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A method of experimentally probing transverse momentum dependent distributions

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Abstract

We calculate the double spin asymmetry A_{LL} for the π^0 production in semi-inclusive deep inelastic lepton–proton scattering with a spectator model of power-law and a model based on the factorization ansatz. We also calculate the double spin asymmetry for the integration over parts of the kinematic range for the setups of the experiments of COMPASS, HERMES, and JLab. We find that the results are characteristically dependent on the model used. Therefore, we suggest that the measurements of the double spin asymmetry provides a method of experimentally probing the transverse momentum dependent distributions.

PACS: 12.39.Ki; 13.60.-r; 13.60.Le; 13.88.+e

1. Introduction

In recent years the role of the transverse momentum of the parton has been more important in the field of the hadron physics since, for example, it provides time-odd distribution and fragmentation functions, and makes the single-spin asymmetries in hadronic processes possible [1,2]. The transverse momentum of the parton inside the proton is also related to the orbital angular momentum carried by the parton, which is an important subject since it is considered as a part of the spin contents of the proton. It is important to probe experimentally how the distribution and fragmentation functions are dependent on the transverse momenta of partons.

Semi-inclusive deep inelastic lepton–nucleon scattering (SIDIS) can be used to extract information on distribution and fragmentation functions. For example, we can explore the transversity distribution of proton from the SIDIS $lp^{\uparrow} \to l\pi X$ in which the initial proton is transversely polarized. In a lot of researches on the transverse momentum dependent distribution (fragmentation) functions, the ansatz which factorizes the longitudinal momentum fraction $x(z)$ and the transverse momentum $k_\perp(p_\perp)$ is adopted. For example, Ref. [3] investigated the double spin asymmetry A_{LL} by using such factorized distribution and fragmentation functions. We study the differences of the distribution and fragmentation functions of the factorized model and those of the spectator model.

Jakob et al. [4] presented a spectator model of power-law, which is based on the scalar and axial-vector diquark models of the nucleon. In this model, a Lorentz invariant form factor of power-law at the vertex of nucleon, quark and diquark is adopted. There are other types of spectator models in which other form factors at the vertex are adopted. However, in this Letter we consider only the spectator model of power-law presented in Ref. [4] and call this model the spectator model throughout this Letter. The important character of the spectator model is that the longitudinal momentum fraction x and the transverse momentum $k_\perp$ of the parton are intimately correlated with each other, since the spectator model is based on Lorentz invariant Feynman diagram. The transverse momentum distributions of the up and down quarks inside the proton are different, since for the proton the up quark is composed of a linear combination of the scalar and axial-vector diquark components and

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the down quark is only composed of the axial-vector diquark component. Power-law wave functions were also used in [5,6] for the study of generalized distribution functions.

We use the well-known scaling variables [3]: $x = Q^2/2P\cdot q$, $y = P\cdot q/P\cdot l$, and $z = P\cdot P_h/P\cdot q$, where l and l' are momenta of the initial and final lepton respectively, $q = l - l'$ is the exchanged virtual photon momentum, P is the initial nucleon momentum, and P_h is the final hadron momentum. The two-dimensional vectors $k_\perp$ and $p_\perp$ are the intrinsic transverse momentum of the parton with respect to the initial nucleon direction and that of the final hadron h with respect to the fragmenting quark direction, respectively. We work in a frame with the z-axis along the virtual photon momentum direction and the final hadron h has transverse momentum P_{hT} with respect to the z-axis. For more details see Ref. [7].

We calculate the dependence of the double spin asymmetry A_{LL} [3] for the π^0 production in semi-inclusive deep inelastic lepton-proton scattering on the variables x, y, z and P_{hT} with the spectator model, and find that the results are characteristically different from those calculated with the model based on the factorization ansatz. For example, the P_{hT}-behavior of A_{LL} is not sensitive to z-value in the case of the spectator model, whereas it is very sensitive to z-value in the case of the model based on the factorization ansatz. We also calculate A_{LL} of π^0 production for the integration over the range of (x, y, z) for the setups of the experiments of COMPASS, HERMES, and JLab. The P_{hT}-behaviors of these results of A_{LL} are also different for the two models. Therefore, it should be possible to use such differences in order to discriminate experimentally the spectator model and the model based on the factorization ansatz. Then, we suggest that we can discriminate experimentally these two models by measuring $A_{LL}(x, y, z, P_{hT})$ to obtain the information on which model is closer to the physical reality.

In Section 2 we present the results of A_{LL} obtained by calculating with the spectator model of Jakob et al. In Section 3 we calculate A_{LL} by using the model based on the factorization ansatz, and compare the results with those of Section 2. Section 4 is conclusion.

2. Spectator model

2.1. Distribution and fragmentation functions

Jakob et al. [4] presented a spectator model, which is based on the scalar and axial-vector diquark models of the nucleon. The important character of the spectator model is that the longitudinal momentum fraction x and the transverse momentum $k_\perp$ of the parton are intimately correlated with each other, since the spectator model is based on Lorentz invariant Feynman diagram. In this model the unpolarized and polarized distribution functions f_1 and g_1 are given by

$$f_{1R}(x, \mathbf{k}_\perp) = N_R(1-x)^3 \frac{(xM+m)^2 + \mathbf{k}_\perp^2}{(\mathbf{k}_\perp^2 + \lambda_R^2)^{2\alpha}},$$
$$g_{1R}(x, \mathbf{k}_\perp) = N_R a_R(1-x)^3 \frac{(xM+m)^2 - \mathbf{k}_\perp^2}{(\mathbf{k}_\perp^2 + \lambda_R^2)^{2\alpha}}, \quad (1)$$

where $\lambda_R^2(x) = (1-x)\Lambda^2 + xM_R^2 - x(1-x)M^2$ and $a_s = 1$, $a_a = -\frac{1}{3}$ for a_R, with $M = 0.94$ GeV, $m = 0.3$ GeV, and $M_s = 0.6$ GeV, $M_a = 0.8$ GeV for M_R. We take $\alpha = 2$ in this Letter. In this subsection we take $\Lambda = 0.5$ GeV. However, in the next subsection we also consider 0.4 and 0.6 GeV for the Λ value. Here the subscripts s and a refer to the scalar and axial-vector diquarks The normalization constant N_R is fixed by the normalization condition of $f_{1R}(x, \mathbf{k}_\perp)$.

From the $SU(4)$ wave function of the proton, we have (also for g_1^q) [4]

$$f_1^u = \frac{3}{2} f_{1s} + \frac{1}{2} f_{1a}, \qquad f_1^d = f_{1a}. \quad (2)$$

That is, transverse momentum distributions of the up and down quarks inside the proton are different, since for the proton the up quark is composed of a linear combination of the scalar and axial-vector diquark components and the down quark is only composed of the axial-vector diquark component. Then, Eqs. (1) and (2) give

$$\frac{f_1^u(x, \mathbf{k}_\perp)}{(1-x)^3} = \frac{3}{2} N_s \frac{(xM+m)^2 + \mathbf{k}_\perp^2}{(\mathbf{k}_\perp^2 + \lambda_s^2)^{2\alpha}} + \frac{1}{2} N_a \frac{(xM+m)^2 + \mathbf{k}_\perp^2}{(\mathbf{k}_\perp^2 + \lambda_a^2)^{2\alpha}},$$
$$\frac{g_1^u(x, \mathbf{k}_\perp)}{(1-x)^3} = \frac{3}{2} N_s \frac{(xM+m)^2 - \mathbf{k}_\perp^2}{(\mathbf{k}_\perp^2 + \lambda_s^2)^{2\alpha}} - \frac{1}{6} N_a \frac{(xM+m)^2 - \mathbf{k}_\perp^2}{(\mathbf{k}_\perp^2 + \lambda_a^2)^{2\alpha}},$$
$$\frac{f_1^d(x, \mathbf{k}_\perp)}{(1-x)^3} = N_a \frac{(xM+m)^2 + \mathbf{k}_\perp^2}{(\mathbf{k}_\perp^2 + \lambda_a^2)^{2\alpha}},$$
$$\frac{g_1^d(x, \mathbf{k}_\perp)}{(1-x)^3} = -\frac{1}{3} N_a \frac{(xM+m)^2 - \mathbf{k}_\perp^2}{(\mathbf{k}_\perp^2 + \lambda_a^2)^{2\alpha}}. \quad (3)$$

The distribution functions given in (3) are plotted in Figs. 1 and 2. Fig. 3 presents the widths of the distribution functions in $\mathbf{k}_\perp$ as functions of x.

We use for both u and d quarks the fragmentation function given in Ref. [8], which is plotted in Fig. 4:

$$D_1(z, \mathbf{p}_\perp) = \frac{1}{z} \frac{g^2}{16\pi^3} \frac{(\frac{\mathbf{p}_\perp}{z})^2 + m^2}{((\frac{\mathbf{p}_\perp}{z})^2 + m^2 + \frac{1-z}{z^2} m_\pi^2)^2}, \quad (4)$$

where m_π is pion mass and $m = 0.3$ GeV.

2.2. Double spin asymmetry

The double spin asymmetry is given by [3]

$$A_{LL}(x, y, z, P_{hT}) = \frac{\Delta\sigma_{LL}}{\sigma_0}, \quad (5)$$

where

$$\Delta\sigma_{LL} = \frac{\pi}{xy^2}\big[y(2-y)\big]\Sigma_q e_q^2 \times \int d^2\mathbf{k}_\perp g_1^q(x, \mathbf{k}_\perp) D_q^h(z, \mathbf{P}_{hT} - z\mathbf{k}_\perp),$$
$$\sigma_0 = \frac{\pi}{xy^2}\big[1 + (1-y)^2\big]\Sigma_q e_q^2 \times \int d^2\mathbf{k}_\perp f_1^q(x, \mathbf{k}_\perp) D_q^h(z, \mathbf{P}_{hT} - z\mathbf{k}_\perp). \quad (6)$$

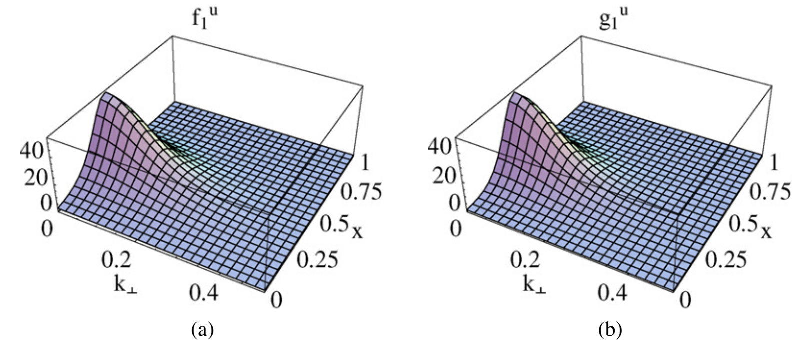

Fig. 1. The distribution functions of u quark $f_1^u(x,\mathbf{k}_\perp)$ (left) and $g_1^u(x,\mathbf{k}_\perp)$ (right) given in (3).

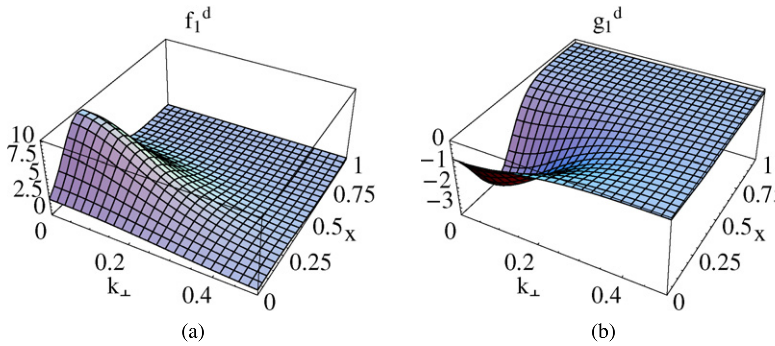

Fig. 2. The distribution functions of d quark $f_1^d(x,\mathbf{k}_\perp)$ (left) and $g_1^d(x,\mathbf{k}_\perp)$ (right) given in (3).

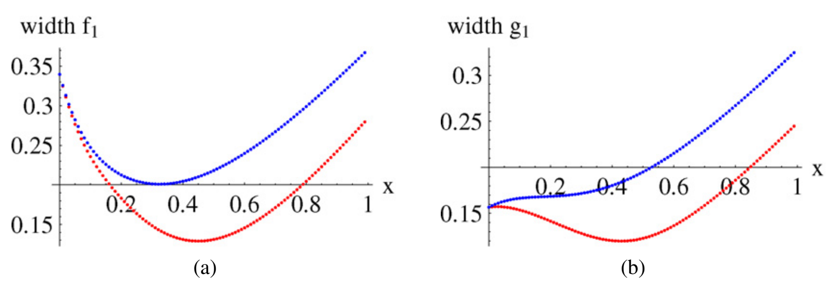

Fig. 3. The width in $\mathbf{k}_\perp$, which is defined as the value of $\mathbf{k}_\perp$ which satisfies $f_1(g_1)(x,\mathbf{k}_\perp)=\frac{1}{2}f_1(g_1)(x,\mathbf{0}_\perp)$ for u quark (lower line) and d quark (upper line).

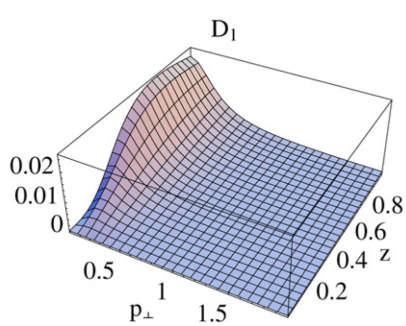

Fig. 4. The fragmentation function $D_1(z,\mathbf{p}_\perp)$ given in (4).

We study $A_{LL}(x,y,z,P_{hT})$ with the model of Ref. [4] by using $f_1^q(x,\mathbf{k}_\perp)$ and $g_1^q(x,\mathbf{k}_\perp)$ given in (3), and $D_q^h(z,\mathbf{p}_\perp)$ given in (4) for both u and d quarks. We note that in this Letter we consider only the contributions from the valence quarks u and d, and ignore the contributions from sea quarks. Therefore, the results are reliable in the range where x is not very small. We calculate the dependence of the double spin asymmetry $A_{LL}(x,y,z,P_{hT})$ of π^0 production on the variables x, y, z and P_{hT} with the spectator model. We consider three values 0.4, 0.5, 0.6 GeV for Λ existing in (3) through λ_R, in order to see the sensitivity of the results to the parameter value of Λ. The results of the calculation are presented in Fig. 5.

We also calculate A_{LL} of π^0 production for the integration over the range of (x,y,z) for the setups of the experiments of COMPASS, HERMES, and JLab. The following ranges are covered by the setup of each experiment,

(A) COMPASS: $0.003<x<1.0$, $0.1<y<0.9$, and $0.2<z<1.0$;

(B) HERMES: $0.02<x<0.4$, $0.25<y<0.85$, and $0.2<z<0.7$;

(C) JLab: $0.15<x<0.48$, $0.45<y<0.85$, and $0.4<z<0.7$.

The results are presented in Fig. 6 for three values of Λ: 0.4, 0.5, 0.6 GeV.

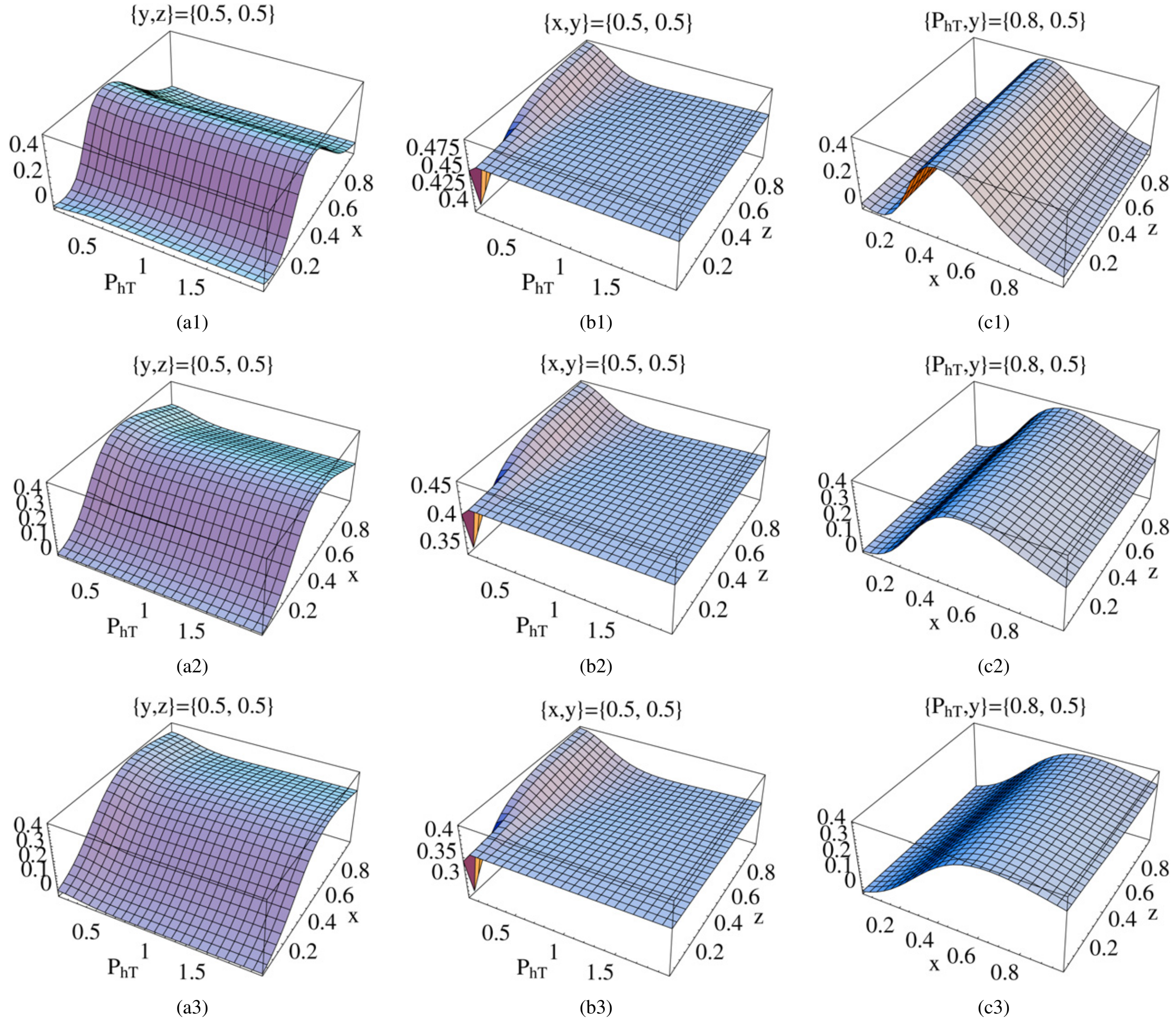

Fig. 5. For the spectator model, A_{LL} of π^0 production as a function of P_{hT} and x (a) with fixed $y = 0.5$ and $z = 0.5$, that of P_{hT} and z (b) with fixed $x = 0.5$ and $y = 0.5$, and that of x and z (c) with fixed $P_{hT} = 0.8$ and $y = 0.5$. $\Lambda = 0.4$ GeV (a1), (b1), (c1), $\Lambda = 0.5$ GeV (a2), (b2), (c2), and $\Lambda = 0.6$ GeV (a3), (b3), (c3).

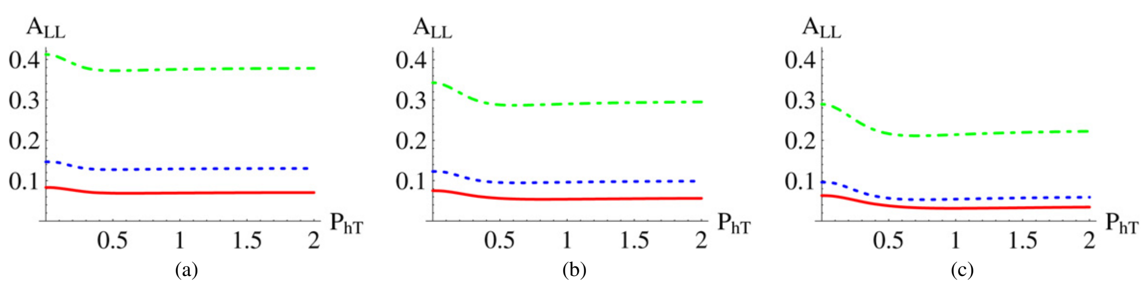

Fig. 6. For the spectator model, A_{LL} of π^0 production for the integration over the ranges of (x, y, z) for the setups of the experiments of COMPASS (solid), HERMES (dotted), and JLab (dash-dotted line). (a) For $\Lambda = 0.4$ GeV; (b) for $\Lambda = 0.5$ GeV; (c) for $\Lambda = 0.6$ GeV.

3. Comparison with model based on factorization

3.1. Distribution and fragmentation functions

For the transverse momentum dependent distribution (fragmentation) functions, the ansatz which factorizes $x(z)$ and $k_\perp(p_\perp)$ is often adopted. For example, Ref. [3] used the factorized functions given by

$$f_1^q(x, \mathbf{k}_\perp) = f_1^q(x) \frac{1}{\pi \mu_0^2} \exp\left(-\frac{\mathbf{k}_\perp^2}{\mu_0^2}\right),$$

$$g_1^q(x, \mathbf{k}_\perp) = g_1^q(x) \frac{1}{\pi \mu_2^2} \exp\left(-\frac{\mathbf{k}_\perp^2}{\mu_2^2}\right),$$

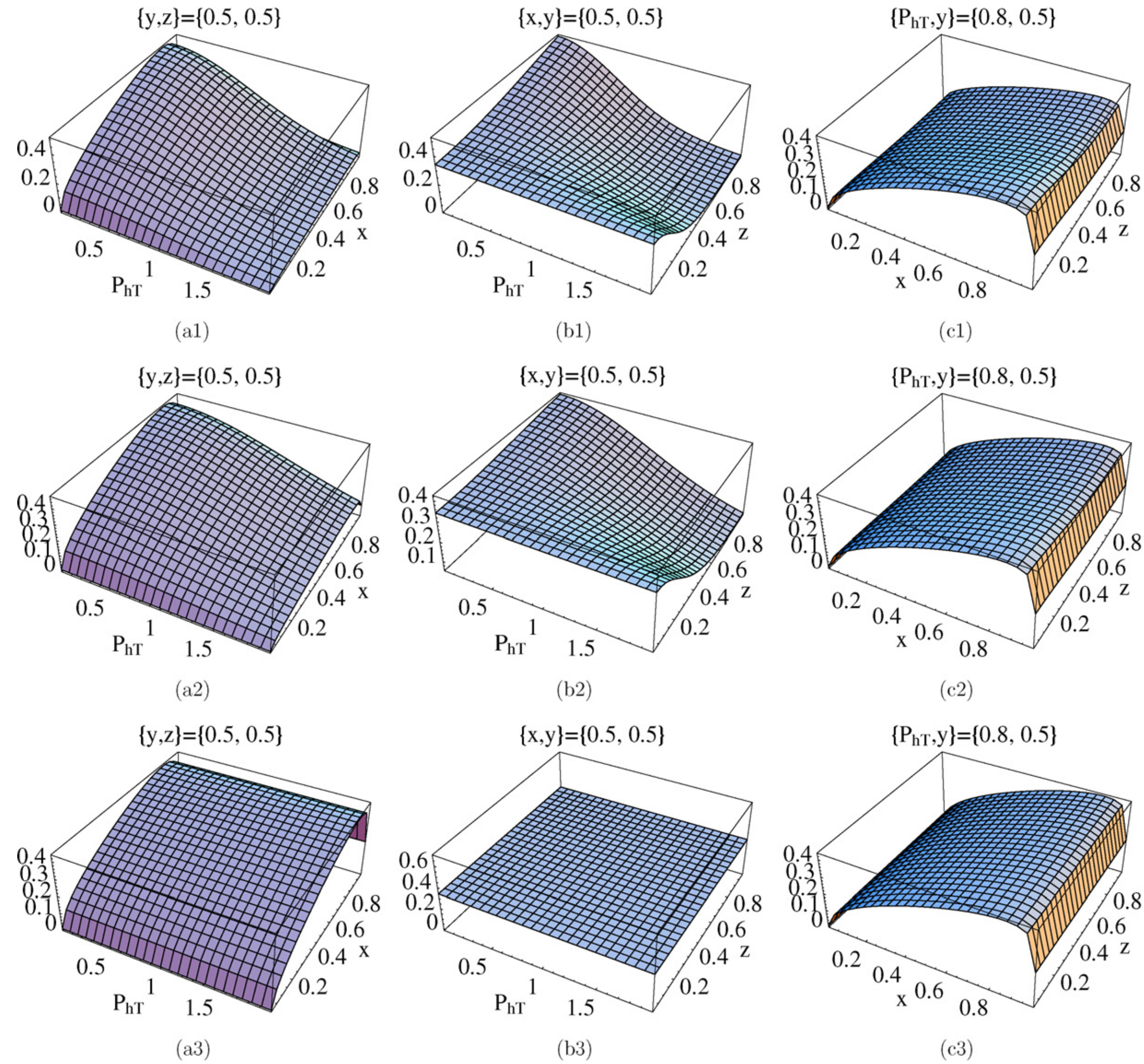

Fig. 7. For the model based on the factorization Ansatz, A_{LL} of π^0 production as a function of P_{hT} and x (a) with fixed $y = 0.5$ and $z = 0.5$, that of P_{hT} and z (b) with fixed $x = 0.5$ and $y = 0.5$, and that of x and z (c) with fixed $P_{hT} = 0.8$ and $y = 0.5$. (a1), (b1), (c1) for $\mu_2^2 = 0.10\ \mathrm{GeV}^2$; (a2), (b2), (c2) for $\mu_2^2 = 0.17\ \mathrm{GeV}^2$; (a3), (b3), (c3) for $\mu_2^2 = 0.25\ \mathrm{GeV}^2$.

$$D_q^h(z, \mathbf{p}_\perp) = D_q^h(z)\frac{1}{\pi\mu_D^2}\exp\left(-\frac{\mathbf{p}_\perp^2}{\mu_D^2}\right). \tag{7}$$

If we draw graphs for the width in $\mathbf{k}_\perp$ corresponding to Fig. 3 in the case of (7), we would get graphs of constants. If we use x dependent $\mu^2(x)$ which are sometimes used, the widths in $\mathbf{k}_\perp$ of distribution and fragmentation functions would not be constant and the results in this section would be modified. Such models, which are not factorized ones, are not considered in this Letter. Here, we only compare a factorized model given by (7) with constant μ^2 and a spectator model of power-law presented by Jakob et al. together with the fragmentation function give in (4).

For the integrated parton distribution functions appearing in (7), we use the following functions [9,10],

$$xf_1^u(x) = xu_v\big(x, \mu_{\mathrm{NLO}}^2\big) = 0.632x^{0.43}(1-x)^{3.09}(1+18.2x),$$
$$xf_1^d(x) = xd_v\big(x, \mu_{\mathrm{NLO}}^2\big) = 0.624(1-x)^{1.0}xu_v\big(x, \mu_{\mathrm{NLO}}^2\big),$$
$$g_1^u(x) = \delta u\big(x, \mu^2\big) = 1.019x^{0.52}(1-x)^{0.12}u_v\big(x, \mu_{\mathrm{NLO}}^2\big),$$
$$g_1^d(x) = \delta d\big(x, \mu^2\big) = -0.669x^{0.43}d_v\big(x, \mu_{\mathrm{NLO}}^2\big). \tag{8}$$

For the integrated fragmentation function appearing in (7), we use the following function [11],

$$D_q^h(z) = D_{u\bar{d}}^{\pi^+}\big(z, \mu_0^2\big) = N_u^\pi z^{-0.829}(1-z)^{0.949}. \tag{9}$$

3.2. Double spin asymmetry

When one uses the factorized distribution and fragmentation functions given in (7) for the calculation of $\Delta\sigma_{LL}$ and σ_0 in (6), one has

$$\Delta\sigma_{LL} = \frac{y(2-y)}{xy^2}\frac{1}{\mu_D^2 + z^2\mu_2^2} \times \exp\left(-\frac{\mathbf{P}_{hT}^2}{\mu_D^2 + z^2\mu_2^2}\right)\Sigma_q e_q^2 g_1^q(x) D_q^h(z),$$

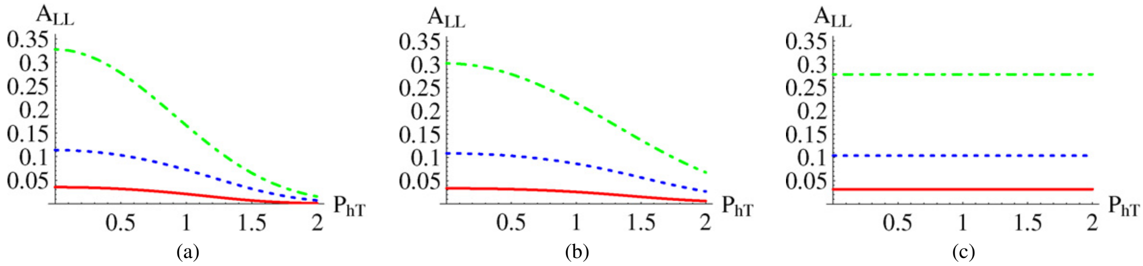

Fig. 8. For the model based on the factorization Ansatz, A_{LL} of π^0 production for the integration over the ranges of (x, y, z) for the setups of the experiments of COMPASS (solid), HERMES (dotted), and JLab (dash-dotted line). (a) For $\mu_2^2 = 0.10\ \text{GeV}^2$; (b) for $\mu_2^2 = 0.17\ \text{GeV}^2$; (c) for $\mu_2^2 = 0.25\ \text{GeV}^2$.

$$\sigma_0 = \frac{1+(1-y)^2}{xy^2}\frac{1}{\mu_D^2 + z^2\mu_0^2} \times \exp\left(-\frac{\mathbf{P}_{hT}^2}{\mu_D^2 + z^2\mu_0^2}\right)\Sigma_q e_q^2 f_1^q(x) D_q^h(z). \tag{10}$$

Then, using $\Delta\sigma_{LL}$ and σ_0 in (10), one can calculate the double spin asymmetry $A_{LL}(x, y, z, P_{hT})$ from (5). Following Ref. [3] we use $\mu_0^2 = 0.25\ \text{GeV}^2$, $\mu_D^2 = 0.20\ \text{GeV}^2$, and three different values: $\mu_2^2 = 0.10, 0.17, 0.25\ \text{GeV}^2$. The results are presented in Fig. 7. We find that the graphs in Fig. 7 are characteristically different from the graphs in Fig. 5 which were obtained by using the spectator model. For example, the P_{hT}-behavior of A_{LL} is not sensitive to z-value in the case of the spectator model, whereas it is very sensitive to z-value in the case of the model based on the factorization ansatz.

We also calculate A_{LL} of π^0 production for the integration over the range of (x, y, z) for the setups of the experiments of COMPASS, HERMES, and JLab. The results are presented in Fig. 8, which agree with the graphs in Fig. 1 of Ref. [3]. The P_{hT}-behaviors of the integrated A_{LL} presented in Figs. 6 and 8 are also different for the two models. Therefore, it should be possible to use such differences for discriminating experimentally the spectator model and the model based on the factorization ansatz. Then, we suggest that we can discriminate experimentally these two models by measuring $A_{LL}(x, y, z, P_{hT})$ to obtain the information on which model is closer to the physical reality.

4. Conclusion

Recently it is realized that it is important to know the transverse momentum dependence of the distributions of partons inside the nucleon. At first it should be useful to know how realistic the factorization ansatz is. In this context, it should be useful to be able to discriminate the spectator model and the model based on the factorization ansatz. In this Letter we found that the double spin asymmetries $A_{LL}(x, y, z, P_{hT})$ obtained by using the spectator model of power-law presented by Jakob et al. and the model based on the factorization ansatz are characteristically different from each other. Therefore, we suggest that the measurement of $A_{LL}(x, y, z, P_{hT})$ can be used as an experimental discrimination of the two models.

We note that Ref. [6] studied a related subject in the generalized parton distributions (GPDs). It showed that the GPDs derived from the spectator model of power-law and those from the model based on factorizing the t-dependence of GPDs give different properties of the form factors and the reaction amplitudes.

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