Ecological optimization for a generalized irreversible Carnot engine with an universal heat transfer law

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Abstract

In this paper, the optimal ecological performance of an irreversible Carnot engine with the losses of heat-resistance, heat leak and internal irreversibility, in which the heat transfer between the working fluid and the heat reservoirs obeys a generalized law \( Q \propto (\Delta T)^n \), is derived by taking an ecological optimization criterion as the objective, which consists of maximizing a function representing the best compromise between the power and entropy production rate of the heat engine. The results obtained corroborate those in the literature.

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1. Introduction

The thermal efficiency of a reversible Carnot cycle is an upper limit of efficiency for heat engines. In classical thermodynamics, the Carnot efficiency is

\[ \eta_c = 1 - \frac{T_L}{T_H}, \]

where \( T_L \) and \( T_H \) are the temperatures of the hot and cold reservoirs between which the heat engine operates. However, the classical thermodynamics bounds (only achievable in the reversible limit) are usually far away from typical real values. In order to seek a more realistic upper bound on the efficiency of a heat engine, the concept of finite-time thermodynamics was introduced by Curzon and Ahlborn[11] with a novel work on Carnot heat engine in which they remarked that the efficiency of an engine operating at maximum power is by

\[ \eta_{c,s} = 1 - \sqrt[\gamma]{\frac{T_L}{T_H}}, \]
where the only source of irreversibility in the engine is a linear finite rate heat transfer between the working fluid and its two heat reservoirs.

In the past decade some new models of irreversible Carnot heat engines which include other irreversibilities, besides thermal resistance, have been established: heat leak and internal dissipations of the working fluid (see [2-9] and included references there). In the optimization of Carnot cycles, including those irreversibilities, have appeared four objective functions: power, efficiency, ecological and entropy generation. Angulo-Brown\cite{10} proposed an ecological criterion: $E = P - T_o \sigma$ for finite-time Carnot heat engines, where $T_o$ is the temperature of cold heat reservoir, $P$ is the power output and $\sigma$ is the entropy generation rate. Angulo-Brown et al.\cite{11} derived a general property of non-endoreversible thermal cycles with this ecological criterion. Yan\cite{12} showed that it might be more reasonable to use $E = P - T_o \sigma$, if the cold reservoir temperature $T_o$ is not equal to the environment temperature $T_0$. This criterion function is more reasonable than presented by Angulo-Brown. The optimization of the ecological function represents a compromise between the power output $P$ and the loss power $T_0 \sigma$, which is produced by entropy generation in the system and its surroundings.

In recent years, some authors\cite{13-20} have investigated ecological performance optimization for Carnot heat engine. Chen et al.\cite{13} and Aragón-González et al.\cite{14} derived the finite-time ecological optimal performance of irreversible Carnot engine with the heat transfer law $q \propto \Delta T$. Sui\cite{15} discussed the ecological optimal performance of irreversible Carnot engine with the heat transfer law $q \propto (\Delta T)^m$. Chen et al.\cite{16}, Zhu et al.\cite{17} and Yang et al.\cite{18} investigated the finite-time ecological optimal performance of irreversible Carnot engine with $q \propto (\Delta T)^{m}$. Zhu et al.\cite{19,20} derived the finite-time ecological optimal performance of the generalized irreversible Carnot engine with the heat transfer law $q \propto (\Delta T)^n$. More recently, Chen et al.\cite{21} derived the optimal performance of Carnot heat engine with heat transfer law $q \propto (\Delta T^m)^n$. In this paper, based on the works of Refs.\cite{19} and \cite{21}, we investigate the optimal ecological performance of a generalized irreversible Carnot engine with an universal heat transfer law $q \propto (\Delta T^m)^n$. In the engine model the losses of heat resistance, heat leak and internal irreversibility are considered.

2. Model of the Irreversible Carnot Engine

The generalized irreversible Carnot engine model including the irreversibilities of finite-rate heat transfer between the working fluid and its reservoirs, heat leak between the two reservoirs and internal dissipations is shown in Fig.1. The following assumptions are made for this model:
(1) The working fluid flows through the system in a steady-state fashion. The cycle consists of two isothermal processes and two adiabatic processes. All four processes are irreversible.

(2) Because of the heat-transfer, the working fluid temperatures ($T_1$ and $T_2$) are different from the corresponding reservoir temperatures ($T_H$ and $T_L$). These temperatures satisfy the following inequalities: $T_H > T_1 > T_2 > T_L$. The heat-transfer surface areas ($F_1$ and $F_2$) of high- and low-temperature heat-exchanges are finite. The total heat-transfer surface areas ($F$) of the heat-exchanges is assumed to be a constant: $F=F_1+F_2$.

(3) There exists a constant rate of heat leakage $q$ from the heat source to the heat sink. Thus $Q_H=Q_1+q$, $Q_L=Q_2+q$, where $Q_1$ is due to the deriving force of $(T_H^n-T_1^n)^m$, $Q_2$ is due to the deriving force of $(T_2^n-T_L^n)^m$, $Q_H$ is the rate of heat transfer supplied by the heat source and $Q_L$ is the rate of heat transfer released to the heat sink.

(4) A constant coefficient $\phi$ is introduced to characterize the additional internal miscellaneous irreversibility effect: $\phi = Q_2/Q_1 \geq 1$, where $Q_2$ is the rate of heat flow from the cold working-fluid to the heat sink for the generalized irreversible Carnot engine, while $Q_1$ is that for the Carton engine with the only loss being the heat resistance.

3. Optimal Characteristics

The second law of thermodynamics requires that

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}. \quad (3)$$

The first law of thermodynamics gives that the power output $P$ of the engine is

$$P = Q_H - Q_L = Q_1 - Q_2. \quad (4)$$

The efficiency $\eta$ of the engine is

$$\eta = \frac{P}{Q_H} = \frac{P}{Q_1+q}. \quad (5)$$

Suppose that the heat transfers between the engine and its surroundings follow a generalized heat transfer law

$$q \propto (\Delta T^n)^m, \quad Q_i = \alpha F_i (T_H^n - T_1^n)^m,$$

$$Q_2 = \beta F_2 (T_2^n - T_L^n)^m, \quad (6)$$

where $\alpha$ is the overall heat transfer coefficient of the hot-side heat exchanger and $\beta$ is the overall heat transfer coefficient of the cold-side heat exchanger.

The heat transfer surface area ratio $f$ and working fluid temperature ratio $x$ are defined as

$$f = \frac{F_1}{F_2}, \quad x = \frac{T_1}{T_2}. \quad (7)$$

Combining (6) and (7) gives
\[ T_2 = \left( \frac{(\phi r)^{1/n} T_H^n + x^n T_L^n}{x^n + (\phi r)^{1/m} x^m} \right)^{1/n}, \]  
\[ (8) \]

where \( r = \alpha / \beta \) Combining above equations gives

\[ P = \frac{(1 + \phi / x) \alpha f A_t}{(1 + f) A_2}, \]
\[ (9) \]

\[ \eta = \frac{(1-\phi / x) A_t}{A_t + q(1 + f) A_2 / (\alpha f F)}, \]
\[ (10) \]

\[ \sigma = \frac{\phi}{x T_L} - \frac{1}{T_H} \frac{\alpha f A_t}{(1 + f) A_2} + q \left( \frac{1}{T_L} - \frac{1}{T_H} \right), \]
\[ (11) \]

\[ E = \alpha f F (a_1 - \phi a_1) \frac{A_t}{x (1 + f) A_2} + q (a_2 - a_1), \]
\[ (12) \]

where \( A_t = (T_H^n - x^n T_L^n)^m, \)
\[ A_2 = [1 + (\phi r x^{m-1})^{1/m} x^m], \]
\[ a_1 = 1 + \frac{T_0}{T_H}, \quad a_2 = 1 + \frac{T_0}{T_L}. \]

Equations (9), (10), (11) and (12) indicate that power output \( P \), efficiency \( \eta \), entropy generation rate \( \sigma \) and ecological function \( E \) of the generalized irreversible Carnot engine are functions of the heat transfer surface area ratio \( f \) for given \( T_H, T_L, T_0, \alpha, \beta, q, \phi \) and \( x \). Taking the derivatives of \( P, \eta \) and \( E \) with respect to \( f \) and setting them equal to zero (\( dP / df = 0, d\eta / df = 0, dE / df = 0 \)) yields the same optimum surface area ratio

\[ f = f_0 = [1/(\phi r x^{m-1})]^{1/(m+1)}. \]
\[ (13) \]

The corresponding optimal power, the optimal efficiency, the optimal entropy generation rate, and the optimal ecological function are as follows:

\[ P = B(1 - \phi / x) A_t, \]
\[ (14) \]

\[ \eta = \frac{(1-\phi / x) A_t}{A_t + q / B}, \]
\[ (15) \]

\[ \sigma = B \left( \frac{\phi}{x T_L} - \frac{1}{T_H} \right) A_t + q \left( \frac{1}{T_L} - \frac{1}{T_H} \right), \]
\[ (16) \]
\[ E = B(a_2 - \frac{\phi_1}{x})A_1 + q(a_2 - a_1), \]  
where \( B = \alpha F / [1 + (\phi r)^{m+1} x^{m+1}]^{m+1}. \)  

4. Discussions

4.1 Effect of different losses on the optimal characteristics

(1) \( q=0 \) and \( \phi = 1 \)
The model is reduced to the endoreversible Carnot engine.

(2) \( q=0 \) and \( \phi > 1 \)
The model is reduced to the irreversible Carnot engine with heat resistance and internal irreversibility.

(3) \( q \neq 0 \) and \( \phi = 1 \)
The model is reduced to the irreversible Carnot engine with heat resistance and heat leak losses.

4.2 Analysis of special cases

(1) \( n=1 \) and \( m=1 \)
In this case, the heat transfer law obeys Newton’s law. The Eqs.(14)-(17) become

\[ P_i = B_i(1 - \phi / x)(T_{H} - xT_{L}), \] 
\[ \eta_i = (1 - \phi / x) \frac{T_{H} - xT_{L}}{T_{H} - xT_{L} + q_i / B_i}, \] 
\[ \sigma_i = B_i(\frac{\phi}{xT_L} \frac{1}{T_H})(T_H - xT_L) + q_i(\frac{1}{T_L} \frac{1}{T_H}), \] 
\[ E_i = B_i(a_2 - \frac{\phi a_1}{x})(T_H - xT_L) + q_i(a_2 - a_1), \] 
where

\[ B_i = \alpha F / \left( \sqrt{\frac{\phi a}{\beta}} + 1 \right)^2, \] 
\[ q_i = C_i(T_H - T_L), \]

where \( C_i \) is the internal conductance of the heat engine. It is clear that the Eqs.(18)-(23) are same results as in Ref.[13].

(2) \( n = 1 \)
In this situation, the heat transfers law obeys \( q \propto (\Delta T)^m \). Setting in Eqs.(14)-(17) one can obtain
\[ P_2 = B_2(1 - \phi / x)(T_H - xT_L)^m, \]  
\[ \eta_2 = (1 - \phi / x)(T_H - xT_L)^m / [(T_H - xT_L)^m + q_i / B_i], \]  
\[ \sigma_2 = B_2 \left( \frac{\phi} {xT_L} \frac{1} {T_H} \right)(T_H - xT_L)^m + q_i \left( \frac{1} {T_L} - \frac{1} {T_H} \right), \]  
\[ E_2 = B_2(a_2 - \frac{\phi a_i} {x})(T_H - xT_L)^m + q_i(a_2 - a_i), \]

where

\[ B_2 = \alpha F / \left[ 1 + (\phi r)^{m-1} x^{m+1} \right]^{m+1}, \]
\[ q_i = C_i(T_H - T_L)^m. \]

The Eqs. (24)-(29) are same outcomes as in Ref.[15].

(3) \( n = -1 \) and \( m = 1 \)

In this case, the heat transfer law obeys another law, i.e., linear phenomenological heat transfer law.

The Eqs.(14)-(17) become

\[ P_3 = B_3(x - \phi)(x / T_H - 1 / T_L), \]
\[ \eta_3 = (x - \phi) \frac{x / T_H - 1 / T_L} {x(x / T_H - 1 / T_L) + q_i / B_i}, \]
\[ \sigma_3 = B_3 \left( \frac{\phi} {T_L} \frac{x} {T_H} \right)(x / T_H - 1 / T_L) + q_i \left( \frac{1} {T_L} - \frac{1} {T_H} \right), \]
\[ E_3 = B_3(a_2 x - a_i \phi)(x / T_H - 1 / T_L) + q_i(a_2 - a_i), \]

where

\[ B_3 = \alpha F / \left( \sqrt{\frac{\phi \alpha} {\beta} + x} \right)^2, \]
\[ q_i = C_i(1 / T_L - 1 / T_H). \]

One can see that the Eqs. (30)-(35) are same results as in Ref.[18].

(4) \( m = 1 \)

In this case, the heat transfers law obeys \( q \propto (\Delta T)^n \) and the Eqs.(14)-(17) become

\[ P_4 = B_4(1 - \phi / x)(T_H^n - x^n T_L^n), \]
\[ \eta_i = (1 - \phi/x)(T^*_{hi} - x^*T^*_{li})/(T^*_{hi} - x^*T^*_{li} + q_i/B_i), \]  

(37)

\[ \sigma_i = B_i \left( \frac{\phi}{xT_L} \right) \left( \frac{1}{T^*_{hi} - x^*T^*_{li}} \right) \left( \frac{1}{T^*_{hi}} \right) \left( \frac{1}{T^*_{hi}} \right), \]  

(38)

\[ E_i = B_i \left( a_2 - \frac{\phi a_1}{x} \right)(T^*_{hi} - x^*T^*_{li}) + q_i(a_2 - a_1), \]  

(39)

where

\[ B_i = aF/\left[ 1 + (\phi r)^{1-x} \right]. \]  

(40)

\[ q_i = C_i(T^*_{hi} - T^*_{li}). \]  

(41)

It is clear that the Eqs. (36)-(41) are same results as in Ref.[20].

5. Conclusion

In conclusion, we investigate the optimal ecological performance of a generalized irreversible Carnot engine with heat transfer law $q \propto (\Delta T^n)^m$. In the engine model the losses of heat resistance, heat leak and internal irreversibility are considered. The results obtained include those in the literature, such as the optimal ecological performance of endoreversible Carnot engine ($q = 0, \phi = 1$), and the optimal ecological performance of irreversible Carnot engine with various heat transfer laws $q \propto \Delta T(n = 1, m = 1)$, $q \propto (\Delta T)^m(n = 1)$, $q \propto (\Delta T^{-1})(n = -1, m = 1)$ and $q \propto (\Delta T^n)(m = 1)$. It is means that our generalized irreversible Carnot engine model in present paper includes those obtained in many literatures and can provide some theoretical guidance for the design of practical engine.

References


