Full Length Article

Numerical study on mixed convection heat transfer in a porous L-shaped cavity

Satyajit Mojumder\textsuperscript{a}, Sourav Saha\textsuperscript{a}, M. Rizwanur Rahman\textsuperscript{a}, M.M. Rahman\textsuperscript{b,\ast}, Khan Md. Rabbi\textsuperscript{a}, Talaat A. Ibrahim\textsuperscript{c,d}

\textsuperscript{a}Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh
\textsuperscript{b}Mathematical and Computing Sciences Group, Faculty of Science, Universiti Brunei Darussalam, BE-1410, Brunei
\textsuperscript{c}King Saud University, P.O. 70908, 11577 Riyadh, Saudi Arabia
\textsuperscript{d}Mechanical Power Dept., Faculty of Engineering-Mattaria, Helwan University, Cairo 11718, Egypt

1. Introduction

Mixed convection flow is a combination of both forced and natural convection. It is the most general case of convection when an outer forcing system and an inner volumetric force act simultaneously, and has received much attention in the literature. In the fields of engineering and science, there are extensive applications of mixed convection in porous media, and so numerous studies have been carried out to analyze the behavior and interaction mechanisms of the thermal and flow fields under different conditions. Applications of mixed convection flow include crystal growth, spray and flash drying, combustion of atomized liquid fuels, thermal–hydraulics of nuclear reactors, the cooling of sophisticated electronics and heat exchanger devices, float gas production, dehydration operations in chemical and food processing, and lubrication technology. Salam Hadi Hussain\textsuperscript{[1]} analyzed heatlines and dehydration operations in chemical and food processing, and lubricated electronics and heat exchanger devices, float gas production, growth, spray and flash drying, combustion of atomized liquid fuels, thermal–hydraulics of nuclear reactors, the cooling of sophisticated electronics and heat exchanger devices, float gas production, dehydration operations in chemical and food processing, and lubrication technology. Zhao and Zhai\textsuperscript{[2]} studied heat transfer in a porous medium with a high Reynolds number (Re = 1–100), and higher Darcy number (Da = 1–100) cause heat transfer to rise. Sameh E. Ahmed\textsuperscript{[3]} studied mixed convection from a discrete heat source in enclosures with two adjacent moving walls and filled with micropolar nanofluids. Numerical modeling of thermal characteristics in a microstructure filled porous cavity with mixed convection conducted by Bhuiyan et al.\textsuperscript{[4]}. Javaheerdeh et al.\textsuperscript{[5]} studied natural convection heat and mass transfer in MHD fluid flow past a moving vertical plate with variable surface temperature and concentration in a porous medium. Kanafer and Vafai\textsuperscript{[6]} carried out a numerical study of mixed convection heat and mass transport in a lid-driven square cavity filled with a non-Darcian fluid-saturated porous medium, and reported that the buoyancy ratio, Reynolds number (Re), Darcy number, and Richardson number have a profound effect on heat transfer. A numerical study on natural convection in porous media-filled an inclined triangular enclosure with heat sources using nanofluid in the presence of heat generation effect is conducted by Mansour and Ahmed\textsuperscript{[7]}. Agarwal et al.\textsuperscript{[8]} investigated double diffusive mixed convection in a lid-driven porous cavity. It was observed that convection flow was significant up to Darcy number of 0.1. Nasseddine Ouertatani et al.\textsuperscript{[9]} studied the intricate three-dimensional flow structure and heat transfer rate in a heated porous lid-driven cavity was investigated by Oztop\textsuperscript{[2]}. He concluded that heat transfer decreases as the Richardson number (Ri) increases, but that increments in the Darcy number (Da) cause heat transfer to rise. Samia M. El Sherbini and M. E. A. Elwakil\textsuperscript{[10]} studied the effect of the Reynolds number (Re) and the Darcy number (Da) on the flow and heat transfer in a partially porous cavity. They found that the heat transfer rate increases with increasing Re and Da. The effect of the thermal Grashof number (Gr) on the flow and heat transfer in a partially porous cavity was investigated by Parameswaran et al.\textsuperscript{[11]}. They found that the heat transfer rate increases with increasing Gr. In the present study, the effect of the thermal Grashof number (Gr) and the Darcy number (Da) on the flow and heat transfer in a partially porous cavity was investigated. The Galerkin weighted residual method is applied to obtain numerical solutions. The effect of the Reynolds number (Re = 1–100), Grashof number (Gr = 10\textsuperscript{5}–10\textsuperscript{6}) and Darcy number (Da = 10\textsuperscript{−5}–10\textsuperscript{−3}) on the velocity and temperature fields is examined. For the vertical wall, a higher heat transfer rate is observed when a low Grashof number, higher Darcy number and higher Reynolds number are applied, but the opposite characteristic is found in the horizontal wall. It is evident that heat transfer decreases up to 63% in the horizontal wall when the flow has a high Reynolds number (Re = 100).

A lid-driven \textit{L}-shaped cavity filled with a porous medium is analyzed. The Galerkin weighted residual method is applied to obtain numerical solutions. The effect of the Reynolds number (Re = 1–100), Grashof number (Gr = 10\textsuperscript{5}–10\textsuperscript{6}) and Darcy number (Da = 10\textsuperscript{−5}–10\textsuperscript{−3}) on the velocity and temperature fields is examined. For the vertical wall, a higher heat transfer rate is observed when a low Grashof number, higher Darcy number and higher Reynolds number are applied, but the opposite characteristic is found in the horizontal wall. It is evident that heat transfer decreases up to 63% in the horizontal wall when the flow has a high Reynolds number (Re = 100).
To the best of the authors’ knowledge, the flow in an L-shaped lid-driven cavity filled with a porous medium is yet to be reported in the literature. From the previous investigations identified above, it is understood that Pr, Gr, Re, Ri, and Da have significant effect on the rate of heat transfer.

This paper numerically investigates the flow characteristics of mixed convection in a lid-driven L-shaped cavity filled with porous medium. The right-hand vertical and horizontal sides of the cavity are heated, while the lower and upper horizontal boundaries are maintained at a low temperature. Detailed investigations of flow characteristics as the function of Da are carried out for different values of Gr and Re.

2. Problem formulation

2.1. Physical model

Details of the physical problem are presented in Fig. 1. The cavity has an L-shape configuration, which has a certain importance from an application point of view. The L-shaped cavity of the present problem is considered to be filled with a porous medium. It is assumed that the solid matrix of the porous media does not undergo deformation and the porous bed is also assumed to be homogeneous, isotropic, saturated with incompressible fluid. It is also assumed that the temperature of the phase of the working fluid is maintained at a low temperature. Detailed investigations of flow characteristics as the function of Da are carried out for different values of Gr and Re.

Double cubic cavity, and mentioned that a higher transfer rate can be achieved optimization between Re and Ri. Sandeep and Subo-chana [19] investigated dual solutions for unsteady mixed convection flow of MHD micropolar fluid over a stretching/shrinking sheet with non-uniform heat source/sink. Steady mixed convection in a square lid-driven cavity under the combined buoyancy effects of thermal and mass diffusion was investigated by Al-Amiri et al. [11]. Mukhopadhyay and Mandal [12] studied magnetohydrodynamic (MHD) mixed convection slip flow and heat transfer over a wavy wall and non-uniform heating on natural convection heat transfer inside porous complex enclosure. Dehnavia and Rezvani [16] numerically investigated natural convection heat transfer and entropy generation inside porous complex enclosure.

Table 1

<table>
<thead>
<tr>
<th>References</th>
<th>Study</th>
<th>Boundary condition</th>
<th>Parameters</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mahmoudi [17]</td>
<td>Free convection, nanofluid</td>
<td>Vertical left and bottom wall heated</td>
<td>Ra, aspect ratio</td>
<td>Decrease of aspect ratio enhances heat transfer</td>
</tr>
<tr>
<td>Mahmud [18]</td>
<td>Free convection</td>
<td>Left vertical heated wall</td>
<td>Gr, aspect ratio</td>
<td>Minimum heat transfer rate at hot wall corner</td>
</tr>
<tr>
<td>Angirasa et al. [19]</td>
<td>Buoyancy-induced convection</td>
<td>Symmetrically heated L-shaped</td>
<td>Ra, Pr</td>
<td>Interesting interactions between the horizontal and vertical</td>
</tr>
<tr>
<td>Kalteh et al. [20]</td>
<td>Free convection, nanofluid</td>
<td>Vertical left and bottom wall heated</td>
<td>Ra, nanoparticle diameter and concentration effects</td>
<td>Heat transfer rate is higher for lower aspect ratio, average Nu decreases with increment in nanoparticle size Hazard</td>
</tr>
<tr>
<td>Sourtiji et al. [21]</td>
<td>MHD natural convection</td>
<td>N/A</td>
<td>Ra, Ha, buoyancy effect</td>
<td>Ha has negative effect on heat transfer at high Ra</td>
</tr>
</tbody>
</table>

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fluid is same as the temperature of the solid phase within the porous region, and local thermal equilibrium (LTE) is applicable in the present investigation [22]. Also, a velocity square term could be implemented in the momentum equations to model the inertia effect which is more important for non-Darcy effect on the convective boundary layer flow over the surface of a body embedded in a high porosity media. However, this term is neglected in the present study because the present study deals with the mixed convection flow in a cavity filled with a porous medium. The left and right vertical walls are kept adiabatic, while the mid-right vertical and horizontal walls are heated \((T = T_b)\). The top and bottom walls of the cavity are kept at constant low temperature \((T = T_c)\). Here, the ratio \(a/b\) is held constant at 1, and \(a/L = b/L = 0.5\). The top wall of the cavity is moving at the lid velocity \((u = U_0)\), while the other boundaries of the cavity are stationary. The co-ordinate system is defined, and gravitational acceleration acts in the negative y-direction. The radiation effect and viscous dissipation are neglected for this study.

2.2. Mathematical modeling

2.2.1. Governing equations

We consider a two-dimensional, steady, laminar incompressible flow in an \(L\)-shaped cavity filled with air. Conservation of mass and the energy momentum equation are used to model the problem. The governing equations [23,24] can be expressed in dimensional form as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\nu e}{K} u - \frac{F_e^2}{k^{1/2}} u^2
\]

\[
\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\nu e}{K} v - \frac{F_e^2}{k^{1/2}} v^2 + g\beta(T - T_c)
\]

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{2}{C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

Here, \(F = \frac{15}{\sqrt{800\pi}}\), and the following scales are used to obtain the non-dimensional form of the governing equations:

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0}, \quad \Theta = \frac{T - T_c}{T_h - T_c}, \quad P = \frac{p}{\rho U_0^2}, \quad Da = \frac{b}{L}, \quad Pr = \frac{\nu}{\lambda}, \quad Gr = \frac{g(\alpha T_h - \alpha T_c)}{\nu^2}
\]

In Eq. (5), thermal diffusivity \((x)\) and kinematic viscosity \((v)\) are defined on the basis of the thermal equilibrium of solids and fluids in a porous medium. These parameters are given by \(x = \frac{x_1}{\nu_1}\) and \(\nu_2\), where \(K_D\) is the effective thermal conductivity of the porous matrix, \(\nu_1\) is the viscosity of the fluid, \(e\) is the porosity, and \(C_P\) is the specific heat of the fluid. The fluid density is assumed to be \(\rho_0 = \rho\) (when \(\Theta = 0\)).

The governing equations are written in non-dimensional form as:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{1}{Pr} \frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{Ue}{Re Da} - \frac{F_e^2}{Re Da} U^2
\]

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{1}{Pr} \frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{Ve}{Re Da} - \frac{F_e^2}{Re Da} V^2 + \frac{G\beta}{Re}
\]

\[
\frac{\partial \Theta}{\partial X} + \frac{\partial \Theta}{\partial Y} = \frac{1}{Pr} \left( \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right)
\]

2.2.2. Boundary conditions

The boundary conditions applied to the present problem are presented in Table 2.

The average temperature [25] inside the cavity is evaluated by

\[
\Theta_{av} = \frac{\int \Theta \, dA}{\int (1 - \frac{\Theta}{\Theta_{av}})}
\]

The average Nusselt number in the vertical and horizontal heated wall is evaluated using the following equations:

\[
Nu_v = \frac{1}{b_L} \int_{b_L}^{1} \frac{\partial \Theta}{\partial X} \, dY, \quad Nu_h = \frac{1}{a_L} \int_{a_L}^{1} \frac{\partial \Theta}{\partial X} \, dX
\]

The stream function is obtained by

\[
U = \frac{\partial \Psi}{\partial Y}, \quad V = \frac{\partial \Psi}{\partial X}
\]

3. Numerical procedure

3.1. Numerical scheme

Numerical simulations were carried out to solve the non-dimensional equations (6)-(9). The momentum and energy balance equations were solved using the Galerkin weighted residual method. A penalty parameter \(\gamma\) was used [25] to eliminate the pressure \(P\). Thus,

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Flow field</th>
<th>Thermal field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top wall: (0 &lt; X &lt; 0.5), (0 &lt; Y &lt; 0.5)</td>
<td>(\Theta = 1), (V = 0)</td>
<td>(\Theta = 0)</td>
</tr>
<tr>
<td>Side wall: (X = 0), (0 &lt; Y &lt; 1); (X = 1), (0 &lt; Y &lt; 0.5)</td>
<td>(U = V = 0)</td>
<td>(\Theta = 0)</td>
</tr>
<tr>
<td>Vertical heated wall: (X = 0.5), (0.5 &lt; Y &lt; 1)</td>
<td>(U = V = 0)</td>
<td>(\Theta = 1)</td>
</tr>
<tr>
<td>Horizontal heated wall: (0.5 &lt; X &lt; 1), (Y = 0.5)</td>
<td>(U = V = 0)</td>
<td>(\Theta = 1)</td>
</tr>
<tr>
<td>Bottom wall: (0 &lt; X &lt; 1), (Y = 0)</td>
<td>(U = V = 0)</td>
<td>(\Theta = 0)</td>
</tr>
</tbody>
</table>

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\[ P = -\gamma \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) \]

(13)

The continuity equation is automatically satisfied. The momentum equation can be reduced and expressed as:

\[
\begin{aligned}
U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} &= -\gamma \frac{\partial}{\partial x} \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) + \frac{1}{Re} \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \\
- \frac{Ue}{ReDa} - \frac{Fe^2}{Re\sqrt{Da}} U^2
\end{aligned}
\]

(14)

\[
\begin{aligned}
U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} &= -\gamma \frac{\partial}{\partial y} \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) + \frac{1}{Re} \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} - \frac{Ve}{ReDa} \frac{\partial^2 V}{\partial x^2} + \frac{Gr}{Re^2} \Theta
\end{aligned}
\]

(15)

The systems described by Eqs. (9), (14), and (15) with the given boundary conditions were solved using the Galerkin finite element method with the basis set \( \{ \phi_i \}_{i=1}^N \) as:

\[ U \approx \sum_{k=1}^{N} U_{k}\phi_k(X,Y) \]

\[ V \approx \sum_{k=1}^{N} V_{k}\phi_k(X,Y) \]

\[ \Theta \approx \sum_{k=1}^{N} \Theta_{k}\phi_k(X,Y) \]

The Galerkin finite element method forms the following nonlinear residual equations at every node of an internal domain \( \Omega \):

\[
R(U,V,\Theta) = \sum_{k=1}^{N} U_k \int_{\Omega} \left[ \left( \frac{\partial U_k}{\partial x} + \frac{\partial V_k}{\partial y} \right) \frac{\partial \phi}{\partial x} + \frac{1}{Re} \frac{\partial^2 U_k}{\partial x^2} \frac{\partial \phi}{\partial x} \right] \, dxdy
\]

(16)

\[
R(U,V,\Theta) = \sum_{k=1}^{N} V_k \int_{\Omega} \left[ \left( \frac{\partial U_k}{\partial x} + \frac{\partial V_k}{\partial y} \right) \frac{\partial \phi}{\partial y} + \frac{1}{Re} \frac{\partial^2 V_k}{\partial x^2} \frac{\partial \phi}{\partial y} \right] \, dxdy
\]

(17)

\[
R(U,V,\Theta) = \sum_{k=1}^{N} \Theta_k \int_{\Omega} \left[ \left( \frac{\partial U_k}{\partial x} + \frac{\partial V_k}{\partial y} \right) \frac{\partial \phi}{\partial y} + \frac{1}{Re} \frac{\partial^2 U_k}{\partial x^2} \frac{\partial \phi}{\partial y} \right] \, dxdy
\]

(18)

<table>
<thead>
<tr>
<th>Gr</th>
<th>Ref. [26]</th>
<th>Nu_{Ref}</th>
<th>Present study</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^3)</td>
<td>1.001</td>
<td>0.982</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>(10^4)</td>
<td>1.231</td>
<td>1.198</td>
<td>2.61</td>
<td></td>
</tr>
<tr>
<td>(10^5)</td>
<td>3.297</td>
<td>3.264</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2. Grid independency test for \( Re = 1, Gr = 10^5 \), and \( Da = 10^{-3} \).

Fig. 3. Code validation against results of Basak et al. [26] for \( Pr = 0.015, Re = 1, Da = 10^{-4} \), and \( Gr = 10^5 \) (red dotted line represents present code). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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This set of nonlinear equations was solved using a reduced integration technique and Newton’s method [25].

3.2. Grid independency test

A grid independence study was performed to ensure the accuracy of the solution of the present study. Fig. 2 shows the change in average Nusselt number in both the horizontal (\( \text{Nu}_h \)) and vertical (\( \text{Nu}_v \)) walls at \( \text{Gr} = 10^5 \), \( \text{Re} = 1 \), and \( \text{Da} = 10^{-3} \). From the figure, it can be seen that the Nusselt number becomes more or less constant when there are more than 4452 elements. While varying the other pertinent parameters, it was observed that a grid having 4452 elements was optimal for satisfying the required numerical accuracy in all cases. Consequently, all computations were carried out with 4452 elements of mesh size and this is considered to be an independent grid for the present problem.

3.3. Code validation

To ensure the validity of the present code, the solution obtained by Basak et al. [26] was taken as the standard for \( \text{Pr} = 0.015 \), \( \text{Re} = 1 \), and \( \text{Da} = 10^{-3} \). The results of this comparison are shown in Fig. 3 in terms of streamlines and isotherms, and in Table 3 in terms of the Nusselt number. From these results, it is evident that the present code generates results that are comparable with those of Basak et al. [26] and thus the present code is considered completely reliable.

4. Results and discussion

Different pertinent parameters such as the Darcy number, Grashof number, and Reynolds number were investigated at Prandtl number, \( \text{Pr} = 6.2 \) and the porosity \( \varepsilon = 0.98 \) to identify their effect on the flow field and heat transfer. Results are presented in the terms of isotherms, streamline contours, and relevant plots.

4.1. Effect of Grashof, Darcy, and Reynolds numbers on streamlines

Figs. 4–6 illustrate the effect of the Darcy number on the streamline for \( \text{Gr} = 10^3 \), \( 10^4 \), and \( 10^5 \) at \( \text{Re} = 1 \), \( 10 \), and \( 100 \). Fig. 4 depicts the streamline formation at \( \text{Re} = 1 \) for the aforementioned Gr values. It can be observed that, when \( \text{Da} = 10^{-3} \), there are two major circulations inside the cavity. There is a larger vortex that covers almost the entire cavity, and a smaller, secondary cell that adheres to the moving lid. The moving lid induces a circulation along its direction of motion. In contrast, the thermal buoyancy effect induces another vortex, because the boundary conditions for these two driving factors oppose one another. As the Grashof number signifies the relative importance of viscous and buoyant forces, we can see that, at lower Grashof numbers, when the effect of viscosity is dominant, the moving lid plays a governing role in determining the flow field. As a result, a small vortex is induced...
next to the moving lid. As the value of $Gr$ increases from $10^3$ to $10^5$, this secondary vortex diminishes. With higher $Gr$ values, the buoyancy force inside the cavity becomes the major factor in determining the rotation of eddy. This observation is independent of the Reynolds number values, and can be seen in Figs. 4–6.

For fixed Reynolds and Grashof numbers, the secondary circulation strengthens as $Da$ increases. The Darcy number is an important parameter for single-phase flow through porous media, and serves as a qualitative measure for a particular flow medium. During flow through highly porous media, the inertia of the flowing particles is the dominant effect. Thus, highly porous media (with high $Da$ values) normally reduce the strength of the flow by introducing instantaneous changes in acceleration. As a result, a lower Darcy number in a single-phase fluid should yield to a better flow scenario. As can be seen from Figs. 4–6, reductions in $Da$ almost always produce a significant increase in strength of the vortices.

Another important parameter for mixed convection heat transfer is the Reynolds number. Figs. 4–6 indicate that higher values of $Re$ produce a more dominant secondary flow, regardless of the $Gr$ and $Da$ values. This is consistent with the theoretical prediction of the flow field, because a higher Reynolds number is related with a higher lid velocity, and hence a stronger secondary flow.

4.2. Effect of Grashof, Darcy, and Reynolds numbers on isotherms

Fig. 7 represents an isotherm contour map that shows the influence of $Re$ and $Gr$ on the heat transfer mode and intensity at $Da = 10^{-3}$. From this figure, it is clear that buoyancy has no significant effect near the horizontal heated wall, as this is at the top of the configuration. Thus, the primary mode of heat transfer adjacent to the horizontal heated wall is conduction. This conclusion is supported by the isotherm contours, which become closely packed and nearly parallel adjacent to the horizontal wall. Because the convection is more or less entirely dependent on the intensity of the lid velocity, viscous effects are dominant here, and increases in $Gr$ have very little effect on the isotherm pattern. However, the isotherms become distorted when the Grashof number is increased for a particular value of $Re$. As the Reynolds number rises from 1 to 100, the isotherm contours become much denser near the moving lid. Increasing the value of $Re$ improves the overall convection. The variation in the isotherms at $Da = 10^{-4}$ and $Da = 10^{-5}$ is also investigated, and a similar pattern as for $Da = 10^{-3}$ was observed. Thus, variation in $Da$ has no significant effect on the isotherms in the present problem.

4.3. Changes in temperature and velocity profile with variation in Reynolds number

Fig. 8 illustrates the local non-dimensional temperature and velocity field information at different Reynolds number values with $Gr = 10^5$ and $Da = 10^{-3}$. As the velocity data is taken at $X = 0.25$ and $Y = 0.25$, it is seen that, for $Re = 1$, there is a hint of two flow fields with opposite directions. For $Re = 10$ and $Re = 100$, the flow field is fairly constant, indicating either the center of a vortex or the meeting place of two opposite vortices. The first scenario is true for $Re = 10$, and the latter is the case for $Re = 100$. The local non-dimensional temperature of the fluid is an indicator of the effective heat transfer. Lowering the local non-dimensional...
Fig. 6. Effect of Do on streamline for Re = 100 at (a) $Gr = 10^3$, (b) $Gr = 10^4$, (c) $Gr = 10^5$.

Fig. 7. Effect of Reynolds and Grashof numbers on isotherms at $Da = 10^{-3}$ for Re = 1, 10, and 100 at (a) $Gr = 10^3$, (b) $Gr = 10^4$, and (c) $Gr = 10^5$, respectively.
Fig. 8. (a) $Y$ velocity, (b) non-dimensional temperature for $Y = 0.25$, (c) $X$ velocity, (d) non-dimensional temperature for $X = 0.25$ for $Gr = 10^3$ and $Da = 10^{-3}$.

Fig. 9. Variation in average fluid temperature with $Da$ and $Gr$ for (a) $Re = 1$, (b) $Re = 10$, (c) $Re = 100$. 

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temperature of the flow field would interpret that a thermal potential was present for heat transfer. In this sense, a lower Reynolds number gives a relatively low local non-dimensional temperature, which is good for convective heat transfer.

4.4. Combined effect of Darcy, Grashof, and Reynolds numbers on average fluid temperature

The combined effect of the Darcy, Grashof, and Reynolds numbers on average fluid temperature inside the cavity is depicted in Fig. 9. At low Re (Re = 1, 10), the average temperature decreases with the increment of Grashof and Darcy numbers indicating better convective heat transfer rate. It is also observed that, when Da = 10^{-5}, there is no significant change in average temperature with Gr. A similar situation occurs when Gr = 10^3: the Darcy number does not have any significant effect on average fluid temperature at low Re (Fig. 9(a) and (b)). Thus, for low Reynolds numbers, it is desirable to have higher Grashof and Darcy numbers for better heat transfer rate. Different order of change is visible at Re = 100 (Fig. 9(c)). For this case, at higher Darcy numbers (Da = 10^{-3}), the fluid temperature is high and decreases as Da decreases. At Da = 10^{-3}, the effect of the Grashof number is quite prominent. Higher Grashof number indicates some augmentation of heat transfer for Da = 10^{-3}. But when the Darcy number decreases to 10^{-5}, there is no significant effect of Gr on average temperature of the working fluid. Importantly, variation in average temperature of the fluid with Darcy and Grashof numbers is affected with the change in Re. This phenomena causes the flow pattern to change. For this higher Re (e.g., Re = 100), better convective heat transfer is obtained with low Darcy and Grashof numbers.

4.5. Combined effect of Darcy, Grashof, and Reynolds numbers on Nusselt number in the heated wall

In the present problem, there are two different heated walls positioned horizontally and vertically. The horizontal wall is at the top of the fluid, so there is no buoyancy effect on this wall. However, the vertically positioned sidewall encounters a significant buoyancy effect. At the horizontal wall, heat transfer is occurred as a result of the fluid flow. For this reason, heat transfer from both the surfaces are analyzed. Fig. 10 portrays the average Nusselt number within the variation of the Darcy and Grashof numbers for different values of Re at the horizontally heated wall. From this figure, we can see that, increment in the Darcy and Grashof numbers produces better heat transfer. At low Grashof number (Gr = 10^3), there is no significant effect of Da. This is also true for a fixed Da = 10^{-5} and different values of Gr for low Reynolds number (Re = 1, 10) (Fig. 10(a) and (b)). However, when the Reynolds number increases to 100, an interesting phenomenon occurs. At low Grashof numbers (Gr = 10^3) (Fig. 10(c)), higher Darcy number can retard the heat transfer. To understand the effect of the Reynolds number, note that the overall Nusselt number.
number decreases as the Reynolds number increases. This is due to the dominance of viscous force which retards the fluid flow, causing the heat transfer rate to decrease at the horizontal heated wall.

Fig. 11 depicts the average Nusselt number with respect to Darcy and Grashof numbers at different Re values at the vertically heated wall. At the vertical wall, Fig. 11(a) and (b) show that heat transfer rate is reduced with the increment of Grashof and Darcy numbers. At the horizontal wall, the opposite is true. For low Gr (Gr = 10^3), there are no significant effects of Darcy number, and a similar trend can be found when Da is fixed at 10^{-3} and Gr is varied. However, for higher Gr and Da (Gr = 10^5 and Da = 10^{-3}), heat transfer becomes reduced significantly. Thus, for low Re (Re = 1, 10), lower Da, Gr values are desirable for better heat transfer in the vertical wall. The situation is quite different when the Reynolds number is increased to 100 (Fig. 11(c)). A lower Darcy number (Da = 10^{-5}) and higher Grashof number cause the heat transfer rate to increase. In this case, increasing the Darcy number reduces heat transfer, and this is independent of the Grashof number. Again, increasing the Reynolds number causes the heat transfer rate to increase. This is due to the fact that the viscous force causes a vortex to form nearer to the vertically heated wall.

5. Conclusions

Heat transfer in an L-shaped geometry requires detailed analysis, as it is widely encountered in many engineering problems. For the convenience of readers and other researchers around the globe, our key findings are recalled in the following points:

- Mixed convection in an L-shaped cavity filled with porous media calls upon optimization of different parameters such as Reynolds, Darcy, and Grashof numbers.
- Higher Darcy and Grashof numbers result in lower average fluid temperatures at low Reynolds number (Re = 1, 10), whereas low Grashof and Darcy numbers are desirable when the Reynolds number is high (Re = 100) to lower the average fluid temperature inside the cavity. The average temperature of the fluid decreases by up to 8% with higher Grashof and Darcy numbers.
- For the horizontally heated wall, higher Darcy and Grashof numbers produce better heat transfer rate. Low Reynolds number gives better heat transfer, as the viscous force becomes more dominant in this flow regime. This phenomenon increases the heat transfer rate by up to 10%.
- For the vertically heated wall, low Darcy and Grashof numbers are preferable when Re is low, and the heat transfer rate can be enhanced by up to 6% in this case. However, for higher Reynolds numbers (e.g., Re = 100), a low Darcy number and high Grashof number are preferable for increasing the heat transfer rate.

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