Renormalization-group running cosmologies and the generalized second law

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Abstract

We explore some thermodynamical consequences of accelerated universes driven by a running cosmological constant (CC) from the renormalization group (RG). Application of the generalized second law (GSL) of gravitational thermodynamics to a framework where the running of the CC goes at the expense of energy transfer between vacuum and matter, strongly restricts the mass spectrum of a (hypothetical) theory controlling the CC running. We find that quantum effects driving the running of the CC should be dominated by a trans-Planckian mass field, in marked contrast with the GUT-scale upper mass bound obtained by analyzing density perturbations for the running CC. The model shows compliance with the holographic principle.

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In ordinary quantum field theory (QFT) the CC is viewed as a parameter subject to RG running and therefore is expected to run with the RG scale, usually identified with an expansion quantity evolving smoothly enough to comply with cosmological data. In such theories, therefore, even a ‘true’ CC cannot be fixed to any definite constant (including zero) owing to RG running effects. In [1], the variation of the CC arises solely from the particle field fluctuations, while [2] represents a complementary approach in which RG running is due to non-perturbative quantum gravity effects and a hypothesis of the existence of an IR attractive RG fixed point. The main theoretical obstacle to treating the CC within QFT in a curved background (the most appropriate tool for studying the problem [1]) is that a derivation of the form of decoupling of heavy-particle species in quantum effects governing the CC running, cannot be obtained in a rigorous way within this framework [3]. If one insists on the familiar quadratic form of decoupling for heavy-matter fields at low energies, then one ends up with a somewhat surprising outcome that more massive fields do play a dominant role in the running at any scale [4]. Consequently, the running in this case becomes stronger than logarithmic, thus providing a viable mechanism for efficient relaxing of the CC from a large value in the early universe to its tiny value observed today. The scenario can therefore shed some light on the hard-pressing CC problem [5].

In addition, the above scenario for the CC running, with the choice for the RG scale μ = H, taken together with the conservation law controlling the continuous transfer of energy between the CC and matter, may provide a viable cosmological model of dark energy of the universe [6]. Indeed, the CC variation law, dρΛ/dz ∝ dH²/dz [1], gives the CC scaling in the form

ρΛ = C₀ + C₂H²,

(1)

thus having a natural appearance of a nonzero constant C₀, while another (dimensionful) constant C₂ represents the effective cumulative mass squared of an underlying QFT and therefore is dominated by the heaviest masses. C₀ represents the ground state of the vacuum, and, of course, cannot be unambiguously set in any theory. The right amount of dark energy at present is obtained for C₂ ∼ Mₚ², if C₀ is subdominant in (1). However, the presence of (even if tiny) C₀ is essential here for phenomenological reasons, since otherwise the scenario is incompatible with a transition from a decelerated to an accelerated era for a spatially flat universe [7]. This is because the matter energy density ρₘ scales with the expansion of the universe in the same way as the variable part in (1) (see below). The parameter C₂ from the variable part of (1) could be poten-
tially observable in the future supernova data [8]. The strongest upper bound on $C_2$ has been obtained recently [9], through the numerical analysis of density perturbations for the running CC $\left(C_2^2\right)^{1/2} \sim 10^{-3} M_{Pl}$.\footnote{Somewhat less stringent bound was obtained in a less rigorous way in [10].} Although these bounds on $C_2$ mean that an interaction between matter and the CC is small enough that (1) is dominated by the constant $C_0$ today, the phenomenological aspect of the model can still be considered viable.

Although the above scenario looks promising in pointing toward both the CC problem and the problem of dark energy of the universe (amongst other advantages because no quintessence-like scalar fields are involved whatsoever), an attempt to bring the above model in accordance with holographic cosmology leads unavoidably to undesired phenomenological implications [11]. In [11], a serious drawback was noticed for the model (1), in conjunction with the corresponding equation of continuity, when trying to accommodate the prediction from the important concept of holographic dark energy (HDE) [12,13]. Namely, such accommodation unavoidably sets the ground state of the vacuum to zero ($C_0 = 0$). Thus, as stated above, a transition between the two cosmological eras for flat space (as suggested by observations as well as by inflation) cannot be obtained. In order to bring the model (1) in agreement with the concept of HDE, a different approach was entertained in [11], in which (1) was investigated together with a different equation of continuity where a transfer of energy is between the CC and a gravitational field, thus also promoting the Newton constant to a varying degree of freedom. As already mentioned, the variable part of $\rho_A \sim \rho_m \sim H^2$.

We begin by considering the entropy of a variable CC inside the future event horizon of a comoving observer. Its entropy inside the horizon can be determined by Gibb’s equation for the zero chemical potential

$$T_A dS_A = dE_A + p_A dV. \quad (7)$$

Although we deal here with a ‘true’ CC, $p_A = -\rho_A (w_A = -1)$, we learn from (7) that, owing to its variable character, it still possesses a nonzero entropy,

$$T_A dS_A = V d\rho_A. \quad (8)$$

The temperature of the CC fluid $T_A$ has to match the temperature of the future event horizon, which, in spite of the fact that the degeneracy between the apparent and the event horizon is broken here, one assumes it to be of a de Sitter form of the Gibbons–Hawking temperature $T_A = \left(2 \pi\right)^{-1} H [17,18]$. With the entropy associated to the event horizon

$$S_E = \pi M_{Pl}^2 d_E^2, \quad (9)$$

the GSL states that\footnote{Note that the entropies of all other stuff inside the horizon do not appear in (10). This is because $\dot{S}_A$ grossly overwhelms all of them. For instance, a contribution of ordinary matter, $S_m \sim (\rho_m/m) d_a^2$, is suppressed in (10) today with respect to the contribution from $S_A$ by a huge factor $m M_{Pl}/\rho_m^{1/2}$. Similarly for other components including the entropy of Hawking particles. However, this may change in the asymptotic region $a \rightarrow \infty$ since various components scale differently with the scale factor [19]. A treatment of relic gravitational waves inside the horizon requires an extra care [20]. For related works, see [21].}

$$\dot{S}_E + \dot{S}_A \geq 0, \quad (10)$$

where overdots represent time derivatives and the future event horizon is given by

$$d_E = a \int_a^\infty \frac{da}{a^2 H}. \quad (11)$$

and the Friedmann equation for flat space read [6]

$$\rho_m = \rho_m \mathcal{A}^{-\xi}. \quad (3)$$

while $\rho_A$ is given by (1) with

$$C_0 = \rho_A - \frac{3}{\sqrt{2}} M_{Pl}^2 H_0^2, \quad (4)$$

$$C_2 = C_0 + \frac{3}{\sqrt{2}} M_{Pl}^2. \quad (5)$$

A dimensionless parameter $\nu$ is defined as

$$\nu = \frac{\sigma M^2}{12 \pi M_{Pl}^2}, \quad (6)$$

where $M$ represents an additive mass contribution of all virtual massive particles, $\sigma = \pm 1$ depending on whether the highest-mass particle is a boson/fermion, $\zeta = 3 \left(1 - \nu\right)$, and the subscript ‘0’ denotes the present-day value. Here $|\nu| \sim 10^{-2}$ means the existence of a particle with Planck mass (or the existence of somewhat less massive particles with large multiplicities), $|\nu| \sim 1$ means the existence of a particle with trans-Planckian mass, $|\nu| \sim 10^{-6}$ means the existence of a particle with GUT-scale mass, whereas much smaller values for $|\nu|$ mean an approximate cancellation between bosonic and fermionic degrees of freedom. As already mentioned, the variable part of $\rho_A \sim \rho_m \sim H^2$.\footnote{Note that the entropies of all other stuff inside the horizon do not appear in (10). This is because $\dot{S}_A$ grossly overwhelms all of them. For instance, a contribution of ordinary matter, $S_m \sim (\rho_m/m) d_a^2$, is suppressed in (10) today with respect to the contribution from $S_A$ by a huge factor $m M_{Pl}/\rho_m^{1/2}$. Similarly for other components including the entropy of Hawking particles. However, this may change in the asymptotic region $a \rightarrow \infty$ since various components scale differently with the scale factor [19]. A treatment of relic gravitational waves inside the horizon requires an extra care [20]. For related works, see [21].}
with $\alpha$ being a scale factor. Upon incorporating the expression (5) for $C_2$ into (10), we obtain the GSL requirement in a compact form

$$vH \geq (d^{-1}_E).$$

(12)

In order to make the presence of cosmic horizons manifest in the above scenario, we restrict ourselves to the parameter space ensuring a de Sitter fate of the universe, i.e., $v < \Omega_0^0$ $\sim 0.7$. Note that this range also covers the interval of compatibility with the LSS data $-10^{-6} \leq v \leq 10^{-6}$ [9]. In addition, to treat the problem completely analytically, in the following we investigate whether the GSL requirement (12) is fulfilled in a dark-energy dominated phase of the expansion only. Thus we expand the Hubble parameter

$$H^2 = \frac{8\pi}{3} M^{2}_{Pl} \left( \rho_{\Lambda_0} - \frac{1}{1 - v} \rho_{m_0} + \frac{1}{1 - v} \mu^{-1} \right)$$

(13)

around its de Sitter value [the constant term in (13)], and keep only the first term in order to obtain the necessary time dependence. Performing so for $d^{-1}_E$ too, we finally end up with the GSL requirement expressed as a simple bound on the dimensionless parameter $v$

$$v(4 - 3v) \geq 1.$$  

(14)

The solution of (14), considering the requirement $v < \Omega_0^0$, $\Omega_0^0 > v \geq \frac{1}{3}$,

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(15)

clearly shows that the requirement from the GSL for a de Sitter phase of the expansion is obeyed only for a small range of trans-Planckian masses. A preferred situation [22] from a view of string/M theory, in which a positive CC at present becomes a negative one sometime in the future (anti-de Sitter fate of the universe), is not covered by our analysis as it corresponds to $v > \Omega_0^0$. However, such large values for $v$ are excluded anyway from recent considerations of the dynamics of density perturbations for the running CC [9].

We also need to check up (for consistency) if the entropy $S_A$ obeys the bound imposed by the holographic principle. For that purpose, we integrate (8) in the manner consistent with our expansion around the constant term in (13). Using

$$\dot{H} \simeq (1 + \xi)(d^{-1}_E),$$

(16)

we arrive at

$$S_A = -v \pi (1 + \xi) d^2_E M^2_{Pl} + C.$$  

(17)

The constant $C$ can be determined by noting that: (i) $\dot{S}_A < 0$, (ii) $S_A \geq 0$ for any sort of nonphantom dark energy, (iii) the adapted third law of thermodynamics tells us that $S_A \rightarrow 0$ for $t \rightarrow \infty$. One obtains

$$S_A \simeq v(1 + \xi) M^2_{Pl} (d^2_{E_{\infty}} - d^2_E).$$

(18)

Since $v(1 + \xi) \geq 1$ at most and $d^2_{E_{\infty}} - d^2_E \ll d^2_E$ by assumption, we conclude that $S_A$ always obeys the holographic bound during a de Sitter phase of the expansion.

One may attempt to bring the bound on $v$, obtained from assuming the validity of the GSL, in accordance with the bound obtained from compatibility with the LSS data by defining the CC temperature up to a constant, $b(2\pi)^{-1} H$, where $b$ is a constant. This would however decrease the temperature of the event horizon by a factor up to $10^6$, which looks unacceptably small. In addition, as warned in [18], any value of $b$ different from 1 would admit a nonzero flow of energy between the horizon and the fluid (or vice versa), thus destroying thermal equilibrium inherent to the FRW geometry.

Another attempt could be to study the law (1) together with the generalized equation of continuity

$$\dot{G}_N (\rho_{\Lambda} + \rho_m) + G_N \dot{\rho}_\Lambda + G_N (\dot{\rho}_m + 3H\rho_m) = 0$$

(19)

which opens up an extra flow of energy now at the expense of variation of $G_N$. To get a consistent theory with a varying $G_N$, one normally goes over some scalar-tensor theory. In the absence of a scalar-tensor theory, the scaling dependence of $G_N$ from the RG entering (19), is used to perform an “RG improvement” [2] instead, either at the level of Einstein’s equation or at the level of the Einstein–Hilbert action. Considering the present bounds on the variation of the gravitational coupling [23], one is however skeptical how this could change the running of the CC to such an extent as to come up with something very different from (14). In any case, any approach of this kind would therefore inevitably require a choice for the RG scale at a significant variance with $\mu = H$. Let us mention though that an analysis of density perturbations for the variable CC and the variable $G_N$ is not available yet.

In conclusion, we have shown how the GSL applied to a model with a variable CC based on the RG effects from standard QFT, sets the lower bound on the heaviest mass in the theory, which cannot be pushed below the Planck scale. The model where the only transfer of energy is between the CC and matter, complies with the holographic bound albeit not with the concept of HDE. If another channel for transfer of energy opens up (between $\rho_{\Lambda}$ and $G_N$), a merging with the concept of HDE density results in moving the heaviest possible masses towards the Planck scale, while the lowest mass sets in around the quintessence-like mass scale [24]. It therefore seems that at least as far as QFT is concerned, the predictions from the GSL and HDE go hand in hand.

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