1st International Conference on Structural Integrity, ICONS-2014

Collapse Pressure Analysis in Torsion of a Functionally Graded Thick-Walled Circular Cylinder under External Pressure

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Abstract

A hollow thick-walled cylinder made of functionally graded material subjected to twist has been analyzed. The objective of this paper is to provide the guidance to design the torsion of thick-walled cylinder made of functionally graded material so that collapse of cylinder due to external pressure can be avoided. The concept of transition theory based on the concept of Lebesgue strain measure has been used to simplify the constitutive equations. Results have been analyzed theoretically and discussed numerically. From the discussion, we can conclude that highly non-homogeneous cylinder is on the safer side of the design as compared to lesser non-homogeneous cylinder. This is because of the reason that percentage increase in pressure required for initial yielding to become fully plastic is very high for non-homogeneous cylinder whose non-homogeneity increases radially as compared to lesser non-homogeneous cylinder. Also, non-homogeneous cylinder whose thickness is very high is on the safer side of the design as compared to less thick-walled cylinders because percentage increase in pressure required for initial yielding to become fully plastic is very high for highly thick-walled cylinders as compared to other cases.

1. Introduction

Torsion of cylindrical bodies such as pressure vessels, pipes, borer, and driving shafts are very common and useful parts of several engineering industry and found in many text books [1-2]. A semi-elliptical crack placed on the outer surface of the circular cross-section of these bodies is considered to model the actual defects. The solution is constructed for the stress and displacement of functions in the form of an infinite integral operator and is established for the shear modulus by Nazarov [3] in his paper on torsion of cylindrically anisotropic inhomogeneous solids of revolution. Nazarov et.al. [4] investigated classes of functions and general solutions in the form of finite integral operator for the stress and displacement functions for shear moduli which depend on a cylindrical system of
coordinates. Taliercio [5] discussed the influence of the different material constants on the stress distribution and the torsional rigidity in his article on torsion of micropolar hollow circular cylinders. Doostfatemeh et.al. [6] developed some analytical formulas for torsion of hollow tube with straight and circular edges and concluded that these formulas can be used for torsion of moderately thick-walled hollow tubes. In analyzing the above problems, these authors assumed incompressibility of the material, a yield condition and power relationship between stress and strain. In most of the cases, it is not possible to find a solution in closed form without this assumption. Transition theory [7] does not require these assumptions and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized principal strain measure [8] and asymptotic solution at the critical points of the differential equations defining the deformed field. It has been successfully applied to several problems i.e. Aggarwal et.al. [9-10] investigated safety factors in thick-walled functionally graded cylinder under internal and external pressure and concluded that functionally graded cylinder is better choice for designers as compared to cylinders made up of homogeneous materials. As transition from one state to another is a natural phenomenon and the aim of our study is to identify the transition state and to eliminate the need of assuming yield conditions, semi empirical laws etc. to analyze the behavior of stresses in transition state. Also, we determine the safety factor for a functionally graded thick-walled circular cylinder subjected to torsion under external pressure using generalized Lebesgue measure to avoid collapse. Seth [8] has defined the generalized principal strain measure $\varepsilon_{ii}$ by taking the Lebesgue integral of the weighted function,

$$
\varepsilon_{ii} = \int_{0}^{1} \left(1 - 2e_{ii}^A \right)^{n-1} de_{ii}^A = \frac{1}{n} \left[ 1 - \left( 1 - 2e_{ii}^A \right)^{n-1} \right] ; \text{ where } n \text{ is the measure and } e_{ii}^A \text{ is the principal Almansi strain component.}
$$

2. Mathematical Formulation

Consider a functionally graded stainless steel composite thick-walled circular cylinder of internal and external radii $a$ and $b$ respectively, subjected to external pressure $p$.

The components of displacement in cylindrical polar coordinates are given by

$$
u = r(1 - \beta) ; \quad v = \alpha r z \quad \text{and} \quad w = d z ,
$$

where $\beta$ is a function of $r = \sqrt{x^2 + y^2}$, $d$ is a constant and $\alpha$ is the angle of twist per unit length.

Non-homogeneity in the material is due to various components. Here, we consider variable compressibility of the material as non-homogeneity in the stainless steel composite cylinder as

$$
C = C_0 \left( \frac{r}{b} \right)^{-k} ; \quad \text{where } a \leq r \leq b ; \quad C = \frac{2\mu}{(\lambda + 2\mu)} ; \quad C_0 \text{ and } k \leq 0 \text{ are constants.}
$$

The generalized components of strain [7] are given as follows:
\[ e_{rr} = \frac{1}{n} \left[ 1 - (r \beta' + \beta)^n \right], \quad e_{\theta \theta} = \frac{1}{n} \left[ 1 - \beta^n \right], \quad e_{zz} = \left( \frac{D^n}{n} \right) \left[ 1 - \left( \frac{\alpha r \beta}{D} \right)^n \right], \quad e_{\theta z} = \frac{1}{n} \left( \alpha^{n/2} r^{n/2} \beta^n \right), \]

\[ e_{r \theta} = e_{z r} = 0, \]

where \( D^n = \left[ 1 - (1 - d)^n \right] \) is a constant, \( n \) is the measure of deformation, \( e_{rr}, e_{\theta \theta}, e_{zz} \) are normal strain components while \( e_{r \theta}, e_{\theta z}, e_{z r} \) are shear strain components and \( \beta' = \frac{d \beta}{dr} \).

The stress strain relation for elastic isotropic material is

\[ T_{ij} = \lambda \delta_{ij} I_1 + 2 \mu e_{ij}, \quad (i, j = 1, 2, 3) \]  

(4)

where \( I_1 = \frac{1}{n} \left[ 3 - (r \beta' + \beta)^n - \beta^n - (1 - d)^n \right] \).

Equation (4) can be written as

\[
T_{r r} = \frac{\lambda + 2 \mu}{n} \left[ 1 - (r \beta' + \beta)^n \right] + \frac{\lambda}{n} (1 - \beta^n) + \lambda \left( \frac{D^n}{n} \right) \left[ 1 - \left( \frac{\alpha r \beta}{D} \right)^n \right],
\]

\[
T_{\theta \theta} = \frac{\lambda}{n} \left[ 1 - (r \beta' + \beta)^n \right] + \frac{\lambda + 2 \mu}{n} (1 - \beta^n) + \lambda \left( \frac{D^n}{n} \right) \left[ 1 - \left( \frac{\alpha r \beta}{D} \right)^n \right],
\]

\[
T_{z z} = \frac{\lambda}{n} \left[ 1 - (r \beta' + \beta)^n \right] + \lambda (1 - \beta^n) + (\lambda + 2 \mu) \left( \frac{D^n}{n} \right) \left[ 1 - \left( \frac{\alpha r \beta}{D} \right)^n \right],
\]

\( T_{\theta z} = \frac{2 \mu}{n} \left( \alpha^{n/2} r^{n/2} \beta^n \right). \)

where \( T_{r r}, T_{\theta \theta}, T_{z z} \) are normal stress components while \( T_{r \theta}, T_{\theta z}, T_{z r} \) are shear stress components. Here \( \lambda \) and \( \mu \) are Lame’s constant and \( \alpha \) is the coefficient of thermal expansion.

The twisting couple \( M \) is given by

\[ M = 2 \pi \int_a^b r^2 T_{\theta z} dr, \quad T_{\theta z} \text{ is shear stress component.} \]

Equations of equilibrium in cylindrical polar coordinates are

\[
\frac{\partial T_{r r}}{\partial r} + \frac{1}{r} \frac{\partial T_{r \theta}}{\partial \theta} + \frac{\partial T_{r z}}{\partial z} + T_{r r} - T_{\theta \theta} = 0
\]

(6)

All equations of equilibrium are satisfied except first equation from above equations and also \( T_{r \theta} = T_{z r} = 0 \), so first equation of equilibrium becomes

\[
\frac{\partial}{\partial r} T_{r r} + \frac{1}{r} T_{r r} - T_{\theta \theta} = 0
\]

The differential equation which comes out to be non-linear at transition state is obtained by substituting equations (4) in equation (6) as,
\[ n \beta^{n+1} p(p+1)^{n-1} \frac{dp}{d\beta} = -n \beta^n p[(p+1)^n + (1-C)] - \frac{(3-4C) r C'}{C(3-2C)} \left[ 1 - \beta^n (p+1)^n \right] - \left(1 - \beta^n \right) \left[ \frac{(2C^2 - 4C + 3) r C'}{C(3-2C)} + C \right] \frac{(2C^2 - 4C + 3) r C'}{C(3-2C)} D^n \left[ 1 - \left( \frac{\alpha r \beta}{D} \right)^n \right] - \frac{n(1-C)(1+p)}{D^{n-1}} \left[ \left( \frac{\alpha r \beta}{D} \right)^n \right] , \]  

where \( r = \beta P \), \( C = \frac{2\mu}{\lambda + 2\mu} \), \( P \) is function of \( r \).

The transition points of \( \beta \) in equation (7) are obtained as \( P \to 1 \) and \( P \to \pm \infty \).

The boundary conditions are \( T_{rr} = 0 \) at \( r = a \); \( T_{rr} = -p \) at \( r = b \).

The resultant axial force in the circular cylinder is given by \( \int_a^b r T_{zz} \, dr = 0 \).

3. Solution Through Principal Stress

To determine the plastic stresses at the transition point \( P \to \pm \infty \), we define the transition function \( R \) in terms of \( T_{rr} \) as

\[ R = T_{rr} - B = \frac{3}{(3-2C)Cn} \left[ 2 - \beta^n \left\{ (p+1)^n + (1-C) \right\} \right] + \frac{3(1-C)D^n}{C(3-2C)n} \left[ 1 - \left( \frac{\alpha r \beta}{D} \right)^n \right] - B. \]  

Taking the logarithmic differentiation of equation (10) w. r. t. \( r \) and substituting the value of \( \frac{dp}{d\beta} \) from equation (7), taking the asymptotic value as \( P \to \pm \infty \) and then by integration we obtain,

\[ R = A \exp f(r) , \]  

where \( A \) is a constant of integration and \( f(r) = \int C r^{-1} \, dr \).

Using equation (10) and (11), we have \( T_{rr} = A \exp f(r) + B \).

Using boundary conditions (8) in equation (12), we have

\[ T_{rr} = A \left[ \exp f(r) - \exp f(b) \right] - p, \quad T_{\theta \theta} = A \left[ (1-C) \exp f(r) - \exp f(b) \right] - p, \]

\[ T_{zz} = \frac{(1-C)}{(2-C)} (T_{rr} + T_{\theta \theta}) + \frac{3}{(2-C)} e_{zz} , \quad M = 2\pi \int_a^b r^2 T_{\theta \theta} \, dr \]

\[ T_{\theta z} = \frac{3\alpha r^{n/2}}{(3-2C)n} \left[ 1 - \frac{nC(3-2C)}{3+6C-4C^2} \left\{ \frac{(3-2C)}{C} (T_{\theta \theta} - T_{rr}) + \frac{1}{C} T_{\theta \theta} \right\} \right] , \]  

where \( e_{zz} = \frac{D^n}{n} \left[ 1 - \left( \frac{\alpha r}{D} \right)^n \left\{ 1 - \frac{nC(3-2C)}{3+6C-4C^2} \left\{ \frac{3-2C}{C} (T_{\theta \theta} + T_{rr}) + \frac{1}{C} T_{\theta \theta} \right\} \right\} \right] , \]

\[ D^n = \left\{ \int_a^b \frac{3\alpha r^{n+1}}{2-C} \left[ 1 - \frac{n(3-2C)^2}{3+6C-4C^2} \left( T_{\theta \theta} + T_{rr} \right) - \frac{n(3-2C)}{3+6C-4C^2} T_{\theta \theta} \right] \, dr \right\} / \left\{ \int_a^b \frac{3r}{n} \frac{1}{(2-C)} \, dr \right\} . \]

Using the non-homogeneity in the cylinder due to variable compressibility, equation (13) gives
\[ T_{rr} = A_1 \left[ \exp\left( \frac{C_0}{k} \left( \frac{r}{b} \right)^{-k} \right) - \exp\left( \frac{C_0}{k} \right) \right] - p, \quad T_{\theta\theta} = A_1 \left[ (1 - C_0 r^{-k}) \exp\left( \frac{C_0}{k} \left( \frac{r}{b} \right)^{-k} \right) - \exp\left( \frac{C_0}{k} \right) \right] - p, \]

\[ T_{zz} = \frac{1}{X} \left[ 1 - C_0 \left( \frac{r}{b} \right)^{-k} \right] \left( T_{rr} + T_{\theta\theta} \right) + \frac{3}{X} e_{zz}, \quad X = \left( 2 - C_0 \left( \frac{r}{b} \right)^{-k} \right), \quad M = 2\pi \int_a^b r^2 T_{\theta\theta} \, dr, \]

\[ T_{\theta z} = \frac{3\alpha r^{n/2}}{(3 - 2C_0 \left( \frac{r}{b} \right)^{-k})} \left[ 1 - \frac{nC_0 \left( \frac{r}{b} \right)^{-k}}{3 + 6C_0 \left( \frac{r}{b} \right)^{-k}} - \frac{(3 - 2C_0 \left( \frac{r}{b} \right)^{-k})}{C_0 \left( \frac{r}{b} \right)^{-k}} \right] (T_{\theta\theta} - T_{rr}) + \frac{1}{C_0 \left( \frac{r}{b} \right)^{-k}} T_{\theta\theta}, \quad (14) \]

where \( e_{zz} = [e_{zz}]_{C=0 \left( \frac{r}{b} \right)^{-k}}, \quad D^\alpha = \left[ D^\alpha \right]_{C=0 \left( \frac{r}{b} \right)^{-k}}. \)

From equation (14), we get
\[ T_{\theta\theta} - T_{rr} = -A_1 C_0 \left( \frac{rb^{-1}}{k} \right)^{-k} \exp\left( C_0 k^{-1} \left( \frac{r}{b} \right)^{-k} \right) \]

(15)

It can be seen from equation (15) that \( T_{\theta\theta} - T_{rr} \) is maximum at \( r = \left( e^2 b^k \right)^{1/k} = r_1 \) and we have
\[ |T_{\theta\theta} - T_{rr}|_{r=r_1} = -A_1 C_0 (r_1 / b)^{-k} \exp\left( C_0 k^{-1} (r_1 / b)^{-k} \right) = Y, \]

(16)

where \( A_1 = p / \exp\left( C_0 k^{-1} (a / b)^{-k} \right) - \exp\left( k^{-1} C_0 \right) \).

Pressure required for initial yielding is given as \( |P_i| = 1 / |A_2| \),

(17)

where \( P_i = p / Y, \quad A_2 = k / e \left[ \exp\left( C_0 k^{-1} (a / b)^{-k} \right) - \exp\left( k^{-1} C_0 \right) \right]. \)

For full plasticity \( (C_0 \to 0) \), equation (15) becomes
\[ |T_{\theta\theta} - T_{rr}|_{r=a} = \left| \frac{p}{(a^{-k} - b^{-k})} \right| = Y_1, \]

(18)

Pressure required for fully plastic is given by \( P_f = |1 / A_3| \),

(19)

where \( P_f = p / Y_1, \quad A_3 = k / \left\{ b^k (a^{-k} - b^{-k}) \right\}. \)

Now components in non-dimensional form are
\[ R = (r / b); \quad R_0 = (a / b); \quad \sigma_{rr} = (T_{rr} / Y); \quad \sigma_{\theta\theta} = (T_{\theta\theta} / Y); \quad \sigma_{zz} = (T_{zz} / Y); \quad M_1 = M / Y. \]

The necessary external pressure required for initial yielding in non-dimensional form is given by
\[ |P_i| = 1 / |A_2|, \quad \text{where} \quad A_2 = k / e \left[ \exp\left( -e^2 R_0^{-k} \right) - \exp\left( -e^2 \right) \right]. \]

(20)

The transitional stresses from equation (14) in non-dimensional form are given as
\[ \sigma_{rr} = \left\{ \frac{P_i}{X_1} \right\} \left[ \exp\left( \frac{C_0 R^{-k}}{k} \right) - \exp\left( \frac{C_0}{k} \right) \right] - P_i, \quad \sigma_{\theta\theta} = \left\{ \frac{1 - C_0 k^{-k}}{(2 - C_0 k^{-k})} \right\} \left( \sigma_{rr} + \sigma_{\theta\theta} \right) + \frac{3}{(2 - C_0 k^{-k})} e_{zz}, \]

\[ \sigma_{zz} = \frac{1}{X_1} \left[ (1 - C_0 R^{-k}) \exp\left( \frac{C_0 R^{-k}}{k} \right) - \exp\left( C_0 / k \right) \right] - P_i, \quad M_1 = 2\pi \int_{\kappa_i}^{1} R^2 b^h \sigma_{\theta z} \, dR, \]
\[
\sigma_{zz} = \frac{3\alpha (Rb)^{n/2}}{(3-2C_0R^{-k})^n} \left[ 1 - \frac{nC_0R^{-k}(3-2C_0R^{-k})}{3+6C_0R^{-k}-4C_0^2R^{-2k}} \left( \frac{3-2C_0R^{-k}}{C_0R^{-k}}(\sigma_{\theta\theta} - \sigma_{rr}) + \frac{1}{C_0R^{-k}}T_{\theta\theta} \right) \right],
\]
(21)

where \( X_1 = \left\{ \exp \left( \frac{C_0R_{0}^{-k}}{k} \right) - \exp \left( \frac{C_0}{k} \right) \right\} \),

\[
e_{zz} = \frac{D^n}{n} \left[ 1 - \left( \frac{Rb\alpha}{D} \right)^n \right] \left[ 1 - \frac{nC_0R^{-k}(3-2C_0R^{-k})}{3+6C_0R^{-k}-4C_0^2R^{-2k}} \left( \frac{3-2C_0R^{-k}}{C_0R^{-k}}(\sigma_{\theta\theta} - \sigma_{rr}) + \frac{\sigma_{\theta\theta}}{C_0R^{-k}} + \frac{3\theta_1 \log R}{(2-C_0R^{-k}) \log R_0} \right) \right],
\]

\[
D^n = \frac{1}{n} \int_{R_0}^{\frac{3bR}{2-C_0R^{-k}}} \frac{3bR}{(2-C_0R^{-k})} dR.
\]

The external pressure required for fully plasticity in non-dimensional form is given as

\[
|P_f| = \left| \frac{1}{A_3} \right|, \quad \text{where} \quad A_3 = k / \left\{ R_0^{-k} - 1 \right\}.
\]
(22)

Fully plastic stresses in non-dimensional form are obtained by taking \( C_0 \to 0 \) in equation (21) as

\[
\sigma_{rr} = (-P_f) \left( \frac{R^{-k} - 1}{R_0^{-k} - 1} \right) - P_f, \quad \sigma_{\theta\theta} = (-P_f) \left( \frac{R^{-k} - 1}{R_0^{-k} - 1} \right) - P_f, \quad \sigma_{zz} = \frac{1}{2} (\sigma_{rr} + \sigma_{\theta\theta}) + \frac{3}{2} e_{zz},
\]

\[
\sigma_{\theta z} = \left( \frac{\alpha Rb} {n} \right)^{n/2} (1 - 4n\sigma_{\theta\theta} + 3n\sigma_{rr}), \quad M_1 = 2\pi \int_{R_0}^{1} R^2 (\alpha Rb)^{n/2} \left( 1 - 4n\sigma_{\theta\theta} + 3n\sigma_{rr} \right) b dR.
\]
(23)

4. Numerical Discussion

To observe the effect of pressure required for initial yielding and fully plastic state against various radii ratios, figure 2 and table 1 have been drawn. It has been observed from figure 2(a), that external pressure required for initial yielding is maximum at internal surface. It has also been noticed that external pressure required for initial yielding increases significantly with the increase in non-homogeneity (i.e., with the increase in value of \( k \)). From figure 2(b), we have noticed that external pressure required for fully plastic state is maximum at internal surface which decreases with the increase in value of \( k \).
It has been noticed from table 1 that with the increase in radii ratio, pressure required for initial yielding decreases significantly. It has also been observed that with the increase in compressibility, pressure required for initial yielding decreases while pressure required for fully plastic state increases. Also, percentage increase in external pressure required for initial yielding to become fully plastic increases with increase in compressibility (i.e. $k = -0.04$) for the cylinder with radii ratio 0.5 as compared to other radii ratio cylinders.

Table 1: External pressure required for initial yielding and fully plastic state:

<table>
<thead>
<tr>
<th>Non-Homo./ Ext. Press.</th>
<th>$Re = 0.1$</th>
<th>$Re = 0.2$</th>
<th>$Re = 0.3$</th>
<th>$Re = 0.4$</th>
<th>$Re = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K = -0.1$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
</tr>
<tr>
<td>$P_t$</td>
<td>0.059596</td>
<td>0.033588</td>
<td>0.022041</td>
<td>0.015282</td>
<td>0.010755</td>
</tr>
<tr>
<td>$P_r$</td>
<td>2.05672</td>
<td>1.4865</td>
<td>1.13432</td>
<td>0.875506</td>
<td>0.66967</td>
</tr>
<tr>
<td>$K = -0.085$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
</tr>
<tr>
<td>$P_t$</td>
<td>0.0593736</td>
<td>0.03107</td>
<td>0.020789</td>
<td>0.014617</td>
<td>0.010399</td>
</tr>
<tr>
<td>$P_r$</td>
<td>2.09126</td>
<td>1.5042</td>
<td>1.14442</td>
<td>0.881517</td>
<td>0.673129</td>
</tr>
<tr>
<td>$K = -0.07$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
</tr>
<tr>
<td>$P_t$</td>
<td>0.046052</td>
<td>0.026735</td>
<td>0.015607</td>
<td>0.013581</td>
<td>0.010659</td>
</tr>
<tr>
<td>$P_r$</td>
<td>2.11473</td>
<td>1.5161</td>
<td>1.15121</td>
<td>0.885515</td>
<td>0.675438</td>
</tr>
<tr>
<td>$K = -0.055$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
</tr>
<tr>
<td>$P_t$</td>
<td>0.043044</td>
<td>0.025673</td>
<td>0.016349</td>
<td>0.013373</td>
<td>0.009724</td>
</tr>
<tr>
<td>$P_r$</td>
<td>2.16275</td>
<td>1.54026</td>
<td>1.16498</td>
<td>0.899585</td>
<td>0.680101</td>
</tr>
<tr>
<td>$K = -0.04$</td>
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<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
<td>$P_t$</td>
</tr>
<tr>
<td>$P_t$</td>
<td>0.038462</td>
<td>0.024575</td>
<td>0.017445</td>
<td>0.012793</td>
<td>0.009404</td>
</tr>
<tr>
<td>$P_r$</td>
<td>2.19973</td>
<td>1.55873</td>
<td>1.17544</td>
<td>0.899702</td>
<td>0.683626</td>
</tr>
</tbody>
</table>

To see the effect of pressure on transitional radial, circumferential and shear stresses, figures 3, 4 and table 2 are drawn. It has been noticed from figure 3 that radial and circumferential transitional stresses are compressive in nature and are maximum at external surface. It has also been observed that these stresses decreases with the increase in non homogeneity (i.e. increase in value of $k$). It has been observed from table 2 that circumferential stress increases significantly with the increase in radii $R$. Also, with the increase in compressibility, these stresses decrease significantly. It has also been noticed that with the increase in external pressure, circumferential stress increase significantly.

Figure 3. Radial and Circumferential transitional stresses for external pressure 10 and 20.

From figure 4 we can see that transitional shear stresses are tensile and are maximum at external surface. It has also been observed that shear stresses decrease at the internal surface while increases at the external surface with the increase in non homogeneity. Shear stresses increases significantly with the increase in external pressure. Also, shear stresses increases along the radii $R$ as can be seen from table 2. At the external surface, these shear stresses increases significantly with the increase in non-homogeneity i.e. increase in value of $k$. 
Table 2. Radial, Circumferential and Shear transitional stresses:

<table>
<thead>
<tr>
<th>External Pressure = 10</th>
<th>R = 0.3</th>
<th>R = 0.5</th>
<th>R = 0.7</th>
<th>R = 1.0</th>
<th>External Pressure = 20</th>
<th>R = 0.3</th>
<th>R = 0.5</th>
<th>R = 0.7</th>
<th>R = 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>σrr</td>
<td>0</td>
<td>-0.30312</td>
<td>-1.73026</td>
<td>-10</td>
<td>σrr</td>
<td>0</td>
<td>-0.60623</td>
<td>-3.46052</td>
<td>-20</td>
</tr>
<tr>
<td>σθθ</td>
<td>-0.12797</td>
<td>-1.94411</td>
<td>-10.436</td>
<td>-57.663</td>
<td>σθθ</td>
<td>-0.25594</td>
<td>-3.88822</td>
<td>-20.872</td>
<td>-115.327</td>
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To see the effect of pressure on fully plastic radial, circumferential and shear stresses, figures 5, 6 and table 3 are drawn. It has been observed from figure 5 that fully plastic circumferential stresses are tensile and are maximum at external surface. These stresses decrease with the increase in non-homogeneity (i.e. increase in value of k). With the increase in external pressure these stresses increases significantly. Also, it has been noticed from the table 3 that circumferential fully plastic stress increase with the increase in radii R. From figure 6, we have noticed that fully plastic shear stresses are uniform at the end points and vary in between the end points. As we increase the external pressure shear stresses increase significantly.
From the discussion, we can conclude that highly non-homogeneous cylinder is on the safer side of the design as compared to lesser non-homogeneous cylinder. This is because of the reason that percentage increase in pressure required for initial yielding to become fully plastic is very high for non-homogeneous cylinder whose non-homogeneity increases radially as compared to lesser non-homogeneous cylinder. Also, non-homogeneous cylinder whose thickness is very high is on the safer side of the design as compared to less thick-walled cylinders because percentage increase in pressure required for initial yielding to become fully plastic is very high for highly thick-walled cylinders as compared to other cases.

Also in case of torsion non-homogeneous cylinder without external pressure is safe for the designer’s point of view because shear stresses are minimum for non homogeneous cylinder without external pressure as compared to non-homogeneous cylinder with external pressure.

**References**