and *F* on concentration profile.

pointed out that temperature dependent heat source affecting significantly by increasing the temperature than space dependent heat source (Figs. 9 and 10). Thus, it is concluded that higher shear rate and viscous dissipation contribute to rise in temperature and hence favors the growth of thermal boundary layer. On the other hand, in the absence of source dependent and temperature dependent parameter ($\alpha = 0, \beta = 0$), the temperature variation attends the minimum (Fig. 11). It is evident from Fig. 11 that *Ec* contributes to enhance the thermal boundary-layer thickness. This is quite consistent with the present model which includes the energy loss due to dissipative heating. The loss of energy resulted in increase in temperature and hence increased the thermal boundary-layer thickness.

Figs. 12 and 13 exhibit solutal variation through concentration profiles. Comparing the profiles of the two figures the study reveals that withdrawing inertia and chemical reaction ($F = 0, Kc = 0$) this profiles are compressed, where as in their presence ($Kc = 1, F = 1.0$) (curves IV and V), profiles are wide apart. This shows, contribution of chemical reaction as well as inertia coefficient is significant in reducing the concentration distribution but on careful observation (Fig. 13, Curves III and IV), it is seen that in the presence of chemical reaction ($Kc = 1$), concentration distribution is not affected significantly (coincidence of curves III & IV), irrespective of presence or absence of inertia effect i.e.$F = 0$ or $F \neq 0$. To sum up, heavier species (higher value of *Sc*), chemical reaction coefficient and magnetic field reduce the solutal boundary layer thickness. Apart from the study of flow, heat and mass transfer phenomena inside the boundary layers i.e. velocity, thermal and solutal, the study at the boundary surface warrants attention as it plays vital role in indicating back flow, heating and cooling of the surface.

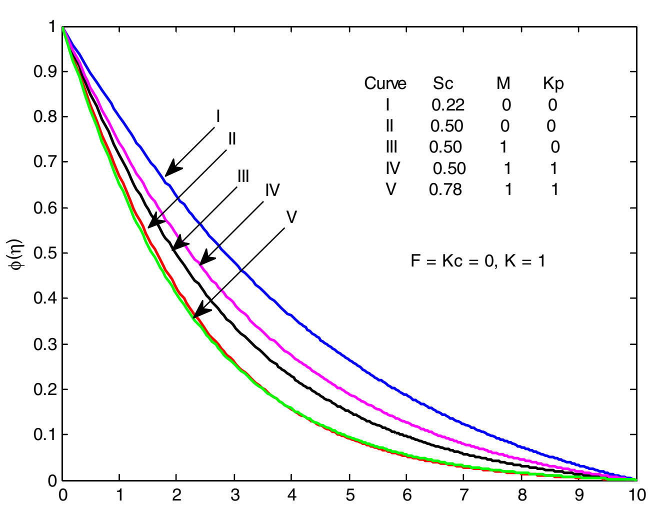

Fig. 12. Variation of *Sc*, *M* and Kp on concentration profile.

In Table 1 a comparison of $\mathrm{Re}_x^{1/2} C_f$ for different values of *K* with considering all other parameters ~0 was presented with the numerical results of Kumar [33] and Qasim et al. [40]. From Table 1 it can be seen that the current study make a good agreement with the previously published numerical results with maximum deviation of about 0.020%. The rate of shear stress is calculated and the coefficient of local skin friction for a fixed value of surface condition parameter ($s = 0.5$) for various values pertinent parameters is presented in Table 2. It is clear that the present method of solution (i.e. Runge–Kutta method) bears a consistency with finite element method which has been adopted by Kumar [33] as the error lies within 2.6% and 3.2% (Table 2). The present model investigates the effects of porous medium with the help of non-Darcian model. It is observed that the increasing value of magnetic parameter is to enhance the skin friction where as material parameter decreases it irrespective of the absence/presence of porous medium. The same observation was made by Kumar [33] in the absence of porous matrix. Therefore, it is concluded that, the presence of porous matrix failed to affect the effects of magnetic field and material property. Additionally, the effects of permeability (*Kp*) and inertial coefficient (*F*) are to increase the skin friction also. On careful observation it is revealed that presence of porous matrix, for both Darcian model: $Kp \neq 0, F = 0$ and non-Darcian model: $Kp \neq 0, F \neq 0$, the skin friction increases. Therefore, the presence of porous matrix is not desirable as it enhances the skin friction. On the other hand the micropolar fluid with higher vortex viscosity and low viscosity (i.e. increasing material parameter *K*) are favorable in reducing the skin friction.

Table 3 presents the rate of heat transfer at the surface in the presence of non-uniform heat source which is characterized by K, M, Kp and *F*. It is seen that all three parameters i.e. M, Kp and *F* enhance the rate of heat transfer at the surface there by cooling of the surface can be accelerated more effectively by increasing M, Kp and *F*. It is also seen that the numerical values of Kumar [33] for $\alpha = 0, \beta = 0$ is in good agreement with the present work for specific values of surface condition and Eckert number. It is also observed that rate of heat transfer is more for non-zero values of α and β i.e. $\alpha = \beta = 0.1$. Thus, it is concluded that presence of porous matrix accelerates the Nusselt number causing the faster cooling of the plate. Further it is revealed that error in Nusselt number is more in comparison with the skin friction (8.67–12.25%). The reason thereof may be attributed to the occurrence of square of the f''^2 and f'^2 in Eq. (10). The error is computing the slope is more than the error in computing the function. In general the higher order of derivatives the more pronounced the error Sen and Krishnamurthy [41].

Tables 4 and 5 present the rate of mass transfer at the plate. It is clear that magnetic parameter permeability parameter and inertial coefficient reduce the rate of mass transfer at the plate whereas increasing *Sc* Schmidt number (heavier species) and chemical reaction parameter enhance the solutal concentration at the plate. The additional criteria, in the present problem, are chemical reaction and permeability of the medium. It is concluded that the presence of $Kc > 0$, favors the rate solutal transfer whereas presence of porous matrix reduce it. Two additional criteria, included here, are of opposite effect on the rate of solutal concentration at the plate.

5. Conclusions

The numerical investigation has been carried out in the present study to analyze the influence of governing over a stretching sheet embedded in a porous medium in the presence of non-uniform heat source and first order chemical reaction. The governing partial differential equations are formulated into ordinary differential equations with the help of similarity transformation and variables. And being solved numerically by using Runge–Kutta method with shooting technique. We have acquired interesting observations graphically for these pertinent parameters which are summarized below:

(1) The parameter M, Kp and F reduce the velocity distribution. Physically, increase in strength of applied magnetic field over uniform magnetic field causes the reduction in the fluid motion. The porous medium characterized by permeability parameter Kp and non-Darcian term F has a decelerating effect on the velocity.
(2) The effect of porosity of the medium combined with inertia effect gives rise to a transverse compression producing a thinner boundary layer.
(3) For microrotation in the presence of inertia coefficient and porosity of the medium flow reversal occur.
(4) Higher values of surface condition parameter, porosity of the medium and inertial effect contribute to enhance the microrotation and absence of the effect of these parameters leads to linearize the variation of microrotation function. Higher share rate and viscous dissipation contribute to growth of thermal boundary layer.
(5) Heavier species, chemical reaction coefficient and magnetic field reduce the solution boundary layer.
(6) Increase in magnetic parameter, porous matrix and inertial effect increases the skin friction while it decreases with increase in material parameter.
(7) Increase in magnetic parameter, porous matrix and inertial effect Sherwood number decreases where as it is an increase with increase in material parameter, Schmidt number and chemical reaction parameter.

It is hoped that present investigation will provide an useful information for many scientific and industrial applications and also serve as a complement to the previous studies.

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