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Material parameter modeling and solution technique using birth–death element for notched metallic panel repaired with bonded composite patch

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Abstract This paper seeks to outline a novel three-layer model and a new birth–death element solution technique to evaluate static strength of notched metallic panel repaired with bonded composite patch and to optimize material parameters. The higher order 3D, 8-node isotropic solid element and 8-node anisotropic layered solid element with three degrees of freedom per node are respectively implemented to model substrate panel, adhesive layer and composite patch to establish three-layer model of repaired panel. The new solving technique based on birth–death element is developed to allow solution of the stress pattern of repaired panel for identifying failure mode. The new model and its solution are used to model failure mode and residual strength of repaired panel, and the obtained results have a good agreement with the experimental findings. Finally, the influences of material parameter of adhesive layer and composite patch on the residual strength of repaired panel are investigated for optimizing material properties to meet operational and environmental constraints.

1. Introduction

The repair for notched metallic structure with adhesively bonded composite patch holds superiority over that with mechanical riveting or fastening in terms of mechanical properties and efficiency, e.g., better geometry flexibility, lighter weight, higher stiffness and strength, improved durability, lower repair time and cost, etc.[1–3] Thus, adhesively bonded composite patch repairs to notched metallic substrate structure have received increasingly attention, and static and fatigue strengths of repaired metallic panels with the thin (less than 12.7 mm) and thick (more than 12.7 mm) thickness have been widely investigated.[4] Generally, thin panels were analyzed by using 2-D models,[5–13] whereas thick panels were done as a 3-D problem.[14–24] With thin panels, Naboulsi and Mall[5] proposed a 2-D model for the analysis of adhesive layer, composite patch and thin notched substrate panel. Sun and Klug[6] developed an effective spring model of adhesive layer from the Mindlin plate theory. Rao et al.[7] conducted the experiments to determine the residual bonding strength of repaired panel with three types of surface treatment subjected to cyclic

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loading. Hosseini-Toudeshky et al.[8] performed experimentally investigation on the influence of layer number of composites patch on crack growth life of single-sided repaired panels. Sabelkin et al.[9] carried out tests to investigate the effect of the stiffener on fatigue lives for both panels with and without adhesively bonded composite patch repair. Xiong and Shenoi[10] investigated experimentally the effects of patch thicknesses and fibre/epoxy prepreg materials on static and fatigue strengths of notched panels repaired with adhesively bonded composite patch. Okafor et al.[11] presented 2-D FE (finite element) model to simulate stress pattern of asymmetrically repaired panel with a central notch. Oterkus[12] and Tsamasphyros et al.[13] took notched substrate panel, composite patch and adhesive layer as individual layers and adhesive layer was regarded as continuous elastomer to establish 2-D two-layer model for calculating stress intensity factor at crackle tip of repaired panel. Ouinas[14] considered notched substrate plate and adhesive layer as continuous elastomers and composite patch as orthotropic elastomer to simulate crack propagation process by means of 2-D two-layer model and J-integration criterion.

Regarding thick panels, because of asymmetric repair to the panels, bending effects have been ignored in experimental investigations.[15] Klug and Sun[16] undertook edge crack propagation tests around central notch of thick panel. Jones and Chiu[17] achieved experimental investigation and numerical analysis on notch repair for thick structures. Due to the difficulty for allowing analytical solution of adhesively bonded composite patch repairs, numerical analysis based on FE and BE (boundary element) models were implemented to simulate stress fields and to evaluate the repair efficiency. Schubbe and Mall[18,19] established three-layer FE model from the 2-D Mindlin-plate element for simulating crack growth in thick panel repaired with bonded composite patch. Though the 3-D FE analysis has been employed to calculate stress intensity factors at the tip of a crack in repaired panels, no numerical analysis has been conducted to simulate crack growth process.[20–23] A combination of BE model and FE method (BEM/FEM) has been presented by Sekine et al.[24] to determine stress intensity factor. Oudad et al.[25] investigated the influences of mechanical properties of adhesive layer and composite patch as well as crack depth on plastic zone size of crack tip in by using 3-D non-linear FEA. It was showed that composite patch resulted to a significant decreasing of plastic zone size of crack tip.

In reality, optimization design on metallic panel repaired with bonded composite patch has received considerable attention recently. Mathias et al.[26] implemented genetic algorithms (GAs) to optimize bonding orientation and stacking sequence of composite patch to reduce stress pattern in repaired structure. Roberto[27] presented optimum design scheme for composite bonding repair by using genetic algorithm and found a significant influence of geometry of composite patch on fracture and fatigue lives of repaired panel. Breitzman et al.[28] conducted optimization design on the thickness and stacking sequence of composite repair patch under tensile loading by checking von Mises stress pattern. Ramji et al.[29] performed geometry optimization of notched panel repaired with bonded symmetrical composite patches with circle, rectangle, square, ellipse, octagon and expanded octagon shapes based on 3-D FEA.

With rapid growth of notched panel repaired with bonded composite patches, an elaborate study on the influences of important parameters such as patch thickness, layer angles and patch material, etc. on the mechanical behavior of repaired parts is urgently needed, because it could provide important information as a basis for technologists to decide what method is the best choice. However, there seems to be precious few works done on this subject; from the above review, most of researchers centered their attention upon the study on individual special issues of each parameter rather than on a comprehensive analysis of important parameters of adhesive layer and composite patch as a whole.[30] The paper, therefore, aims to present new three-layer FE model and a novel solution technique using birth–death element to solve the stress pattern for identifying failure mode of repaired panel and for investigating and comparing the influences of important parameters on the residual strength of the repaired panel.

2. Modified three-layer model and solution algorithm

Notched metallic panels repaired with adhesively bonded composite patch were fabricated to determine static mechanical properties and Fig. 1 shows the geometry and dimensions of repaired panels. The materials of substrate panel, adhesive layer and composite patch were the LY12 aluminum-alloy, SY-24C adhesive system and symmetric T300/3234 prepreg tape respectively. The panels with 350 mm length, 60 mm width and 2.4 mm thickness were manufactured from the substrates of LY12 aluminum-alloy and an edge-notch with 1.4 mm depth and 40 mm diameter was machined through linear cutting and polishing at the side of the panel for modelling corrosion pit. Surface treatment of substrate panel was made through acetone-cleaning and drying prior to adhesive bonding of composite patches. All patches are of rectangular shape. Since patch debonding occurs due to the development of high peel stress at the extremities of the load transfer regions (e.g., at the overlap end), in order to minimize the peel stress, a tapering was made along the longitudinal edge of patch by using plies of decreasing lengths from the bonded surface to the top with a cover ply, as is usually done in the actual applications. The taper of all patches had a constant nominal length with the uniform ply drop-off dependent on the number of plies. The composite single-sided patches were adhesively bonded to the notched panels. All the repaired specimens were cured at 160 °C for 2 h and then at 200 °C for 1 h. All specimen shoulders were adhesively bonded with 'staircase' aluminum-alloy tabs for suppressing the effect of tester grippers and for minimizing stress concentrations at the shoulders roots in the specimens (shown in Fig. 1). Static tensile tests of identical repaired specimens were conducted on MTS880-500 kN servo-hydraulic tester to determine the residual strength at room moisture and temperature as well as at a loading rate of 0.5 mm/min. In this context, the residual strength is defined as the ultimate load of the notched specimen repaired with adhesively bonded composite patch. The P–D (load–displacement) curves of tested specimens were recorded by tester. Fig. 2 shows that all the P–D curves of tested specimens appear almost identical in linear elastic scale. An existence of one peak on all P–D curve marks the debonding of adhesive resin at the interface between substrate panel and composite patch around the notch of repaired specimen, which results in a load-drop on the *P–D* curve. From the experimental observation, it is clear that all tested specimens displayed similar failure mode of

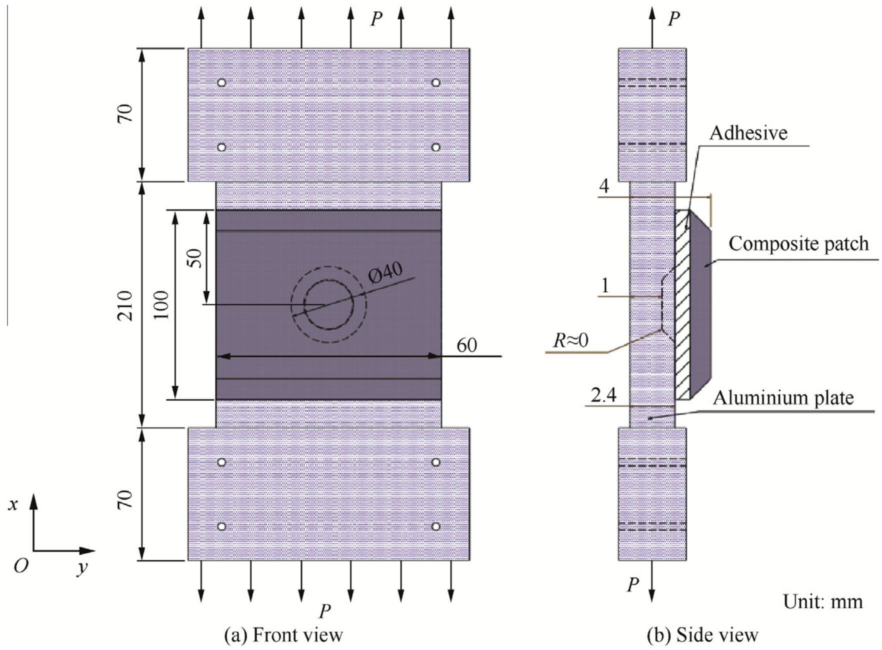

Fig. 1 Notched specimen repaired with bonded fibre/epoxy prepreg.

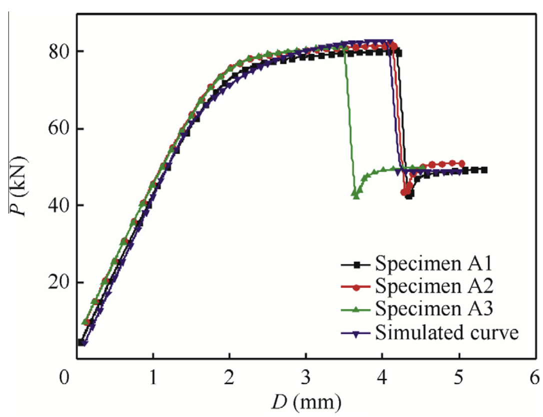

Fig. 2 Load–displacement curves of repaired specimen.

static tensile. With the increasing load, the debonding of adhesive resin first resulted in stress concentration along the notch in a ductile manner, and then caused the delamination in composite patch, which subsequently led to final failure along the notch of repaired specimen.

In order to provide numerical analysis to model material parameters of adhesive layer and composite patch, notched specimen repaired with bonded fibre/epoxy prepreg shown in Fig. 1 is chosen to be modeled and local coordinate systems are then set up to ensure the fibres with correct 3D orientation, i.e., to keep three axial directions 1–3 of the coordinate system consistent with the three normal stresses σ_i (i = 1, 2, 3) as shown in Fig. 3. Fig. 3 illustrates the definitions of three axial directions 1–3 of the coordinate system and three normal stresses σ_i (i = 1, 2, 3) for both the substrate panel, adhesive layer and composite patch, where the coordinate axes 1–3 denote the longitudinal, transverse and through-thickness directions respectively. Based on the definitions of three normal stresses σ_i (i = 1, 2, 3), one has the definitions of three shear stress components τ_{12}, τ_{13} and τ_{23}. From Fig. 3, it can be seen that there are three layers of substrate panel, adhesive layer and composite patch. In order to model them, the same higher or-

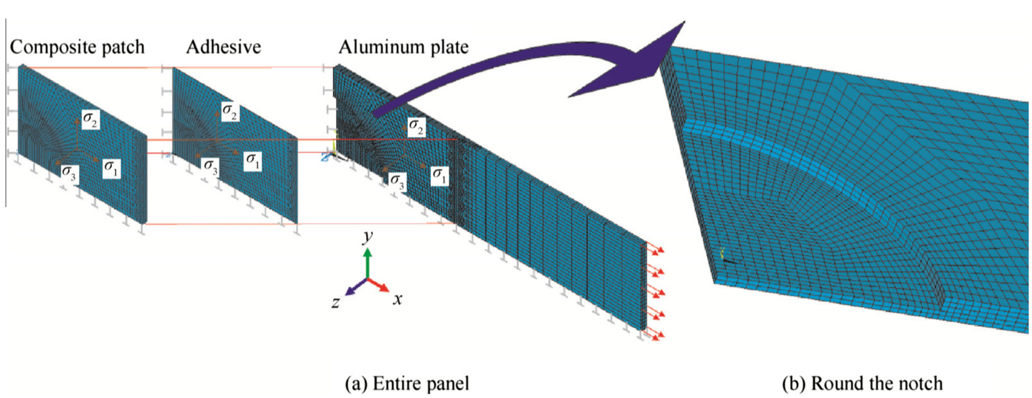

Fig. 3 Three-layer model.

der 3D, 8-node isotropic SOLID45 element of ANSYS code[31] is implemented to model substrate panel and adhesive layer as per the dimensions and 3D, 8-node anisotropic layered solid element SOLID46 with three degrees of freedom per node is employed to model composite patch to attain a high accuracy of simulation (shown in Fig. 3(a)). In order to obtain stress pattern near the notch, super fine meshing has been done around the notch region with a total of 8640 8-node SOLID45 plane elements (144 circumferential, 12 radial; 5 elements through the thickness) (shown in Fig. 3b). Outside the disk, a structured area mesh has been made in the panel as per the dimensions shown in Fig. 1 and all the areas are extruded in thickness direction to generate volume together with all the generated volumes meshed through sweep mode. The adhesive filled up inside the notch during repair is meshed with five elements in thickness direction. The area meshing of patch and adhesive is generated similar to the panel so as to be easily coupled with regard to each other at the interface. As recommended in Ref.[30] the patch is assumed to be perfectly bonded to the panel by the adhesive and all the three degrees of freedom of the nodes are appropriately coupled at the respective interfaces to depict the perfectly bonded behavior. In the thickness direction, the panel is meshed with five elements, adhesive with two elements outside the notch but five elements inside the notch, and patch with three elements. Thus, a 3D three-layer FE model (see Fig. 3) which includes 20,640 hexahedral elements and 23,903 nodes is generated to model stress or strain patterns of notched specimen repaired with bonded fibre/epoxy prepreg in association with relevant material properties listed in Tables 1 and 2. A tensile load is being applied to the top surface of the panel and the bottom face is arrested (see Fig. 3).

As it considers the interaction between longitudinal, transverse and through-thickness strengths of material, the Tsai-Wu criterion[32] seems more appropriate and effective for predicting the failure of E-glass/epoxy composites, etc. as compared with, for example, the maximum stress or strain rule. However, Tsai-Wu is not suitable for isolating individual damage modes and has fallen out of favor over the years for this reason and for its history in metal failure theories. In fact, the maximum stress or strain approach, although simplistic, is effective in identifying damage modes, or at least dominant stress or strain components.[33] Thus the maximum stress or strain rule is used for identifying failure modes associated with the individual stress or strain components in this work. Actually, dominant failure modes of repaired specimens include the debonding of adhesive layer resulted from shear stress, fracture of substrate panel arising from biaxial stress and shear delamination of composite patch caused by interlaminar shear stress. Therefore, maximum shear stress or strain criterion, maximum principal stress rule and maximum interlaminar shear stress criterion are respectively applied for identifying failure modes of adhesive layer, substrate panel and shear delamination of composite patch in association with relevant material strengths listed in Tables 1 and 2.

In order to identify failure mode of repaired panel, a new solution technique based on birth–death element is developed to allow solution of the stress pattern of repaired panel by

Table 1 Material properties of LY12CZ and SY-24C.

Property	LY12CZ	SY-24C
Elasticity modulus E (GPa)	71	10
Poisson's ratio v	0.3	0.3
Tensile strength X_{1t} (GPa)	400	/
Shear strength X_{12} (MPa)	/	26.3

Table 2 Material properties of T300/3234 composites patch.

Property	Value
Elasticity modulus E_1 (GPa)	140
Elasticity modulus E_2 (GPa)	9
Elasticity modulus E_3(GPa)	9
Poisson's ratio v_{12}	0.3
Poisson's ratio v_{13}	0.3
Poisson's ratio v_{23}	0.02
In-plane shear modulus G_{12} (GPa)	4.7
Inter-laminar shear modulus G_{13} (GPa)	4.7
Inter-laminar shear modulus G_{23} (GPa)	4.7
In-plane shear strength X_{12} (MPa)	111
Inter-laminar shear strength X_{13} (MPa)	25
Inter-laminar shear strength X_{23} (MPa)	30

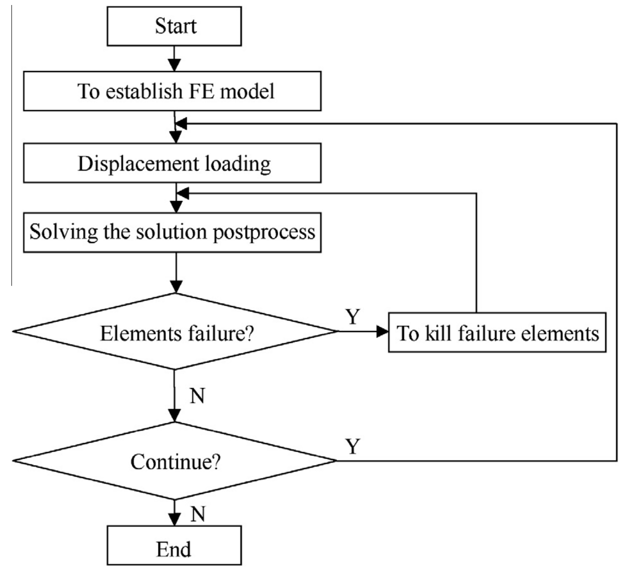

Fig. 4 Flowchart of solution technique involving birth–death element.

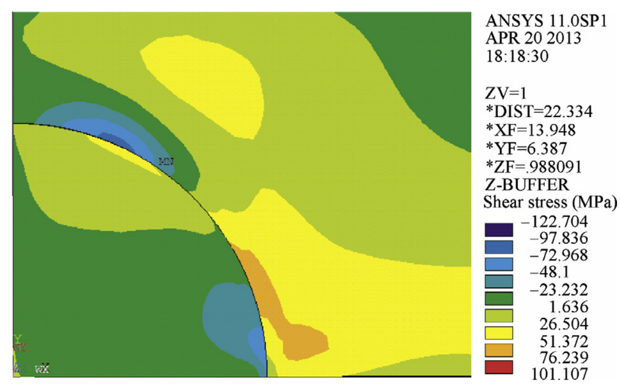

Fig. 5 Shear stress at the interface between adhesive layer and substrate panel (MPa).

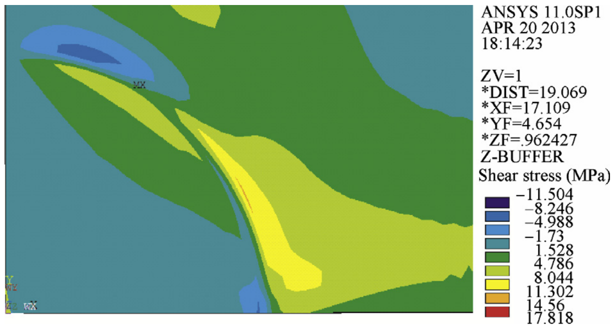

Fig. 6 Shear stress pattern of adhesive layer (MPa).

using the above three-layer FE model and failure criteria. Fig. 4 shows a flowchart of the implemented solution technique involving birth–death element. In the case of displacement loading, stress or strain pattern of repaired panel is simulated and recorded by means of three-layer FE model, and the failure elements are identified from maximum stress or strain criterion and stress pattern; after this, the stiffness matrix of failure element is multiplied by a small number to lose the influence on stiffness matrix of repaired panel, thus the load on failure element becomes zero and this is called the element killing or element death. With such-and-such iterative interpolation calculation, failure process of repaired panel is simulated and failure mode is observed.

In order to verify the numerical results of notched specimen repaired with bonded fibre/epoxy prepreg with above-described FE model, failure criteria and solution technique, a comparison is carried out between the experimental load–displacement curves and the numerical results (see Fig. 2). Fig. 2 demonstrates that the predicted values to construct the *P*–*D* curve from the FE results are in good agreement with those from the experiment. The stress patterns of repaired specimens obtained from the FE model are shown in Figs. 5 and 6. From Figs. 5 and 6, it can be seen that stress concentration occurs around notch edge at the interface between aluminum-alloy substrate panel and composite patch. This implies that the failure initiation likely appears first around notch edge at the interface (i.e., adhesive layer) between substrate panel and composite patch. The damage pattern (see Fig. 7) in adhesive layer is then determined from applying the maximum shear stress or strain criteria to the stress patterns for indicating the failure initiation location around notched edge at adhesive layer between substrate panel and composite patch. This is exactly consistent with the findings about the failure initiation process and mode in tests.

3. Material parameter modeling

Actually, there is some material parameters of adhesive layer and composite patch which significantly influence the residual strength of notched panel repaired with bonding composite patch. It is necessary to understand structural behavior of the repaired panel for providing important information as a

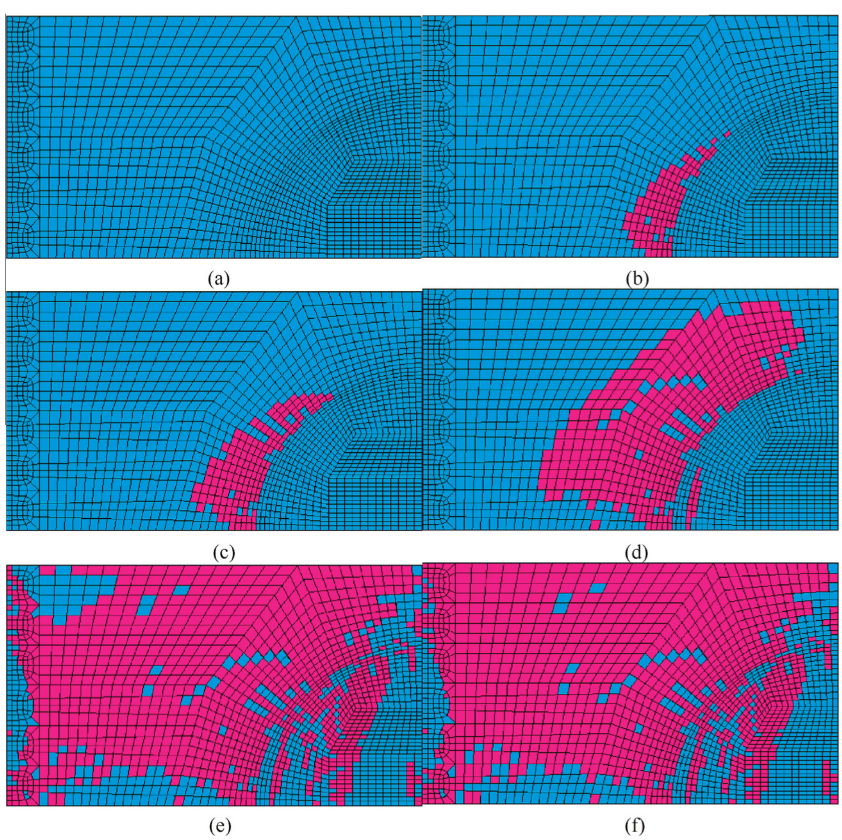

Fig. 7 Failure process of adhesive layer.

basis for designers to decide which material parameters are the best choice to meet operational and environmental constraints. Thus the material parameters of adhesive layer and composite overlay were modeled for use in bonded composites patch repair schemes to cracked aluminum-alloy panel by applying a combination of polymer science, finite element modeling, adhesion and surface engineering as well as mechanics.

For different elastic modulus or shear modulus of adhesive layer, the relationship between elastic modulus of adhesive layer and residual strength of repaired panel is obtained from three-layer FE model, maximum shear strain criteria and solution technique mentioned in Section 2 (shown in Fig. 8). Similarly, for different thickness of adhesive layer, the relationship between the thickness of adhesive layer and residual strength of repaired panel is shown in Fig. 9. From Figs. 8 and 9, it is seen that the residual strength of repaired panel has a significant rise with the increasing elastic modulus as well as adhesive layer thickness until 23 GPa and 0.3 mm, and then becomes constant; alternatively, 23 GPa and 0.3 mm are probably the optimal values of elastic modulus and thickness of adhesive layer for repair applications. Obviously, an appropriate elastic modulus and adhesive layer thickness can significantly enhance the static strength of repaired. It is worth pointing out that some commercially adhesives such as epoxies, phenolic and siliconesresin adhesives, etc. with higher elastic modulus are available for repair application. The thickness increase likely causes higher peel stress in adhesive layer and the peel debonding of adhesive layer, thus the thickness of adhesive layer needs to be elaborately selected by further theoretical or experimental methodology.

The effects of composite thickness (i.e. number of plies), elastic modulus and length–width ratio appear to be a sensitive parameter affecting the stress distribution in the repaired panel. This study is made to establish the relationships between material parameter of composite patch and residual strength of repaired panel (shown in Figs. 10–12). Figs. 10–12 show that the residual strengths of repaired panel increase with the increasing elastic modulus, thickness and length–width ratio of bonding patch until 123 GPa, 20 mm and 3.35, and then become less. This implies that the thickness, elastic modulus and length–width ratio of composite patch have a significant influence on static strengths of repaired specimen. An appropriate thickness, elastic modulus and length–width ratio of bonding patch can significantly enhance static strength of repaired specimen, while an improper thickness, elastic modulus and length–width ratio can cause slight benefit, perhaps even adverse effects. 123 GPa, 20 mm and 3.35 seem to be the optimal values of elastic modulus, thickness and length–width ratio of bonding patch for repair applications. In general, commer-

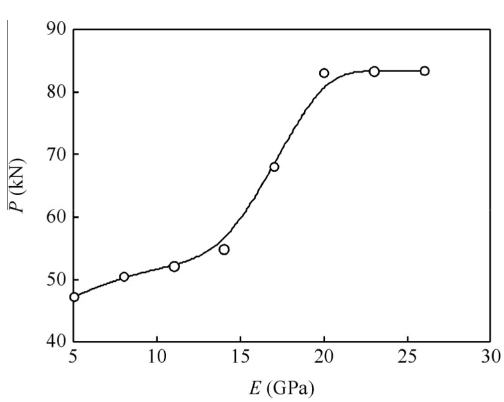

Fig. 8 Residual strength (ultimate load) of repaired panel with elastic modulus of adhesive layer.

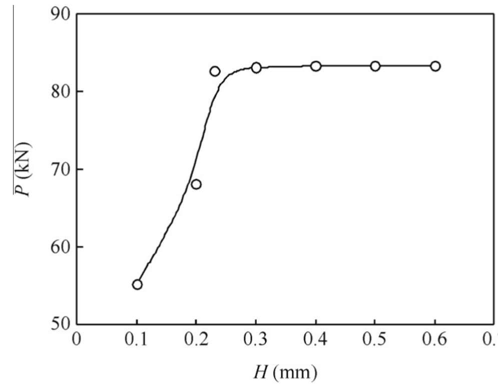

Fig. 9 Residual strength (ultimate load) of repaired panel with the thickness of adhesive layer.

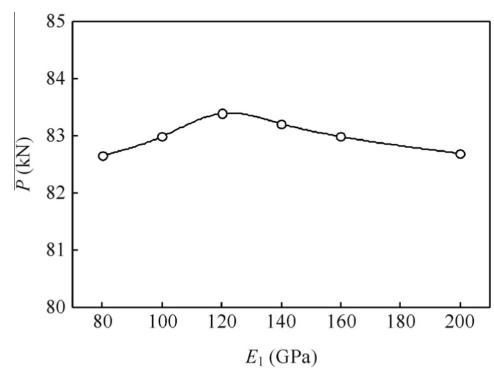

Fig. 10 Residual strength (ultimate load) of repaired panel with elastic modulus of composite patch in first principal direction.

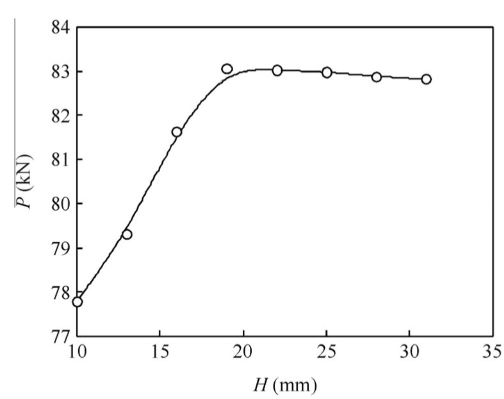

Fig. 11 Residual strength (ultimate load) of repaired panel with the thickness of composite patch.

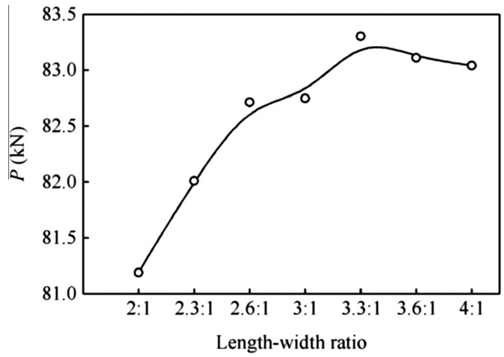

Fig. 12 Residual strength (ultimate load) of repaired panel with length–width ratio of composite patch.

cially available balanced Graphite/Epoxy laminate (e.g., T300/3234 lamina, etc.) with elastic modulus variation of 100–200 GPa can be considered as the patch materials for repair application.

4. Conclusions

The focus of this paper has been to present a new three-layer FE model with many degrees of freedom and a novel solution technique using birth–death element to solve the stress pattern for identifying failure mode of notched metallic panel repaired with bonded composite patch. The applicability of the new model and solution technique has been shown for a bonded composites patch repair to cracked aluminum-alloy panel based on static strength. The significant features of this work are threefold.

(1) A new three-layer FE model with many degrees of freedom is implemented using a higher order 3D 8-noded isotropic solid element and 8-noded anisotropic layered solid element to model the notched substrate panel repaired with adhesively bonded composite patch. The predicted *P–D* curve from the FE results is in good agreement with those from the test.
(2) Novel solving technique based on birth–death element is developed to analyze the stress pattern of repaired panel for identifying failure mode. The simulated damage pattern in adhesive layer is exactly consistent with the findings about the failure initiation process and mode from tests.
(3) An FE based study is done to predict the influences of material parameter of adhesive layer and composite patch on the residual strength of the repaired panel. From the simulations, it can be observed that an appropriate elastic modulus and thickness of adhesive layer can significantly enhance static properties of repaired panel, whereas an improper elastic modulus and thickness can cause slight benefit, perhaps even adverse effects. Similarly, an appropriate thickness, elastic modulus and length–width ratio of bonding patch can significantly enhance static properties of repaired specimen, while an improper thickness, elastic modulus and length–width ratio can cause slight benefit, perhaps even adverse effects.

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