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Connections between two cycles – a new design of dense processor interconnection networks

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Abstract

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In this paper we attempt to maximize the order of graphs of given degree Δ and diameter D. These graphs, which are known as (Δ, D) graphs, are used as dense interconnection networks, i.e., processors with relatively few links are connected with relatively short paths. The method described in this paper uses periodic connections between two cycles of the same length. The results obtained give a significant improvement of the known lower bounds in many cases. Large bipartite graphs with a given degree and diameter were also obtained by our method. Again, the improvement of the lower bounds is significant.

1. Introduction

In this paper we discuss the problem of constructing large interconnection networks of given degree and diameter. The recent advances in technology such as very large scale integrated (VLSI) circuit technology, make it possible to construct large interconnection networks. Nevertheless, it is still desirable that the number of links from each processor be as small as possible. Also, short paths between the processors are needed to achieve fast communication. The topology of networks which achieve this goal can be modeled by graphs. For previous works on such graphs the reader is referred to the survey of Bermond, Delorme and Quisquater [2].

In this paper we present a new method to design large (Δ, D) graphs with a given degree and diameter. These graphs are based on two cycles of the same length n.

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The connection between the two cycles is periodic and based on some permutation of the integers $0, 1, \ldots, s-1$, where s is a small integer which divides n. The well-known Petersen graph, shown in Fig. 1, which is an optimal (3,2) graph is a special case of our construction. Our method can be viewed as a combination of the chordal rings [1], the generalized chordal rings [8], and the torus networks [11]. The graphs obtained by this method are significantly larger than the largest previously known (Δ, D) graphs [2]. Our method and the new lower bounds are presented in Section 2.

Another important problem is to find large (Δ, D) bipartite graphs [7], especially since these can be used to construct large (Δ, D) graphs [9]. The method described can be also used to obtain large (Δ, D) bipartite graphs. In Section 3 we give the conditions for which our method generates bipartite graphs and tables for the new lower bounds.

2. The general construction

Let $G=(V,E,F,s,\Delta)$ be a graph with degree Δ, with a set F of $\Delta-2$ functions on the integers $0, 1, \ldots, s-1$, having order $2n$, where n is divisible by s. For each function F_i, $1 \le i \le \Delta-2$, $F_i(0) \bmod s$, $F_i(1)+1 \bmod s$, $F_i(2)+2 \bmod s, \ldots, F_i(s-1)+(s-1) \bmod s$ is a permutation of the integers $0, 1, \ldots, s-1$. The vertices of G are defined by $V=\{(i,0) \mid 0 \le i \le n-1\} \cup \{(i,1) \mid 0 \le i \le n-1\}$. E is union of the sets E_i, $0 \le i \le \Delta-2$ defined by

$$E_0 = \{[(i,j), ((i+1) \bmod n, j)] \mid 0 \le i \le n-1,\ j=0,1\}$$

and

$$E_i = \{[(j,0), ((j+F_i(j \bmod s)) \bmod n, 1)] \mid 0 \le j \le n-1\}, \quad 1 \le i \le \Delta-2.$$

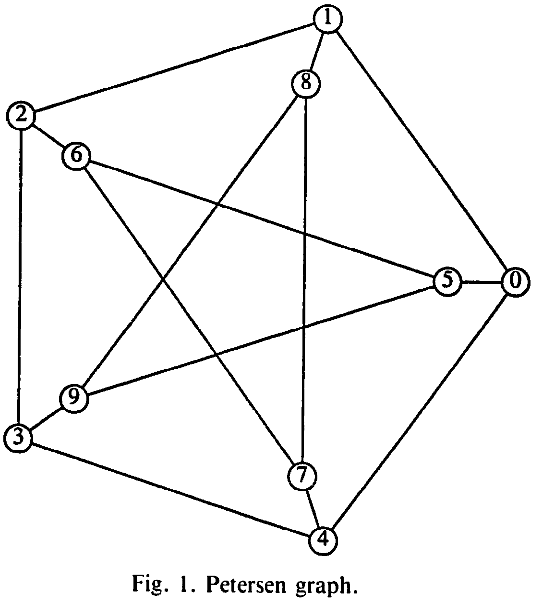

Fig. 1. Petersen graph.

The structure of the graph is two cycles of the same length defined by the edges of E_0. The edges $[(j,0),(j+F_i(j \bmod s),1)]$, $0 \le j \le s-1$, $1 \le i \le \Delta-2$, connect the vertices $(j,0)$, $0 \le j \le s-1$, of the first cycle to vertices of the second cycle. This picture from the s vertices $(j,0)$, $0 \le j \le s-1$, is repeated from any other s vertices $(j,0)$, $sk \le j \le s(k+1)-1$ for $0 \le k < n/s$. Hence, the picture from the s vertices $(j,1)$, $0 \le j \le s-1$, of the second cycle is repeated from any other s vertices $(j,1)$, $sk \le j \le s(k+1)-1$ for $0 \le k < n/s$. In order to check whether such a graph has a diameter D, it suffices to check the distance from $2s$ vertices to the other vertices. Those $2s$ vertices are s consecutive vertices on the first cycle and s consecutive vertices on the second cycle. We don't have to check the distances from the other vertices, since the connection between the two cycles has period s.

Computer search and some heuristic methods to find "good" values for the functions F_i, $1 \le i \le \Delta-2$, were used to obtain a significant improvement over the previous known bounds [2]. The first "good" values were found by using some random search. Then, to increase the number of vertices in the graph we used tuning on the previous values of the F_i to obtain new values. Another simple heuristic method to obtain large graphs from small ones (not necessarily with the same diameter) was to multiply the values of the previous F_i's by some factor and then to use some tuning on the new values. The search was done on SUN04 and CONVEX. For small graphs we used sometimes several days of CPU time mainly on the SUN04, while for larger graphs with more than 1,000,000 vertices we used CONVEX because of space limitations of the SUN04. Table 1 contains the known lower bounds for the order of (Δ, D) graphs, where our new lower bounds are marked by *. In Appendix A we give values of the functions F_i for the new graphs which attain the bounds of Table 1. The data is given as follow. In the first row we have $(\Delta, D) = N$, where N is the number of vertices in the graph. Then the functions are given, where two functions are separated by an /, and for function i the values are

Table 1. New lower bounds on (Δ, D) graphs

Δ	D							
	3	4	5	6	7	8	9	10
3	20	38	70	130	184	320	540*	938*
4	41	95	364	734	1081	2943	7439	15657
5	70	184*	532	2742	4378	12000*	40000*	132000*
6	105	355	1085	7832	13222*	51170*	210000*	900000*
7	136*	504*	2162	10554	39732	150000*	911088	4773696
8	203	842	3300*	39258	103776	480000*	2400000*	7738848
9	585	1254	6120*	74954	215688	1320000*	4773696	19845936
10	650	1820	12144	132932	486837	3000000*	9000000*	47059200
11	715	3200	14625	156864	898776	4200000	21345930	179755200
12	780	4680	24360	354422	1727180	10000000*	48493900	466338600
13	845	6560	33345	531440	2723040	15000000*	72541560	762616400

Table 2. New lower bounds on (Δ, D) bipartite graphs

Δ	D							
	3	4	5	6	7	8	9	10
3	14	30	56^x	126	156	248^x	432	672^x
4	26	80	152*	728	834	1540*	3440*	8000*
5	42	170	280*	2730	2984	6100*	19000*	60000*
6	62	312	540*	7812	8310	21000*	80000*	300000*
7	78	346	936	8992	23436	63000*	270000*	1400000*
8	114	800	1560	39216	40586	195300	900000*	3000000*
9	146	1770	2400	74898	117648	392160	2100000*	9000000*
10	182	1640	4000	132860	224694	1176480	4200000*	14979600

given in the order $F_i(0)$, $F_i(1)$, $F_i(2)$, and so on. We wish to remark that for considerably large graphs we have limited our search, though with more computer search the bounds obtained in Table 1 can be improved. We also didn't handle larger graphs because of limitations of computer space.

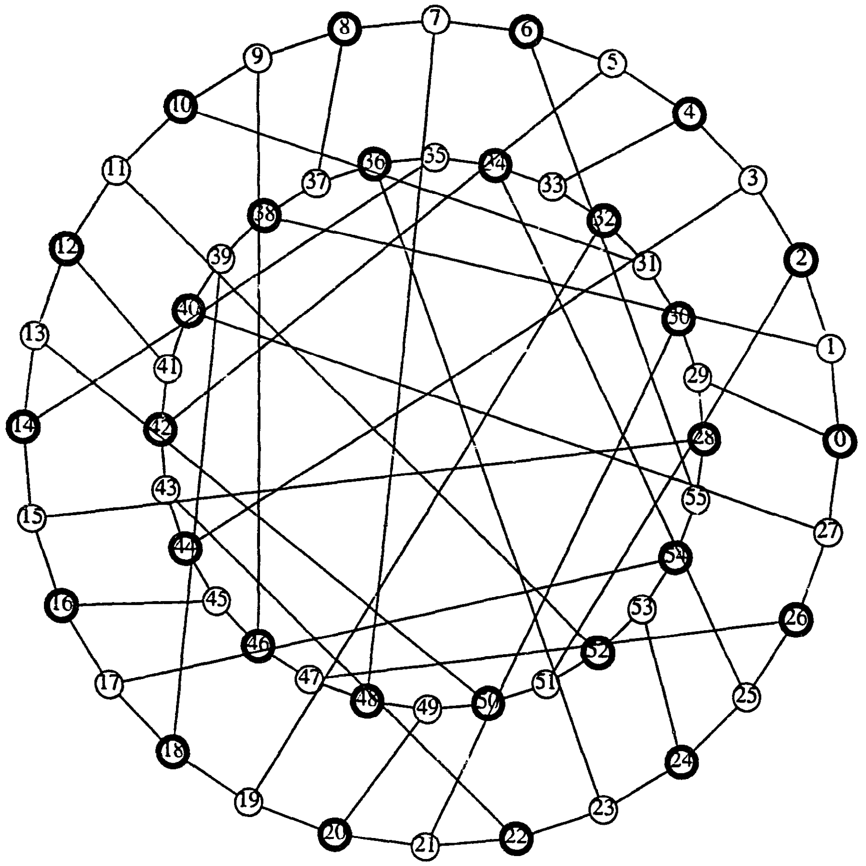

Fig. 2. A (3,5) bipartite graph with 56 vertices.

3. Large bipartite graphs

For generating large bipartite graphs with a given degree and diameter we use the same method. Since the graph should be bipartite, the lengths of the cycles should be even. Without loss of generality, the two partite sets of vertices for the bipartite graph are $V_1=\{(2i,j) \mid 0\leq i\leq n/2-1,\ j=0,1\}$ and $V_2=\{(2i+1,j) \mid 0\leq i\leq n/2-1,\ j=0,1\}$. Hence, each value $F_i(j)$, $1\leq i\leq \Delta-2$, $j=0,1,\ldots,s-1$ should be odd. Again we used computer search to find large graphs which satisfy the conditions described above. Table 2 contains the known lower bounds, where our new bounds are marked by * and bounds which are the same as the previous lower bounds are marked by x. The best previous lower bounds appear in [3]. In Appendix B we give the data for the new graphs which attain the bounds of Table 2. Finally, in Fig. 2 we show the (3,5) graph with 56 vertices obtained by our method, where instead of $(j,1)$, $0\leq j\leq 27$, we put $28+j$.

Note. Originally, we had some more lower bounds, for general (Δ,D) graphs, which are better from the ones mentioned in [2]. But, Jean-Claude Bermond informed us about references [3], [4], [6] and [10] which had better bounds from the ones we have obtained.

Appendix A: Graphs which attain the bounds of Table 1

(3,9) = 540
194, 33, 23, 108, 236, 258

(3,10) = 938
42, 29, 333, 183, 123, 147, 375

(5,4) = 184
10, 30, 38, 22 / 83, 43, 51, 15 / 77, 1, 61, 69

(5,8) = 12000
5336, 356, 2696, 4826, 4811, 5411, 4616, 5156, 5636, 1931, 1181, 5426, 4136, 1691, 4601 / 2356, 3661, 4471, 2461, 4261, 4606, 4921, 5536, 481, 1576, 4786, 226, 151, 4306, 3226 / 4073, 3878, 4298, 1058, 713, 4343, 68, 4883, 3773, 3623, 5213, 8, 1433, 983, 4298

(5,9) = 40000
1201, 17937, 7009, 1553, 14865, 7281, 3697, 19121, 3585, 1041, 14913, 16433, 1441, 1633, 6609, 15953 / 2710, 17254, 12246, 8518, 14262, 8198, 17974, 7206, 5062, 13686, 18566, 6630, 9462, 758, 870, 6150 / 11437, 8397, 7917, 3357, 11037, 17373, 5405, 3005, 19997, 6653, 13293, 9437, 6493, 19773, 11485, 15853

(5,10) = 132000
35454, 58209, 43839, 8079, 54774, 12009, 20424, 53379, 62919, 26469, 29694, 3804, 21414, 40674, 53049 / 24283, 6418, 41968, 42418, 59593, 1768, 41143, 44578, 8533, 63073, 35548, 44818, 27343, 55933, 53518 / 55082, 25337, 14582, 24827, 32237, 18857, 65462, 52487, 29597, 47882, 39272, 23117, 15632, 16847, 47042

(6,7) = 13222
6077, 2260, 115, 1919, 3580, 1666, 2381, 2821, 335, 3701, 2062 / 3758, 2713, 6013, 2614, 2361, 535, 4176, 1008, 1580, 5661, 1954 / 5182, 3444, 5314, 1552, 3697, 199, 2696, 4093, 4698, 804, 1079 / 2507, 681, 5037, 10, 1726, 5356, 802, 3992, 2089, 2067, 5928

(6,8) = 51170
1733, 16, 2974, 2073, 21538, 14449, 4521, 6952, 4878, 12834, 8992, 17917, 1750, 7292, 18580, 10131, 4079 / 11511, 7074, 121, 18413, 13177, 18039, 14367, 22884, 3963, 23904, 24227, 5136, 14979, 12004, 5425, 18583, 716 / 17974, 22088, 9593, 209, 13333, 13877, 3388, 25454, 17481, 3252, 7213, 11939, 21289, 21527, 17192, 25420, 17311 / 19545, 15754, 15125, 3837, 14071, 5622, 19290, 11453, 22843, 23489, 22231, 7832, 20429, 15312, 23319, 7152, 3174

(6,9) = 210000
66092, 31802, 85742, 27392, 49337, 1562, 73952, 87692, 38087, 80882, 38657, 67307, 37742, 100082, 36947 / 91273, 7258, 16153, 68638, 24808, 25273, 86488, 14938, 103108, 508, 47698, 75598, 4588, 43843, 35803 / 51788, 53093, 14753, 50243, 82493, 11378, 35438, 32093, 68603, 14378, 84308, 48248, 7958, 30548, 41903 / 6769, 20929, 79189, 72709, 49954, 41059, 20299, 69844, 32539, 92089, 94669, 21619, 79174, 55564, 3814

(6,10) = 900000
105729, 405684, 7554, 357609, 331314, 18369, 239034, 390669, 352134, 76404, 297534, 259239, 313059, 276729, 71184 / 162235, 19225, 91150, 252225, 109825, 422935, 210280, 123280, 322090, 410725, 55015, 30370, 366235, 182950, 98575 / 108752, 154247, 66857, 164372, 189167, 238697, 36767, 225212, 112217, 259847, 327122, 105902, 219362, 225617, 35252 / 126073, 808, 319603, 253393, 141403, 197593, 441928, 406348, 343663, 218893, 129163, 195808, 115708, 16438, 302278

(7,3) = 136

65, 37, 57, 65 / 48, 40, 4, 40 / 0, 12, 48, 20 / 29, 17, 45, 53 / 5, 1, 61, 13

(7,4) = 504

126, 69, 135 / 4, 158, 228 / 79, 115, 109 / 101, 35, 29 / 171, 48, 15

(7,8) = 150000

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(8,5) = 3300

1452, 681, 1635 / 1535, 50, 278 / 1201, 1321, 1228 / 804, 504, 1155 / 1092, 1443, 1356 / 1208, 1595, 818

(8,8) = 480000

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(8,9) = 2400000

910023, 1019688, 904833, 1143738, 809388, 857553, 430308, 414183, 979563, 156483, 892668, 467823, 865743, 1085238, 1032603 / 548173, 160273, 93508, 388198, 58033, 790303, 457963, 1188208, 1142128, 1003033, 1038928, 647818, 1002193, 141988, 50713 / 1140472, 87652, 609172, 420997, 202522, 774982, 1044322, 894457, 1099837, 247957, 18487, 278287, 314752, 169342, 476377 / 796161, 1161006, 391836, 854241, 772491, 1188336, 1061946, 536001, 755046, 1029591, 1025901, 333291, 870201, 545841, 401061 / 68305, 759250, 819160, 1124470, 182560, 499000, 792250, 971290, 307225, 185725, 602140, 153535, 52810, 708535, 894400 / 640936, 484186, 854026, 551431, 691141, 348691, 95011, 510316, 316276, 147226, 1092301, 605926, 30106, 375391, 872956

(9,5) = 6120

1503, 827, 1191, 955 / 525, 865, 353, 89 / 932, 2356, 2440, 2132 / 1273, 1473, 1285, 2609 / 1234, 1306, 1522, 2666 / 2294, 2138, 2618, 986 / 707, 887, 1215, 2195

(9,8) = 1320000

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(10,8) = 3000000

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(10,9) = 9000000

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(12,8) = 10000000

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(13,8) = 15000000

4609947, 2421627, 2269602, 6144507, 7015407, 6975402, 4866072, 6703227, 6284382, 6399792, 2516667, 4959882, 1529127, 5293977, 5738082 / 2779786, 639601, 521131, 7016236, 227476, 7209556, 769006, 4769761, 3991921, 4673701, 1136791, 597676, 3460906, 3228991, 3695371 / 5996707, 2069677, 613237, 3168652, 6060232, 2460397, 456277, 4895182, 6708937, 599092, 6393457, 3077602, 4422427, 1296367, 6995782 / 3654837, 3852357, 4727172, 4457742, 5436057, 2566482, 5002242, 6653292, 353472, 4656882, 5913192, 3450927, 3901827, 3272457, 1681017 / 4039346, 2304671, 3535781, 3258716, 2742176, 6874976, 3268481, 1798646, 4969856, 5177651, 1015046, 813011, 861281, 4068386, 929291 / 6018032, 645947, 1949177, 6454277, 967592, 44342, 649172, 5109362, 1432667, 2662592, 1232732, 5421587, 655202, 5821742, 4131977 / 1787785, 1083025, 2555905, 505930, 3430255, 1412245, 5359915, 1984765, 6324970, 3384730, 6597925, 4328560, 1186705, 6819355, 3415900 / 2924119, 6156919, 727294, 1718779, 3014554, 551539, 1519069, 4636324, 4319629, 1936969, 526144, 6237184, 6921574, 4282594, 1328719 / 5304682, 7246147, 5996377, 31672, 228307, 6839797, 314287, 2285632, 2562382, 2010877, 6784327, 5661682, 2041627, 1047172, 5516722 / 4854913, 766153, 532963, 5663803, 460138, 4147873, 3353098, 4860808, 2305783, 4088308, 67108, 659548, 3710038, 5296648, 4834738 / 7162284, 7438119, 1575459, 1696164, 2564664, 5911659, 741789, 4050534, 2093634, 2610744, 295749, 5270094, 2618304, 7088109, 5729289

Appendix B: Graphs which attain the bounds of Table 2

(3,5) = 56
12, 0, 8, 20

(3,8) = 248
48, 92, 80, 28

(3,10) = 672
77, 63, 39, 125, 301, 175, 221

(4,5) = 152
65, 25, 41, 9 / 57, 13, 33, 73

(4,8) = 1540
157, 477, 117, 567, 137 / 529, 669, 319, 439, 449

(4,9) = 3440
215, 835, 435, 1385, 295, 1655, 1685, 225, 565, 145 / 1499,
1309, 189, 719, 1119, 1449, 1019, 1169, 1399, 539

(4,10) = 8000
933, 3637, 245, 2277, 869, 437, 469, 309, 3045, 837, 933, 597,
821, 3189, 229, 277 / 1007, 1903, 2655, 2943, 1071, 1183, 607,
1743, 655, 1647, 3727, 2463, 2783, 1103, 1407, 2559

(5,5) = 280
71, 91, 31, 23 / 19, 35, 55, 131 / 83, 75, 3, 127

(5,8) = 6100
1591, 101, 1901, 671, 3011, 2111, 3021, 1031, 2661, 381 / 115,
245, 1225, 1255, 1565, 1485, 2305, 1875, 1295, 855 / 2139, 189,
859, 1269, 939, 2079, 299, 519, 99, 389

(5,9) = 19000
3021, 8781, 581, 7001, 7881, 5551, 3441, 5111, 8151, 7181 /
1315, 165, 5465, 2685, 135, 9095, 5685, 6355, 6195, 1935 /
4853, 3563, 1573, 6033, 2003, 1293, 5303, 3403, 913, 2243

(5,10) = 60000
17531, 23371, 7431, 5111, 871, 25591, 2831, 10131, 22811,
16051, 16671, 5251, 16831, 6671, 13191, 21451, 8631, 851,
29151, 25791 / 20299, 27019, 14899, 17739, 11459, 12419, 4259,
19759, 15399, 13359, 24659, 26599, 15939, 14339, 899, 11619,
2699, 8759, 6179, 16959 / 29687, 9367, 3607, 5587, 25387, 1307,
22367, 6467, 21107, 28227, 28347, 7807, 10247, 10827, 20387,
23927, 24227, 11167, 19567, 16727

(6,5) = 540
203, 53, 227 / 93, 39, 147 / 19, 109, 193 / 27, 171, 57

(6,8) = 21000
3375, 5865, 7455, 5305, 5735, 6425, 7445, 5005, 6785, 2325 /
1093, 9373, 2413, 5943, 5933, 9203, 1233, 423, 6923, 6913 /
6611, 5151, 7841, 7511, 4931, 3651, 3601, 1391, 3951, 8941 /
8321, 1941, 4021, 3901, 3821, 4311, 7681, 8011, 951, 8791

(6,9) = 80000
1779, 26299, 25879, 6799, 59, 13359, 1879, 37099, 26879, 32819,
35219, 37239, 13539, 21339, 7299, 38259, 33979, 16919, 24759,
2779 / 9757, 16457, 32437, 29137, 37837, 18677, 38477, 36337,
16017, 8937, 30797, 31317, 20197, 24437, 8437, 2457, 38837,
31137, 497, 8237 / 8791, 3971, 10111, 9291, 24131, 1391, 11431,
7771, 6311, 28391, 23351, 22031, 11491, 8811, 18651, 2811,
23591, 7111, 31831, 28151 / 18095, 25495, 9375, 4415, 9055,
5055, 6215, 12975, 35875, 7355, 31935, 22435, 28075, 26075,
29935, 17655, 19055, 8635, 21955, 5675

(6,10) = 300000
81541, 75451, 80371, 136501, 3931, 148171, 3721, 25831,
141121, 69811, 24301, 26221, 122641, 89671, 110161 / 139075,
4465, 142645, 58735, 148855, 33235, 128035, 4045, 147715,
43795, 85585, 110305, 86935, 22135, 47605 / 75981, 1491,
55311, 143691, 86751, 84801, 120201, 34761, 16911, 80061,
71661, 34701, 92871, 87531, 27921 / 20161, 127471, 83461,
119191, 106411, 101311, 81451, 78001, 83371, 122521, 42451,
42361, 105931, 108421, 134161

(7,8) = 63000
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29741, 2777, 23, 27095, 12587, 12155, 6467, 26663, 13091 /
29185, 14587, 19789, 7801, 18277, 5857, 12643, 7117, 18277,
28537, 30985, 7801, 11617, 25351, 7225, 27295, 29113, 4687 /
4617, 15489, 6345, 639, 19575, 14967, 25461, 13851, 5589, 3717,
17469, 12285, 30411, 23787, 12735, 2907, 13293, 8955 / 31013,
18251, 22553, 31265, 25199, 17171, 8639, 19889, 28061, 7127,
22103, 21131, 17027, 12113, 2807, 14795, 20735, 17639 / 25663,
23809, 20317, 9409, 20623, 20101, 14161, 4459, 9787, 20191,
22585, 5899, 26851, 6097, 553, 18463, 7645, 22765

(7,9) = 270000
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95119, 110689, 128899, 57319, 114949, 75499, 52399 / 119481,

120591, 102471, 70041, 36021, 74661, 58581, 115131, 45771,
57891, 104061, 90711, 53061, 9891, 95181 / 110081, 131861,
77951, 114881, 5141, 42641, 100811, 1781, 134261, 68111,
71891, 122411, 105131, 14441, 67511 / 12521, 124391, 128081,
76121, 118931, 124301, 129491, 6071, 2531, 110471, 25481,
113141, 117461, 110741, 29591 / 91365, 39255, 57165, 120915,
60375, 74955, 68235, 55455, 15525, 122655, 31815, 28635,
124455, 91275, 118935

$(7,10)=1400000$

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663869, 2141, 87789, 473949, 3165, 332877, 339917, 11629 /
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485185, 109793, 562769, 506257, 525537, 662417, 398177,
277777 / 364021, 79173, 428597, 567061, 86517, 635013, 31909,
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224101, 225717 / 460451, 86803, 129987, 66291, 206179, 426259,
358963, 193331, 320547, 41491, 613267, 419571, 178355, 328915,
82691, 263107 / 89497, 404617, 419401, 646921, 581657, 311369,
565113, 625497, 555049, 641033, 63401, 384137, 599753, 304713,
478249, 301433

$(8,9)=900000$

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387837, 176577, 379587, 76167, 82587, 137397, 44547, 388017 /
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148875, 374895, 168495, 18375, 105375, 367095, 295125, 147675
/ 449321, 394991, 359261, 339281, 296201, 20081, 371981,
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300845, 192155, 235445, 240455, 400925, 57665, 422075, 110285,
226565, 315785, 288365, 131405, 329045, 366545, 194405 /
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420309, 323739, 246129, 83019, 388869, 301479, 127539 / 69887,
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11297, 370697, 114737, 304097, 367937, 109997

$(8,10)=3000000$

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(9,9) = 2100000

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(9,10) = 9000000

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(10,9) = 4200000

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1267125, 1887465

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