A Numerical Study of Moving Reference Planes Associated with Unit Cells of Reciprocal Lossy Periodic Transmission-Line Structures by Using the Equivalent BCITL Model

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Abstract

A moving reference plane of unit cell of reciprocal lossy periodic transmission-line (TL) structures (RLSPTLSs) by using the equivalent bi-characteristic-impedance transmission line (BCITL) model is studied numerically in this paper. Applying the BCITL theory, only the equivalent BCITL parameters, characteristic impedances for wave propagating in forward and reverse directions and associated complex propagation constant, are of interest. In this study, a unit cell of an infinite RLSPTLSs is obtained by shifting a reference position of unit cells along TLs of interest. For illustration, an example of symmetrical RLSPTLSs is initially considered. Then, unsymmetrical structures are subsequently investigated by shifting a reference position of unit cells. It is found that the equivalent BCITL complex propagation constant remains unchanged as expected, while the equivalent BCITL characteristic impedances are generally different, depending on the reference position of unit cells. Numerical results will be provided and discussed in this paper.

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1. Introduction

Periodic structures of transmission lines (TLs) have several practical applications in microwave technology; e.g., microwave filters, slow wave components, traveling-wave amplifiers, phase shifters, antennas and metamaterials1-4. They are composed of identical cascaded two-port networks, referred to as unit cells. To analyze and design associated problems of periodically loaded TL structures, there are several methods having been used in the literature. Recently,
the theory based on bi-characteristic-impedance TLs (BCITLs) has been successfully employed to terminated finite reciprocal lossy periodic TL structures (RLSPTLSs)\(^5\). To apply the equivalent BCITL model, only the equivalent quantities associated with each unit cell of RLSPTLSs are employed instead; i.e., the characteristic impedances for waves propagating in forward and reverse directions, defined as \(Z_0^f\) and \(Z_0^r\) respectively, and the corresponding complex propagation constants for waves propagating in the forward (\(\gamma^f\)) and reverse (\(\gamma^r\)) reverse directions, respectively. It should be pointed out that conjugately characteristic-impedance TLs (CCITLs)\(^6\) are the special case of BCITLs when \(Z_0^f\) and \(Z_0^r\) are conjugated of each other.

2. Theory

A finite RLSPTLS of \(M\) unit cells at each unit-cell terminal in \(^5\) can be effectively modeled as a BCITL of length \(l=Md\), as shown in Fig. 1(a), where \(d\) is the length of each unit cell. Note that \(V_m\) and \(I_m\) are the phasor voltage and the phasor current at the terminal of the \(m\)th unit cell (where \(m = 1, 2, \ldots, M\)), respectively. Generally, reciprocal BCITLs possess the complex propagation constant \(\gamma\) with the characteristic impedances \(Z_0^f\) and \(Z_0^r\). Note that both forward and reverse complex propagation constants are identical (\(\gamma^f = \gamma^r = \gamma\)) for reciprocal lossy periodic TL structures. In this paper, an infinite RLSPTLS is considered, which can be obtained from Fig.1 by letting both ends approach infinity.

Using the \(ABCD\) matrix technique\(^1,5\), \(Z_0^\pm\) can be expressed in terms of the total \(ABCD\) parameters of the unit cell of interest as

\[
Z_0^\pm = \frac{\mp 2B}{(A - D \mp j\sqrt{4 - (A + D)^2})^2}, \tag{1}
\]

In addition, \(\gamma\) can be determined from the following dispersion relation:

\[
cosh \gamma d = \frac{A + D}{2}. \tag{2}
\]
where $\gamma$ is defined as $\alpha + j\beta$, $\alpha$ is the attenuation constant and $\beta$ is the propagation constant.

To analyze RLSPTLSs associated with moving reference planes of unit cells, a symmetrical unit cell of length $d$ is initially considered for convenience as shown in Fig. 1(b) (on the left end). In Fig. 1(b), the RLSPTLS consists of two identical lossy TLs, with the unloaded complex propagation constant $\gamma_0$ and the characteristic impedance $Z_0$, loaded with lumped elements at the center. It should be pointed out that these lumped elements are dimensionless.

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The reference position ($s$) of the unit cell is shifted along TLs of interest, where $0 \leq s \leq d$. Since it is on the same RLSPTLS, $\gamma$ is expected to remain unchanged. However, the equivalent characteristic impedances $Z_0^s$ of different reference planes are expected to be distinct ($Z_0^s = Z_0^o$), where $Z_0^o$ and $Z_0^s$ are the equivalent characteristic impedances of the original and shifted unit cells, respectively. Numerical results of moving reference planes associated with unit cells of the equivalent BCITL model are discussed in the next section.

3. Numerical Results

Consider an example of RLSPTLSs, implemented by two identical lossy TLs with the attenuation constant of 0.73 neper/m and the propagation constant of 115.43 radian/m. It is periodically loaded with equal series capacitors $C_{s1}$ and $C_{s2}$ of 10 pF and a shunt inductor $L_{sh}$ of 5.7 mH at the center of the TLs as shown in Fig. 1(b). The characteristic impedance of TLs is $61.24 + j0.26$ and the length $d$ of the unit cell is 6 cm. It should be pointed out that only the operating frequency of 1.5 GHz is considered in this paper.

Figure 2 illustrates the plot of the total $ABCD$ parameters of unit cells of interest as a function of $s/d$. For an example, at $s = 0$ and $s = d$, the original unit cell is symmetrical, where $A$ and $D$ are identical as expected. It is interesting to point out that $A$ and $D$ are discontinuous at $s/d = 0.5$, while $B$ and $C$ are still continuous. However, the parameter $(A+D)/2$ keeps a constant value even at $s/d = 0.5$, implying that $\gamma$ is constant (see Eq. 2), as expected. It should be pointed out that the imaginary parts of $A$, $C$ and $D$ are very small in this example. However, the imaginary part of $B$ is dominant as shown in Fig. 2(b). These $ABCD$ parameters contribute to the behavior of equivalent BCITL parameters ($Z_0^s$ and $\gamma$) as discussed next.

Using Eq. 1, the equivalent BCITL characteristic impedances can be determined. Figure 3 shows the plot of the magnitudes and the arguments of $Z_0^s$ versus $s/d$. Note that $\phi^o$ and $\phi^s$ are the arguments of $Z_0^s$ and $Z_0^o$, respectively. It is found that $Z_0^o$ are different as $s/d$ is varied as expected, while $\gamma$ remains constant; i.e., $\alpha$ and $\beta$ are equal to 0.78 neper/m and 7.35 radian/m, respectively. Note that the arguments $\phi^o$ of $Z_0^o$ are identical and equal to -1.77 degrees due to the symmetrical unit cell, while the argument $\phi^s$ of $Z_0^s$ are different as expected since the shifted unit cell is unsymmetrical in general. It is also observed that $Z_0^s$ are discontinuous at $s/d = 0.5$ as a consequence of the discontinuities of $A$ and $D$ as shown in Fig. 2.
4. Conclusions

A numerical study of moving reference planes associated with unit cells of infinite RLSPTLSs by using the equivalent BCITL model is presented in this paper. It is found that the equivalent BCITL complex propagation constant remains unchanged for both original and shifted unit cells as expected. In addition, the equivalent BCITL characteristic impedances of both unit cells are generally different depending on the moving reference planes as expected. Therefore, to select a reference plane of unit cells for a given RLSPTLS problem, we can choose any suitable reference position for the analysis and design since the original and shifted unit cells are related. These results will be extended to the case of nonreciprocal lossy periodic TL structures in the future.

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References