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Tidal Love and Shida numbers estimated by geodetic VLBI

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ABSTRACT

Frequency-dependent Love and Shida numbers, which characterize the Earth response to the tidal forces, were estimated in a global adjustment of all suitable geodetic Very Long Baseline Interferometry (VLBI) sessions from 1984.0 to 2011.0. Several solutions were carried out to determine the Love and Shida numbers for the tidal constituents at periods in the diurnal band and in the long-period band in addition to values of the Love and Shida numbers common for all tides of degree two. Adding up all twelve diurnal tidal waves that were estimated, the total differences in displacement with respect to the theoretical conventional values of the Love and Shida numbers calculated from an Earth model reach 1.73 ± 0.29 mm in radial direction and 1.15 ± 0.15 mm in the transverse plane. The difference in the radial deformation following from the estimates of the zonal Love numbers is largest for the semi-annual tide S_{sa} with 1.07 ± 0.19 mm.

1. Introduction

Tidal forces induced by the gravitational attraction of the Moon and Sun cause a periodic deformation of the Earth, called Earth tides. The total displacement of the solid Earth's surface can reach up to 40 cm in radial direction during one day. To meet today's goal of 1 mm position accuracy with global space geodetic techniques (Plag and Pearlman, 2009), it is essential to use a priori models in the data analysis that describe the Earth's deformation by one order of magnitude better, i.e., with an accuracy of 0.1 mm. On the other hand, the ever increasing precision of the space geodetic techniques allows accessing the theoretically computed parameters of the models and providing their validation from observations. These parameters, the Love and Shida numbers (h and l), are proportionality factors describing the Earth's anelastic response to the tidal potential. The recommended representation of the Love and Shida numbers (Mathews et al., 1995, 1997), adopted by the recent Conventions 2010 of the International Earth Rotation and Reference Systems Service (IERS) (Petit and Luzum, 2010), includes multiple complex Love and Shida parameters which depend on the frequency of the tidal constituents.

The first estimates of Love numbers from Very Long Baseline Interferometry (VLBI) measurements were provided by Herring et al. (1983) and Ryan et al. (1986) followed, e.g. by Mitrovica et al. (1994), Haas and Schuh (1996) and Petrov (2000). Determination of Love numbers using satellite laser ranging was done, e.g. by Wu et al. (2001) and a recent estimation of station displacement residuals on tidal frequencies in a regional network from global positioning system was published by Yuan and Chao (2012). The motivation for our paper was to use the complete geodetic VLBI data set that exists, spanning 27 years (1984.0–2011.0), which is significantly longer than in previous studies. In our analysis we determine the diurnal and long-period Love and Shida numbers within global adjustments following the latest recommendations on the geodetic VLBI data analysis.

2. Theory

In the classical theory of a spherical, non-rotating, elastic and isotropic Earth model, (e.g. Melchior, 1978), the Love and Shida numbers are dependent on the degree n of the tidal potential V^t. The displacement vector Δd in the local coordinate system (radial $\hat{r}$, east $\hat{e}$, north $\hat{n}$) induced by the tidal potential is expressed as:

$$\Delta d = \frac{1}{g}\sum_{n=2}^{\infty} h_n \cdot V^t \quad \hat{r} + \frac{1}{g\cos\Phi}\sum_{n=2}^{\infty} l_n \cdot \frac{\partial V^t}{\partial \Lambda}\hat{e} + \frac{1}{g}\sum_{n=2}^{\infty} l_n \cdot \frac{\partial V^t}{\partial \Phi}\hat{n}, \qquad (1)$$

where Φ and Λ are geocentric latitude and longitude of the station and g is the gravitational acceleration. Only the tidal potential of the second and third degree generated by the Moon and of

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second degree generated by the Sun is taken into account when the displacement of the Earth surface caused by the tidal forces is calculated to achieve sub-millimeter accuracy (Petit and Luzum, 2010).

The recent theory of solid Earth tidal displacements is based upon the model of Wahr (1981) who considered the effects of rotation and ellipticity of the Earth. The deformation of the Earth's surface caused by lunisolar tides is based on the sum of the tidal potential with spherical harmonic degrees n and orders m, where the effective values of Love and Shida numbers additionally depend on the frequency of the tidal wave. Whereas Wahr (1981) used older Earth models such as 1066A (Gilbert and Dziewonski, 1975) for the resonance expansions of the Love and Shida numbers, Mathews et al. (1995) based their computations on the modified PREM (Dziewonski and Anderson, 1981), where one of the changes is the increase of the dynamical ellipticity of the fluid outer core by about 4.5%. As a consequence, the period of Free Core Nutation (FCN) decreases from 460 days to about 430 days which is consistent with the values from VLBI estimates, see, e.g. Roosbeek et al. (1999), Mathews et al. (2002) and Krásná et al. (2013b) and also with the results from gravity observations, e.g. Ducarme et al. (2007) and Rosat et al. (2009). This free rotational mode of the Earth, also known as Nearly Diurnal Free Wobble (NDFW) in the terrestrial reference frame, causes a strong resonance behavior of the Love and Shida numbers in the diurnal band.

The computational procedure for the calculation of the station displacements due to solid Earth tides recommended by the recent IERS Conventions 2010 (Petit and Luzum, 2010) was suggested by Mathews et al. (1997) and consists of two steps. In the first step the displacement with frequency-independent nominal values of the Love and Shida numbers for the second and third degree of the tidal potential is computed. The vector displacement is expressed in the time domain in terms of the station location and the instantaneous position of the Moon and the Sun obtainable from ephemeris. In our analysis we use the Development Ephemeris DE421 provided by Jet Propulsion Laboratory (Folkner et al., 2009). In the second step corrections to the results of the first step are computed in the frequency domain. They take account of the frequency-dependent deviations of the Love and Shida numbers and also of the variations arising from mantle anelasticity. The number of spectral components for which such computations have to be done may be minimized by appropriate choice of nominal values used in the first step (see Mathews et al., 1997).

Accounting for the frequency dependence of the Love and Shida numbers in the diurnal band, the respective terms can be separated from the general equation for diurnal tides given in Mathews et al. (1997, Eq. (3)). The frequency-dependent corrections δd_f to the displacement vector can then be written as:

$$\delta d_f = -3\sqrt{\frac{5}{24\pi}} H_f \left\{ \delta h_f \frac{1}{2} \sin 2\Phi \sin(\theta_f + \Lambda)\hat{r} + \delta l_f \sin\Phi \cos(\theta_f + \Lambda)\hat{e} + \delta l_f \cos 2\Phi \sin(\theta_f + \Lambda)\hat{n} \right\}. \quad (2)$$

H_f is the amplitude of a tidal term of frequency f defined by the convention of Cartwright and Tayler (1971), θ_f is the argument for the tidal constituent with the frequency f, and δh_f and δl_f are the corrections to the Love and Shida numbers of degree two.

In the long-period band the frequency dependence is mainly due to mantle anelasticity. The frequency-dependent correction of the displacement caused by the long-period tides follows from Mathews et al. (1997, Eq. (2)):

$$\delta d_f = \sqrt{\frac{5}{4\pi}} H_f \left\{ \left(\frac{3}{2}\sin^2\Phi - \frac{1}{2}\right)(\delta h_f \cos\theta_f)\hat{r} + \frac{3}{2}\sin 2\Phi(\delta l_f \cos\theta_f)\hat{n} \right\}. \quad (3)$$

The generalization to a complex form of Eqs. (2) and (3) is done by the following replacement, where L is a generic symbol for the Love and Shida numbers and m equals to 1 for diurnal tides and to 0 for long-period tides:

$$L\cos(\theta_f + m\Lambda) = L^R\cos(\theta_f + m\Lambda) - L^I\sin(\theta_f + m\Lambda), \quad (4)$$

$$L\sin(\theta_f + m\Lambda) = L^R\sin(\theta_f + m\Lambda) + L^I\cos(\theta_f + m\Lambda). \quad (5)$$

3. VLBI analysis

VLBI is a space geodetic technique based on the simultaneous observation of microwave signals from extragalactic radio sources at at least two Earth-based antennas. The difference between the arrival times of the signal at both stations, i.e. the group delay τ, is the primary observable of the VLBI technique (Sovers et al., 1998; Schuh and Böhm, 2013). The measured time delay reflects any changes in the positions of the antennas and enables herewith the access to the parameters contained in the a priori models, such as Love and Shida numbers. To access these parameters, the partial derivative of the baseline b between the two stations with respect to those numbers L in the VLBI basic observation equation has to be formulated:

$$\begin{aligned} \frac{\partial\tau}{\partial L} &= k \cdot QRW \cdot \frac{\partial b}{\partial L} \\ &= k \cdot QRW \cdot \left(\frac{\partial x_2}{\partial L} - \frac{\partial x_1}{\partial L}\right), \end{aligned} \quad (6)$$

where k is a unit vector in the direction of the radio source, defined in the barycentric celestial reference system, Q, R and W are the transformation matrices between the celestial and terrestrial reference frame, and x_1, x_2 are the position vectors of station 1 and 2 in the terrestrial reference system.

We used the Vienna VLBI Software VieVS (Böhm et al., 2012) to analyze 4.6 million observations from 1984.0 to 2011.0 included in 3360 24-h sessions of the International VLBI Service for Geodesy and Astrometry (IVS; (Schuh and Behrend, 2012)). These sessions fulfil two criteria: (1) they involve more than two stations, and (2) the a posteriori sigma of unit weight obtained from a single-session adjustment does not exceed the value of 2. For the modeling of the theoretical time delays the IERS Conventions 2010 (Petit and Luzum, 2010) are followed, with the exception of applying a priori corrections on station coordinates due to non-tidal atmospheric loading (Petrov and Boy, 2004) which is a common procedure in the VLBI analysis. We apply ocean loading corrections (provided by M.S. Bos and H.-G. Scherneck, http://holt.oso.chalmers.se/loading/) based on the ocean tide model FES2004 (Letellier, 2004) in our standard solution. For each session the normal equation (NEQ) system is formulated including the station coordinates and velocities, source coordinates, Earth Orientation Parameters (EOP), zenith wet delays, tropospheric gradients, clock parameters, and the Love and Shida numbers. In the module Vie_GLOB (Krásná et al., 2013a) of VieVS a common adjustment of all sessions is carried out after local parameters (connected only to a single session) are reduced from the normal equations per session in a first step. The latter parameters are the EOP, zenith wet delays, tropospheric gradients, and clock parameters. The number of parameters included in the NEQ that enters the global adjustment then decreases significantly and

Table 1
Complex Love numbers for twelve diurnal waves. The second column shows the recommended theoretical values from Mathews et al. (1997), the third and fourth columns contain real and imaginary parts of the diurnal Love numbers estimated in this work. The respective changes in the displacement amplitudes are given in the fifth and sixth columns.

Tidal wave	h_f (Mathews et al., 1997)	h_f^R this work	h_f^I this work	$\Delta\delta R_f^{ip}$ [mm]	$\Delta\delta R_f^{op}$ [mm]
135.655 (Q_1)	$0.6036 - 0.0026i$	0.6147 ± 0.0043	-0.0087 ± 0.0043	0.22 ± 0.08	-0.12 ± 0.08
145.555 (O_1)	$0.6028 - 0.0025i$	0.6026 ± 0.0009	-0.0013 ± 0.0008	0.00 ± 0.09	0.11 ± 0.08
155.655 (M_1)	$0.6004 - 0.0023i$	0.5888 ± 0.0101	-0.0084 ± 0.0101	0.09 ± 0.08	0.05 ± 0.08
162.556 (π_1)	$0.5878 - 0.0015i$	0.5083 ± 0.0289	-0.0321 ± 0.0290	-0.22 ± 0.08	-0.08 ± 0.08
163.555 (P_1)	$0.5817 - 0.0011i$	0.5816 ± 0.0017	0.0037 ± 0.0017	-0.03 ± 0.08	0.26 ± 0.08
165.555 (K_1)	$0.5236 + 0.0030i$	0.5267 ± 0.0007	0.0041 ± 0.0007	-0.08 ± 0.10	-0.56 ± 0.10
165.565 (K_1')	$0.5182 + 0.0036i$	0.5294 ± 0.0043	0.0223 ± 0.0043	-0.16 ± 0.08	-0.42 ± 0.08
166.554 (ψ_1)	$1.0569 + 0.0036i$	1.1224 ± 0.0704	0.3291 ± 0.0704	-0.09 ± 0.08	-0.36 ± 0.08
167.555 (ϕ_1)	$0.6645 - 0.0059i$	0.7707 ± 0.0392	0.0007 ± 0.0392	-0.22 ± 0.08	-0.01 ± 0.08
173.655 (θ_1)	$0.6117 - 0.0030i$	0.8093 ± 0.0515	0.1562 ± 0.0515	-0.30 ± 0.08	-0.24 ± 0.08
175.455 (J_1)	$0.6108 - 0.0030i$	0.5988 ± 0.0098	-0.0194 ± 0.0098	0.09 ± 0.08	0.13 ± 0.08
185.555 (OO_1)	$0.6080 - 0.0028i$	0.6594 ± 0.0176	-0.0182 ± 0.0176	-0.23 ± 0.08	0.07 ± 0.08

allows easy matrix manipulation in terms of the computer memory capacity. The NEQ system of the global solution contains only the station coordinates, station velocities, source coordinates, and the Love and Shida parameters. In this work the period of FCN was fixed to its recommended value of −431.39 sidereal days (Mathews et al., 2002) which is only slightly different from our estimated value of −431.18 ± 0.10 sidereal days obtained in a global VLBI solution (see Krásná et al. (2013b)).

3.1. Love and Shida numbers for the diurnal tides

The complex frequency-dependent Love and Shida numbers for twelve diurnal tides were estimated together with the nominal values for the Love and Shida numbers of degree two. In addition to the three strongest diurnal waves (K_1, O_1, P_1), four waves (Q_1, M_1, π_1, K_1') with a lower frequency than the resonance frequency of the NDFW and five waves (ψ_1, ϕ_1, θ_1, J_1, OO_1) with a higher frequency were included. The complex Love and Shida numbers by Mathews et al. (1997) are listed in the second column of Tables 1 and 2. The estimates of the real parts obtained within our work are summarized in the third column and the imaginary parts in the fourth column. To get an impression how the computed displacement changes when using the estimated parameters instead of the conventional values, the differences in the radial δR_f and transverse δT_f amplitudes are computed according to Eqs. (7) and (8). They follow from Eq. (2) and include both in-phase (*ip*) and out-of-phase (*op*) parts. Columns five and six of Tables 1 and 2 give the differences expressed in millimeters.

$$\begin{pmatrix} \delta R_f^{ip} \\ \delta R_f^{op} \end{pmatrix} = -\frac{3}{2}\sqrt{\frac{5}{24\pi}}H_f \begin{pmatrix} \delta h_f^R \\ \delta h_f^I \end{pmatrix}, \qquad (7)$$

$$\begin{pmatrix} \delta T_f^{ip} \\ \delta T_f^{op} \end{pmatrix} = -3\sqrt{\frac{5}{24\pi}}H_f \begin{pmatrix} \delta l_f^R \\ \delta l_f^I \end{pmatrix}. \qquad (8)$$

It is evident that our estimates of Love and Shida numbers are close to their theoretical values. A slightly larger deviation from its theoretical value is found for the wave θ_1 with a difference of 0.197 ± 0.052 in the real part and 0.159 ± 0.052 in the imaginary part of the Love number. The estimated Shida number for this tide differs by 0.051 ± 0.013 in the real part, whereas the imaginary part does not show any significant deviation from the theoretical model (difference 0.003 ± 0.013). The large formal error of the estimate for this tide may have been caused by the small amplitude, which obstructs an accurate estimation. The resulting differences in $\delta R_{\theta_1}^{ip}$, $\delta R_{\theta_1}^{op}$, $\delta T_{\theta_1}^{ip}$, and $\delta T_{\theta_1}^{op}$ are in the sub-millimeter range: −0.30 ± 0.08 mm, −0.24 ± 0.08 mm, −0.15 ± 0.04 mm, and −0.01 ± 0.04 mm respectively. The total difference with respect to the theoretical displacement summed over the absolute value of the twelve diurnal waves reaches 1.73 ± 0.29 mm in vertical direction and 1.15 ± 0.15 mm in horizontal direction.

In Figs. 1 and 2 the solid black line represents the theoretical values from Mathews et al. (1997), and in red we show the complex Love and Shida numbers from our standard solution where we applied ocean loading corrections from the ocean tide model FES2004 (Letellier, 2004). For comparison, a second solution (in light blue) is computed with the AG06a (Andersen, 2006) ocean tide model for the correction of ocean tidal loading. The difference in the station displacement in the height component between these two models can vary up to a few millimeters. The model FES2004 is a numerical hydrodynamic model with assimilated altimetry data from TOPEX/Poseidon with a 0.125° resolution. The model

Table 2
Complex Shida numbers for twelve diurnal waves. The second column shows the recommended theoretical values from Mathews et al. (1997), the third and fourth columns contain real and imaginary parts of the diurnal Shida numbers estimated in this work. The respective changes in the displacement amplitudes are given in the fifth and sixth columns.

Tidal wave	l_f (Mathews et al., 1997)	l_f^R this work	l_f^I this work	$\Delta\delta T_f^{ip}$ [mm]	$\Delta\delta T_f^{op}$ [mm]
135.655 (Q_1)	$0.0846 - 0.0006i$	0.0870 ± 0.0010	-0.0027 ± 0.0010	0.09 ± 0.04	-0.08 ± 0.04
145.555 (O_1)	$0.0846 - 0.0006i$	0.0858 ± 0.0002	-0.0006 ± 0.0002	0.20 ± 0.04	0.02 ± 0.04
155.655 (M_1)	$0.0849 - 0.0006i$	0.0815 ± 0.0025	-0.0040 ± 0.0025	0.05 ± 0.04	0.05 ± 0.04
162.556 (π_1)	$0.0853 - 0.0006i$	0.0827 ± 0.0072	-0.0028 ± 0.0072	-0.01 ± 0.04	-0.01 ± 0.04
163.555 (P_1)	$0.0853 - 0.0006i$	0.0864 ± 0.0004	-0.0009 ± 0.0004	0.08 ± 0.04	-0.02 ± 0.04
165.555 (K_1)	$0.0870 - 0.0006i$	0.0881 ± 0.0003	-0.0008 ± 0.0003	-0.27 ± 0.08	0.02 ± 0.08
165.565 (K_1')	$0.0872 - 0.0006i$	0.0912 ± 0.0011	0.0027 ± 0.0011	-0.15 ± 0.04	-0.13 ± 0.04
166.554 (ψ_1)	$0.0710 - 0.0020i$	0.0832 ± 0.0175	0.0409 ± 0.0175	-0.03 ± 0.04	-0.10 ± 0.04
167.555 (ϕ_1)	$0.0828 - 0.0007i$	0.1052 ± 0.0098	-0.0273 ± 0.0098	-0.09 ± 0.04	0.11 ± 0.04
173.655 (θ_1)	$0.0845 - 0.0006i$	0.1352 ± 0.0129	0.0026 ± 0.0129	-0.15 ± 0.04	-0.01 ± 0.04
175.455 (J_1)	$0.0845 - 0.0006i$	0.0833 ± 0.0025	0.0043 ± 0.0025	0.02 ± 0.04	-0.08 ± 0.04
185.555 (OO_1)	$0.0846 - 0.0006i$	0.0856 ± 0.0045	-0.0050 ± 0.0044	-0.01 ± 0.04	0.04 ± 0.04

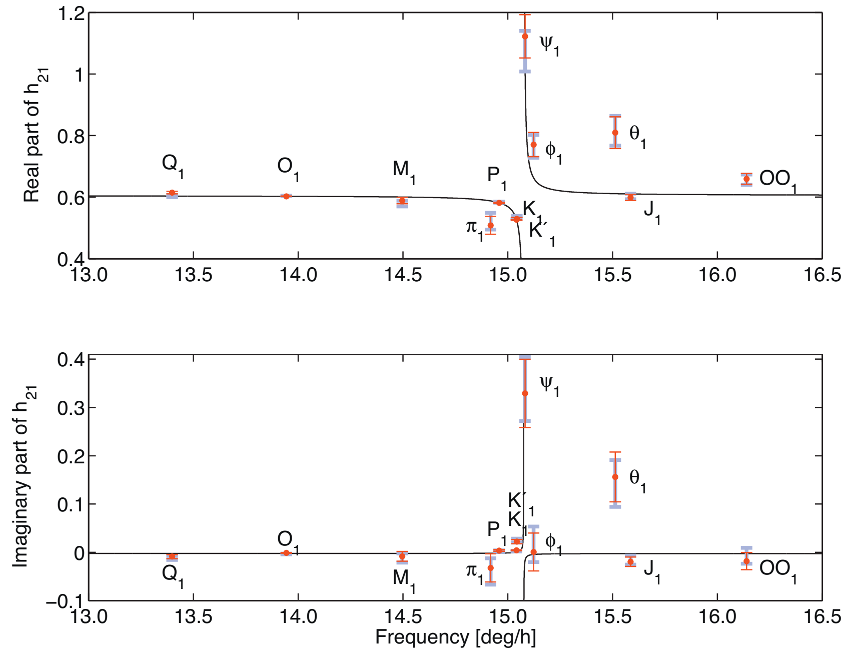

Fig. 1. Real and imaginary part of the Love numbers for twelve diurnal waves estimated from the "FES2004 solution" in red and from the "AG06a solution" in light blue. The black line denotes the theoretical values from Mathews et al. (1997).

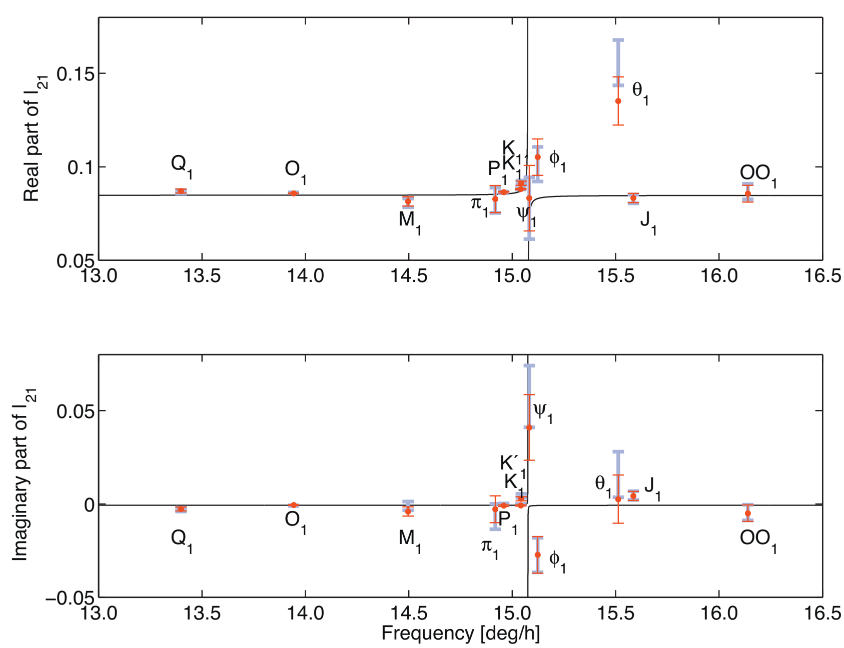

Fig. 2. Real and imaginary part of the Shida numbers for twelve diurnal waves estimated from the "FES2004 solution" in red and from the "AG06a solution" in light blue. The black line denotes the theoretical values from Mathews et al. (1997).

Table 3

Complex Love numbers for six long-period waves. The second column shows the recommended theoretical values from Mathews et al. (1997), the third and fourth columns contain real and imaginary parts of the long-period Love numbers estimated in this work (solution S3). The respective changes in the displacement amplitudes are given in the fifth and sixth columns.

Tidal wave	h_f (Mathews et al., 1997)	h_f^R this work	h_f^I this work	$\Delta\delta R_f^{ip}$ [mm]	$\Delta\delta R_f^{op}$ [mm]
55.565 (Ω_1)	$0.6344 - 0.0093i$	0.6372 ± 0.0199	-0.2710 ± 0.0154	0.05 ± 0.35	4.61 ± 0.27
56.554 (S_a)	$0.6207 - 0.0060i$	0.5708 ± 0.0612	-0.5137 ± 0.0583	0.15 ± 0.19	-1.57 ± 0.18
57.555 (S_{sa})	$0.6182 - 0.0054i$	0.5635 ± 0.0095	-0.0145 ± 0.0093	1.07 ± 0.19	-0.18 ± 0.18
65.455 (M_m)	$0.6126 - 0.0041i$	0.5905 ± 0.0079	0.0357 ± 0.0083	0.49 ± 0.18	0.88 ± 0.18
75.555 (M_f)	$0.6109 - 0.0037i$	0.6049 ± 0.0043	0.0137 ± 0.0044	0.25 ± 0.18	0.73 ± 0.19
75.565 (M_f')	$0.6109 - 0.0037i$	0.5893 ± 0.0105	-0.0090 ± 0.0107	0.38 ± 0.18	-0.09 ± 0.19

Table 4
Complex Shida numbers for six long-period waves. The second column shows the recommended theoretical values from Mathews et al. (1997), the third and fourth columns contain real and imaginary parts of the long-period Shida numbers estimated in this work (solution S3). The respective changes in the displacement amplitudes are given in the fifth and sixth columns.

Tidal wave	l_f (Mathews et al., 1997)	l_f^R this work	l_f^I this work	$\Delta\delta T_f^{ip}$ [mm]	$\Delta\delta T_f^{op}$ [mm]
55.565 (Ω_1)	$0.0936 - 0.0028i$	0.1078 ± 0.0047	-0.0603 ± 0.0035	0.37 ± 0.12	1.52 ± 0.09
56.554 (S_a)	$0.0894 - 0.0018i$	0.1079 ± 0.0146	-0.1562 ± 0.0139	-0.09 ± 0.07	-0.72 ± 0.06
57.555 (S_{sa})	$0.0886 - 0.0016i$	0.0984 ± 0.0023	0.0057 ± 0.0022	-0.28 ± 0.07	0.21 ± 0.06
65.455 (M_m)	$0.0870 - 0.0012i$	0.0825 ± 0.0019	0.0027 ± 0.0020	0.15 ± 0.06	0.13 ± 0.07
75.555 (M_f)	$0.0864 - 0.0011i$	0.0864 ± 0.0010	0.0018 ± 0.0011	0.01 ± 0.06	0.18 ± 0.07
75.565 (M_f')	$0.0864 - 0.0011i$	0.0772 ± 0.0025	-0.0064 ± 0.0025	0.24 ± 0.07	-0.14 ± 0.07

AG06a is based on the theoretical model FES94.1 (Le Provost et al., 1994) but is adjusted to multi-mission altimetry measurements (TOPEX/Poseidon, Jason-1, ERS/ENVISAT, GFO) (Andersen, 2006). The ocean tidal loading is modeled at four diurnal waves: K_1, O_1, P_1, and Q_1. Therefore a change in the estimated Love and Shida numbers is primarily expected for these tides. That is confirmed by the values visible in Fig. 1 where the largest differences expressed in $\Delta\delta R_f^{ip}$ can be found for waves K_1 and Q_1 with -0.26 ± 0.14 mm and -0.21 ± 0.11 mm, respectively, in the sense “AG06a – FES2004”. The corresponding differences for the waves O_1 and P_1 are within their formal errors. Similar results are noticed by the imaginary parts and horizontal displacement, where most of the estimated differences are smaller than their formal errors. From this investigation it is evident that the choice of an a priori tide model for ocean loading corrections to station coordinates causes a difference in the $\Delta\delta R_f^{ip}$ for the strongest tides at the sub-millimeter level. Please note that as discussed, e.g., by Ryan et al. (1993) and Gipson and Ma (1998) the formal errors which are an outcome of the VLBI data analysis should be scaled by a factor of 1.5 to obtain realistic values for the determined parameters. The a posteriori variances of unit weight are 0.79 for both solutions. They are computed as weighted residuals divided by the degree of freedom of the adjustment. As weights we used the squared reciprocal values of the a priori uncertainty of the observations increased quadratically by 1 cm. In our analysis we do not apply any further re-weighting approach.

The estimated values for the degree two Love and Shida numbers are identical in both solutions, and their values are 0.6072 ± 0.0003 and 0.0843 ± 0.0000, respectively.

3.2. Love and Shida numbers for the long-period tides

To ensure an accuracy of 0.05 mm for the computed radial displacements of the crust in the long-period band, five tidal waves (M_f', M_f, M_m, S_{sa}, and Ω_1) have to be taken into account (Petit and Luzum, 2010). Three solutions for the estimation of the zonal Love and Shida numbers are performed. In the first solution S1 the default parametrization is applied and Love and Shida numbers for the five main zonal tidal waves are estimated. In the second solution S2 hydrology loading corrections were applied a priori to the station coordinates. The hydrology loading is provided by the NASA GSFC VLBI group (Eriksson and MacMillan; http://lacerta.gsfc.nasa.gov/hydlo) as monthly time-series in a local coordinate system for the VLBI sites. These corrections mainly contain annual and semi-annual signals. Solution S3 is identical to solution S2, but the Love and Shida numbers for the annual tidal wave S_a are estimated additionally. The corresponding results are summarized in Tables 3 and 4.

The second column of each table contains the theoretical complex Love and Shida numbers (Mathews et al., 1997; Petit and Luzum, 2010). Columns three and four list the real and the imaginary parts of the estimated Love and Shida numbers, and the last

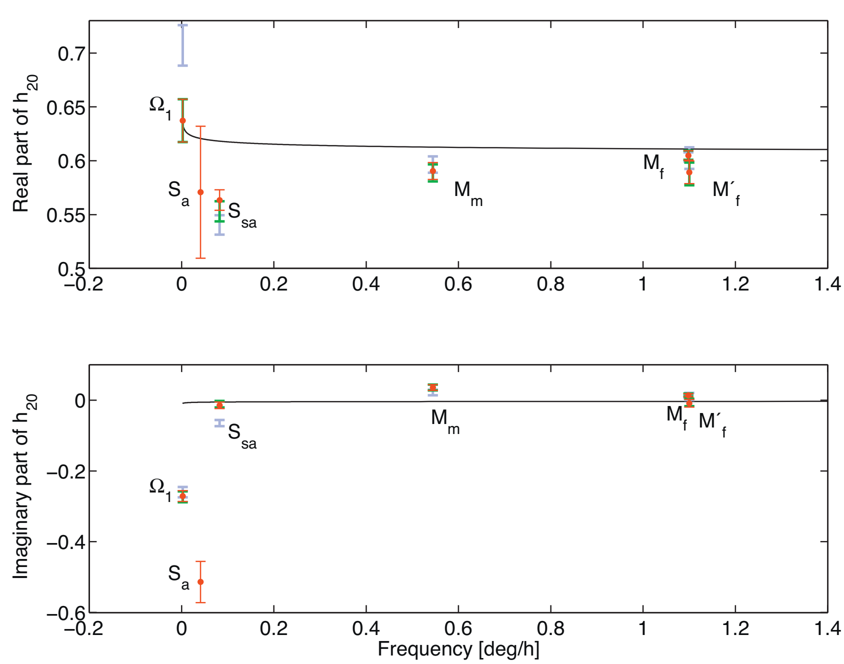

Fig. 3. Real and imaginary part of the zonal Love numbers estimated from solution S1 (light blue), S2 (green), and S3 (red). The black line denotes the theoretical values from Mathews et al. (1997).

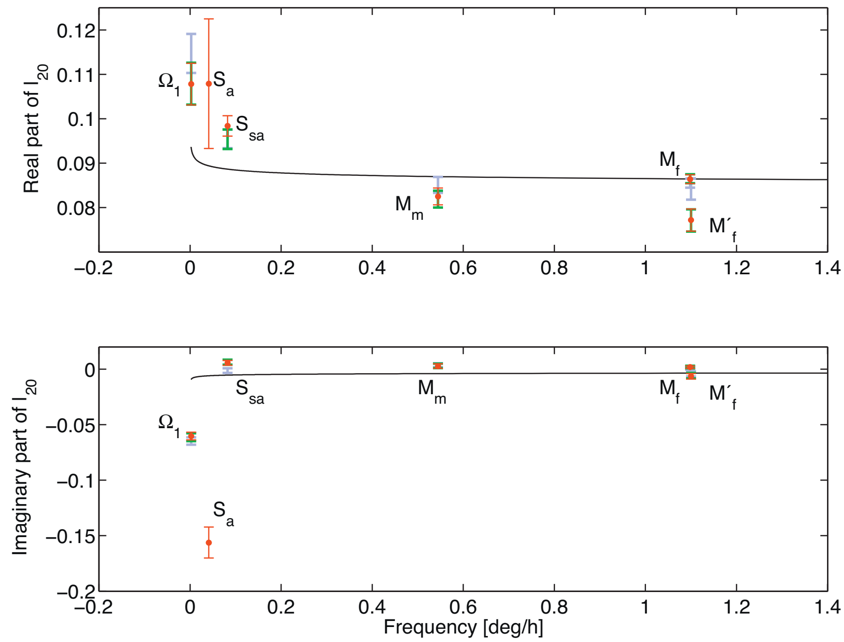

Fig. 4. Real and imaginary part of the zonal Shida numbers estimated from solution S1 (light blue), S2 (green), and S3 (red). The black line denotes the theoretical values from Mathews et al. (1997).

two columns the differences between the a priori and the estimated Love and Shida numbers from solution S3, expressed as differences in amplitudes of the tidal term in millimeters with (Petit and Luzum, 2010):

$$\begin{pmatrix} \delta R_f^{ip} \\ \delta R_f^{op} \end{pmatrix} = \sqrt{\frac{5}{4\pi}} H_f \begin{pmatrix} \delta h_f^R \\ -\delta h_f^I \end{pmatrix}, \quad (9)$$

$$\begin{pmatrix} \delta T_f^{ip} \\ \delta T_f^{op} \end{pmatrix} = \frac{3}{2} \sqrt{\frac{5}{4\pi}} H_f \begin{pmatrix} \delta l_f^R \\ -\delta l_f^I \end{pmatrix}. \quad (10)$$

The estimates of the complex Love and Shida numbers from all three solutions are displayed in Figs. 3 and 4. The real parts of the Love numbers from solution S1 (light blue) show a relatively large difference of about 0.073 ± 0.019 and -0.078 ± 0.009 with respect to their theoretical values for the tidal waves Ω_1 and S_{sa}. The application of hydrology loading corrections on station coordinates (solution S2 – green) leads to a decrease of the difference between the theoretical and estimated values of the real part of the Love number for the Ω_1 wave (0.003 ± 0.020), whereas the expected improvement of the estimated Love number of the semi-annual tide S_{sa} is small (the difference to the theoretical value is now -0.065 ± 0.009). The imaginary part of the Love number for the wave S_{sa} agrees in solution S2 with the theoretical value (-0.005) within the formal error (-0.012 ± 0.01). In solution S1 the estimated value is smaller (-0.065 ± 0.009) than the theoretical prediction. The imaginary part for the wave Ω_1 is considerably smaller in both solutions (-0.260 ± 0.015 and -0.273 ± 0.015) than is expected (-0.009).

In the third solution S3 (red color) the additional estimation of the Love number for the annual tide S_a causes another slight decrease of the difference between estimated and theoretical Love number for the semi-annual term S_{sa} (-0.055 ± 0.010). The larger formal error of the estimated Love number for the annual tide S_a is related to its small amplitude. The estimated Love number of the semi-annual tidal wave S_{sa}, which corresponds to a 1.07 ± 0.19 mm difference in the radial amplitude of the crustal displacement with respect to the theoretical value, may reflect deficiencies in the a priori station displacement modeling of long-period origin. The larger formal errors of the displacement amplitudes for the Ω_1 tidal wave are likely due to the not sufficiently long interval of observations. In terms of a posteriori variance of unit weight we got values of 0.79, 0.78, and 0.78 for the solutions S1, S2, and S3, respectively.

4. Conclusions

Complex frequency-dependent Love and Shida numbers of degree two of the solid Earth tide model are estimated with unprecedented accuracy from 27 years of precise and globally distributed geodetic VLBI data. Our estimates of the constant values of the degree two Love and Shida numbers (0.6072 ± 0.0003 and 0.0843 ± 0.0000) slightly deviate from the conventional values (0.6078 and 0.0847) and we provide diurnal Love and Shida estimates which are significantly improved in terms of precision and accuracy compared to an earlier solution by Haas and Schuh (1996). The radial station displacement computed from the theoretical and estimated values of the diurnal Love numbers reaches 1.73 ± 0.29 mm if summed up. The largest differences in the displacement amplitudes in the long-period band are found for the semi-annual term S_{sa} with 1.07 ± 0.19 mm and for the monthly tide M_m with the difference of 0.49 ± 0.18 mm.

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