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Full one-loop radiative corrections to the asymmetry parameter of polarised neutron beta decay

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Abstract

We present a calculation of full one-loop radiative corrections, including the constant term, to the asymmetry parameter of polarised neutron beta decay. This gives the radiative correction to the axial coupling constant g_A extracted from the beta asymmetry so that it ties to g_A that appears in neutron decay lifetime in a consistent renormalisation scheme. We find that the ratio of axial-vector to vector couplings determined from the beta asymmetry, after taking account of the outer radiative correction, is related to the bare value as $G_A/G_V = 1.0012\, G_A^0/G_V^0$.

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Keywords: Neutron beta decay asymmetry; Radiative corrections

Beta decay asymmetry of the polarised neutron has been used to determine the axial vector coupling constant g_A of the nucleon. One-loop radiative corrections to the asymmetry parameter have been calculated by several authors [1–4], but their calculations do not include the constant term, or so-called inner corrections, which require a special care in the treatment of the UV divergence of the radiative correction. This makes the identification of g_A extracted from β decay asymmetry with that which appears in the nucleon beta decay rate ambiguous.[1] The radiative correction to beta decay is UV divergent and it is rendered finite only with the use of Weinberg–

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[1] It is included in Sirlin's proof [7] that the inner radiative corrections can be factored out for the processes concerning the unpolarised neutron. As far as the authors know there is no proof which shows that the inner radiative correction for the polarised neutron and that for the unpolarised neutron should agree.

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Salam's theory of electroweak interaction. The complication arises from the fact that one must deal with quarks in electroweak theory, and one must continue to calculations with hadrons at low energies [5].

In this Letter we calculate one-loop radiative corrections including the inner correction: hereby the coupling constant g_A that appears in the β asymmetry parameter is unambiguously tied to that in the decay rate in a consistent renormalisation scheme. The key point of the calculation is the clarification of the universal and non-universal UV divergent parts by using the current algebra technique and the proof that the same combinations of the renormalisation factors appear in the beta asymmetry as in the beta decay rate. The separation of the UV divergences into universal and non-universal parts was done first by Abers et al. [6] for the Fermi transition of nuclear beta decay, and then used by Sirlin [5,7] to develop a practical scheme of one-loop radiative corrections for the $0^+ \to 0^+$ transition. The scheme was extended to the Gamow–Teller transition by our recent publication (Paper I) [8]. The present work is an application of the formalism developed in Paper I. We are content with the outline of the calculations in this Letter, since the bulk of lengthy calculations are parallel to the ones presented in Paper I. We refer the readers who are interested in technical details of calculations to Paper I.

The tree amplitude for beta decay of the polarised neutron is given by

$$\mathcal{M}^{(0)} = \frac{G_V}{\sqrt{2}}\left[\bar{u}_e(\ell)\gamma^\lambda\left(1-\gamma^5\right)v_\nu(p_\nu)\right]\left[\bar{u}_p(p_2)W_\lambda(p_2,p_1)\frac{1}{2}\left(1+s\gamma^5\gamma^\mu n_\mu\right)u_n(p_1)\right], \tag{1}$$

where $G_V = G_{\mathrm{F}}\cos\theta_C$ with the universal Fermi coupling G_{F} and the Gell-Mann–Lévy–Cabibbo angle θ_C, n_μ is the polarisation vector of the neutron with $n^2=-1$, $n\cdot p_1 = 0$, and $s=\pm 1$, and $W_\lambda(p_2,p_1)$ is a general form of the weak vertex of hadrons and reads

$$W_\lambda(p_2,p_1) = \gamma_\lambda\left(f_V - g_A\gamma^5\right) \tag{2}$$

at low energies. We retain $f_V = 1$ to trace the vector coupling in the calculation. The spinors of the neutron, proton, electron and antineutrino are denoted by u_n, u_p, u_e, and v_ν, respectively, with the momenta specified in parentheses. After spin summation and integration over $\vec{p}_\nu$ the amplitude square reads

$$\sum_{\mathrm{spin}}\left|\mathcal{M}^{(0)}\right|^2 = 16G_V^2 m_n m_p E_\nu\left[\left(f_V^2+3g_A^2\right)E + 2s\left(f_V g_A - g_A^2\right)\left(\vec{n}\cdot\vec{\ell}\right)\right], \tag{3}$$

where E and E_ν are energies of the electron and the antineutrino. Therefore, the asymmetry parameter is given by

$$A = \frac{2(f_V g_A - g_A^2)}{f_V^2 + 3g_A^2}, \tag{4}$$

the electron velocity factor $\beta = |\vec{\ell}|/E$ being removed as a convention.

To evaluate full one-loop corrections, we divide the integration region of the virtual gauge bosons into long- and short-distance parts [5]:

$$\text{(i)}\quad 0 < |k|^2 < M^2, \qquad \text{(ii)}\quad M^2 < |k|^2 < \infty, \tag{5}$$

where k is the momentum of the virtual gauge bosons, and the mass scale M, introduced by hand, divides the low- and high-energy regimes and is supposed to lie between the proton–neutron masses (m_p and m_n) and the W and Z boson masses, m_W and m_Z. Old-fashioned four-Fermi interactions are applied to the proton and neutron in region (i), and the mass scale M is regarded as the ultraviolet cutoff of the QED (i.e., purely photonic) correction. In region (ii), electroweak theory is used for quarks and leptons, and M is the mass scale that describes the onset of the asymptotic behaviour. The concern is to connect the results in (i) and (ii) smoothly. Abers et al. [6] proved that the logarithmic divergences that are proportional to f_V^2 are universal for the Fermi transition on the basis of the conserved vector current with the use of the current algebra technique. The same was proven for the g_A^2 terms for the Gamow–Teller transition for which current conservation is broken only with soft operators [8,9]. This guarantees smooth connection of the logarithmic divergence for the corrections of f_V^2 and g_A^2. There appear,

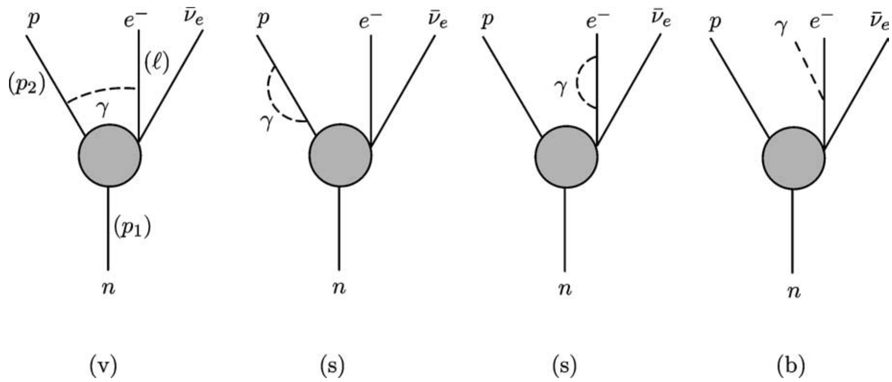

Fig. 1. Radiative corrections to neutron beta decay.

however, interference terms of the order $f_V g_A$, for which logarithmic divergences depend on the model of hadrons. Marciano and Sirlin [10] proposed the prescription to evaluate the high and low energy contributions separately by rendering the UV divergence in the low-energy contribution milder by taking account of form factors of hadrons. We follow the same prescription [10,11] in the present calculation of the asymmetry parameter.

The diagrams of QED one-loop corrections are depicted in Fig. 1, where (v) is the vertex correction, (s) is the self energy correction and (b) is bremsstrahlung. We write the one-loop amplitude

$$\mathcal{M}' = \mathcal{M}^{(v)} + \mathcal{M}^{(s)}. \tag{6}$$

The bremsstrahlung contribution is added separately. We consider the static limit for nucleons, $q^2 = (p_1 - p_2)^2 \ll m_p^2$. The bremsstrahlung from the proton is suppressed by the nucleon mass and does not contribute in this limit. Our calculation is done in the Feynman gauge.

We start with the vertex correction, which is given by

$$\begin{aligned}\mathcal{M}^{(v)} = {} & \frac{i}{2\sqrt{2}} G_V e^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(\ell - k)^2 - m_e^2} \frac{1}{(p_2 + k)^2 - m_p^2} \frac{1}{k^2 - \lambda^2} \\ & \times \bar{u}_e(\ell)\gamma^\mu \{\gamma \cdot (\ell - k) + m_e\}\gamma^\lambda (1 - \gamma^5) v_\nu(p_\nu) \\ & \times \bar{u}_p(p_2)\gamma_\mu \{\gamma \cdot (p_2 + k) + m_p\} W_\lambda(p_2 + k, p_1)(1 + s\gamma^5\gamma \cdot n) u_n(p_1),\end{aligned} \tag{7}$$

where λ is the photon mass to regulate the infrared divergence. Using identities,

$$\bar{u}_e(\ell)\gamma^\mu \{\gamma \cdot (\ell - k) + m_e\} = \bar{u}_e(\ell)\{(2\ell - k)^\mu + i\sigma^{\mu\nu}k_\nu\}, \tag{8}$$

$$\bar{u}_p(p_2)\gamma_\mu \{\gamma \cdot (p_2 + k) + m_p\} = \bar{u}_p(p_2)\{(2p_2 + k)_\mu - i\sigma_{\mu\nu}k^\nu\}, \tag{9}$$

we decompose (7) into three parts,

$$\mathcal{M}^{(v)} = \mathcal{M}^{(v1)} + \mathcal{M}^{(v2)} + \mathcal{M}^{(v3)}. \tag{10}$$

Here $\mathcal{M}^{(v1)}$ picks up the product of $(2\ell - k)^\mu$ in (8) and $(2p_2 + k)_\mu$ in (9), and at the same time $W_\lambda(p_2 + k, p_1)$ is replaced by $W_\lambda(p_2, p_1)$. It has the same gamma matrix structure as the Born term (1), and is then written as a multiplicative correction factor. This correction has both UV and IR divergences and depends on the electron velocity. The UV divergence in $\mathcal{M}^{(v1)}$ is cancelled by that in the self energy correction of $\mathcal{M}^{(s)}$,

$$\mathcal{M}^{(s)} = \left\{\sqrt{Z_2(m_e)} - 1 + \sqrt{Z_2(m_p)} - 1\right\}\mathcal{M}^{(0)}, \tag{11}$$

and the IR divergence, along with that arising from $\mathcal{M}^{(s)}$, is cancelled when the contribution of bremsstrahlung $\mathcal{M}^{(b)}$ is added.

The term $\mathcal{M}^{(v2)}$ represents the combination of $i\sigma^{\mu\nu}k_\nu$ in (8) and $(2p_2+k)_\mu$ in (9), and $W_\lambda(p_2+k, p_1)$ is again replaced with $W_\lambda(p_2, p_1)$. This term is UV and IR finite, but gives an electron-velocity dependent factor. In the static limit of the nucleon the correction is also a multiplication on the tree amplitude.

A straightforward calculation yields

$$\sum_{\text{spin}}\left\{\left(\mathcal{M}^{(v1)}+\mathcal{M}^{(v2)}\right)\mathcal{M}^{(0)*}+\text{c.c.}\right\}+\int\frac{d^3\vec{k}}{(2\pi)^3 2\omega}\sum_{\text{spin}}\left|\mathcal{M}^{(b)}\right|^2\frac{(E_0-E-\omega)}{(E_0-E)}$$
$$=16G_V^2 m_n m_p E_\nu\frac{e^2}{8\pi^2}\left[\left(g(E)-\frac{3}{4}\right)\left(f_V^2+3g_A^2\right)E+2\left(\hat{g}(E)-\frac{3}{4}\right)\left(f_V g_A-g_A^2\right)s\left(\vec{n}\cdot\vec{\ell}\right)\right], \tag{12}$$

where $g(E, E_0)$ is the conventional g function that appears in the radiative correction for the beta decay rate and is defined with an additional constant $3/4$ [7,12].

$$g(E,E_0)=3\ln\left(\frac{m_p}{m_e}\right)-\frac{3}{4}+\frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right)+4\left(\frac{1}{\beta}\tanh^{-1}\beta-1\right)\left[\frac{E_0-E}{3E}-\frac{3}{2}+\ln\frac{2(E_0-E)}{m_e}\right]$$
$$+\frac{1}{\beta}\tanh^{-1}\beta\left\{2\left(1+\beta^2\right)+\frac{(E_0-E)^2}{6E^2}-4\tanh^{-1}\beta\right\}, \tag{13}$$

where E_0 is the end point energy of the electron, and $\hat{g}(E, E_0)$ is a similar function for the spin-dependent term,

$$\hat{g}(E,E_0)=3\ln\left(\frac{m_p}{m_e}\right)-\frac{3}{4}+\frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right)$$
$$+4\left(\frac{1}{\beta}\tanh^{-1}\beta-1\right)\left[\frac{E_0-E}{3E\beta^2}-\frac{3}{2}+\frac{(E_0-E)^2}{24E^2\beta^2}+\ln\frac{2(E_0-E)}{m_e}\right]$$
$$+\frac{4}{\beta}\tanh^{-1}\beta\left(1-\tanh^{-1}\beta\right). \tag{14}$$

Here

$$L(z)=\int_0^z\frac{dt}{t}\log(1-t), \tag{15}$$

is the Spence function. We also define $\hat{g}(E, E_0)$ with an additional constant $3/4$ as a convention. These are the outer corrections, which we may write

$$\delta_{\text{out}}=\frac{e^2}{8\pi^2}g(E,E_0),\qquad \hat{\delta}_{\text{out}}=\frac{e^2}{8\pi^2}\hat{g}(E,E_0) \tag{16}$$

and agree with the formulae given by Shann [1] and by García and Maya [3]. They also agree with the expression derived by Yokoo et al. [2] up to a constant.

We now consider the remaining term $\mathcal{M}^{(v3)}$,

$$\mathcal{M}^{(v3)}=\frac{i}{2\sqrt{2}}G_V e^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{(\ell-k)^2-m_e^2}\frac{1}{(p_2+k)^2-m_p^2}\frac{1}{k^2-\lambda^2}$$
$$\times\bar{u}_e(\ell)\left\{(2\ell-k)^\mu+i\sigma^{\mu\nu}k_\nu\right\}\gamma^\lambda\left(1-\gamma^5\right)v_\nu(p_\nu)\bar{u}_p(p_2)R_{\mu\lambda}(p_2,p_1,k)\left(1+s\gamma^5\gamma.n\right)u_n(p_1), \tag{17}$$

where

$$R_{\mu\lambda}(p_1,p_2,k)=(2p_2+k)_\mu\left\{W_\lambda(p_2+k,p_1)-W_\lambda(p_2,p_1)\right\}-i\sigma_{\mu\nu}k^\nu W_\lambda(p_2+k,p_1)$$
$$\simeq -i\sigma_{\mu\nu}k^\nu W_\lambda(p_2+k,p_1)$$
$$\simeq -i\sigma_{\mu\nu}k^\nu\gamma_\lambda\left(f_V-g_A\gamma^5\right), \tag{18}$$

in the approximation of the point nucleons. It is only this $\mathcal{M}^{(v3)}$ term that depends on the details of the hadronic part of the weak current. It is clear from the powers of k that this term is IR convergent, whereas it is UV divergent.

A straightforward calculation leads to

$$\sum_{\text{spin}}\{\mathcal{M}^{(v3)}\mathcal{M}^{(0)*}+\mathcal{M}^{(v3)*}\mathcal{M}^{(0)}\}$$
$$=16G_V^2 m_n m_p\frac{e^2}{8\pi^2}\Bigg[\left\{\frac{3}{2}(f_V^2+f_V g_A)\log\left(\frac{M}{m_p}\right)^2+\left(\frac{3}{4}f_V^2+\frac{9}{4}f_V g_A\right)\right\}(EE_\nu+\vec{\ell}\cdot\vec{p}_\nu)$$
$$+\left\{\frac{3}{2}(g_A^2+f_V g_A)\log\left(\frac{M}{m_p}\right)^2+\left(\frac{7}{4}g_A^2+\frac{5}{4}f_V g_A\right)\right\}(3EE_\nu-\vec{\ell}\cdot\vec{p}_\nu)$$
$$+2s\left\{\left(\frac{3}{4}f_V^2+\frac{3}{4}g_A^2+\frac{3}{2}f_V g_A\right)\log\left(\frac{M}{m_p}\right)^2+\left(\frac{5}{8}f_V^2+\frac{9}{8}g_A^2+\frac{5}{4}f_V g_A\right)\right\}\left(E(\vec{n}\cdot\vec{p}_\nu)+E_\nu(\vec{n}\cdot\vec{\ell})\right)$$
$$+2s\left\{\frac{3}{2}(g_A^2+f_V g_A)\log\left(\frac{M}{m_p}\right)^2+\left(\frac{7}{4}g_A^2+\frac{5}{4}f_V g_A\right)\right\}\left(E(\vec{n}\cdot\vec{p}_\nu)-E_\nu(\vec{n}\cdot\vec{\ell})\right)\Bigg]. \tag{19}$$

We use the current algebra technique to classify logarithmic divergences into those that have universal coefficients irrespective of the model of hadrons and those that are model dependent. Repeating the same calculation as in Paper I but including the spin projection operator, we find that $(3/2)f_V^2\log(M/m_p)^2$ in the first, $(3/2)g_A^2\log(M/m_p)^2$ in the second, $(3/2)f_V g_A\log(M/m_p)^2$ in the third and $(3/2)g_A^2\log(M/m_p)^2$ in the fourth curly brackets are universal. This observation tells us that the terms are summarised as

$$\sum_{\text{spin}}\{\mathcal{M}^{(v3)}\mathcal{M}^{(0)*}+\mathcal{M}^{(v3)*}\mathcal{M}^{(0)}\}$$
$$=16G_V^2 m_n m_p\Bigg[f_V^2\left(\delta_{\text{in}}^{\text{F}'}+\frac{3}{4}\cdot\frac{e^2}{8\pi^2}\right)(EE_\nu+\vec{\ell}\cdot\vec{p}_\nu)+g_A^2\left(\delta_{\text{in}}^{\text{GT}'}+\frac{3}{4}\cdot\frac{e^2}{8\pi^2}\right)(3EE_\nu-\vec{\ell}\cdot\vec{p}_\nu)$$
$$+2sf_V g_A\left(\frac{1}{2}\delta_{\text{in}}^{\text{F}'}+\frac{1}{2}\delta_{\text{in}}^{\text{GT}'}+\frac{3}{4}\cdot\frac{e^2}{8\pi^2}\right)\{E(\vec{n}\cdot\vec{p}_\nu)+E_\nu(\vec{n}\cdot\vec{\ell})\}$$
$$+2sg_A^2\left(\delta_{\text{in}}^{\text{GT}'}+\frac{3}{4}\cdot\frac{e^2}{8\pi^2}\right)\{E(\vec{n}\cdot\vec{p}_\nu)-E_\nu(\vec{n}\cdot\vec{\ell})\}\Bigg]. \tag{20}$$

Adding the tree term and after integration over p_ν

$$\sum_{\text{spin}}|\mathcal{M}^{(0)}|^2+\sum_{\text{spin}}\{\mathcal{M}^{(v3)}\mathcal{M}^{(0)*}+\mathcal{M}^{(v3)*}\mathcal{M}^{(0)}\}$$
$$=16G_V^2 m_n m_p E_\nu\left(1+\frac{3}{4}\cdot\frac{e^2}{8\pi^2}\right)$$
$$\times\left[\{f_V^2(1+\delta_{\text{in}}^{\text{F}'})+3g_A^2(1+\delta_{\text{in}}^{\text{GT}'})\}E+2s\left\{f_V g_A\left(1+\frac{1}{2}\delta_{\text{in}}^{\text{F}'}+\frac{1}{2}\delta_{\text{in}}^{\text{GT}'}\right)-g_A^2(1+\delta_{\text{in}}^{\text{GT}'})\right\}(\vec{n}\cdot\vec{\ell})\right], \tag{21}$$

where

$$\delta_{\text{in}}^{\text{F}'}=\frac{e^2}{8\pi^2}\left[\frac{3}{2}\log\left(\frac{M^2}{m_p^2}\right)+\frac{g_A}{f_V}\left\{\frac{3}{2}\log\left(\frac{M^2}{m_p^2}\right)+\frac{9}{4}\right\}\right], \tag{22}$$

$$\delta_{\text{in}}^{\text{GT}'}=\frac{e^2}{8\pi^2}\left[\frac{3}{2}\log\left(\frac{M^2}{m_p^2}\right)+1+\frac{f_V}{g_A}\left\{\frac{3}{2}\log\left(\frac{M^2}{m_p^2}\right)+\frac{5}{4}\right\}\right] \tag{23}$$

are the inner corrections for Fermi and Gamow–Teller transitions that are defined in Paper I and the factor $[1 + (3/4)e^2/8\pi^2]$ is to be included in $\delta_{\rm out}$. In Eqs. (22) and (23) the first logarithms are model-independent and the second with the coefficients g_A/f_V or f_V/g_A are model dependent. The correction from $\mathcal{M}^{(v3)}$ is written as multiplicative factors on the coupling constants for both Fermi and Gamow–Teller parts while they are divergent within QED.

The short distance correction from the integration region (ii) in (5) is evaluated using electroweak theory [5]. When we consider corrections relative to muon decay, we only need to consider the box diagrams of photon (or Z) and W exchanges (see Fig. 2 of Paper I). In order to connect the quark-level amplitudes with hadronic ones, we assume that the ratio of the tree and loop amplitudes for beta decays of the assembly of quarks is the same as that for neutron beta decay [5]. This is justified at least for the universal logarithmic divergent part. With this prescription the correction amounts to a multiplication factor

$$\frac{e^2}{8\pi^2}\left[\left\{\frac{3}{2}\log\left(\frac{m_W^2}{M^2}\right) + 3\bar{Q}\log\left(\frac{m_Z^2}{M^2}\right) + \frac{5}{2\tan^4\theta_W}\log\left(\frac{m_Z^2}{m_W^2}\right)\right\} - \left(-\frac{3}{2} + \frac{5}{2\tan^4\theta_W}\right)\log\left(\frac{m_Z^2}{m_W^2}\right)\right]\left|\mathcal{M}^{(0)}\right|^2, \tag{24}$$

where $\bar{Q} = 1/6$ is the mean charge of the isodoublet of quarks, and the second line is the correction that appears in muon decay and thus subtracted when we consider the radiative correction relative to muon decay that determines G_F.

This electroweak one-loop correction amounts to adding to $\delta_{\rm in}^{\rm F'}$ extra terms,

$$\begin{aligned}\delta_{\rm in}^{\rm F} \equiv \delta_{\rm in}^{\rm F'} &+ \frac{e^2}{8\pi^2}\left[\frac{3}{2}\log\left(\frac{m_W^2}{M^2}\right) + 3\bar{Q}\log\left(\frac{m_Z^2}{M^2}\right) + \frac{5}{2\tan^4\theta_W}\log\left(\frac{m_Z^2}{m_W^2}\right)\right] \\ &- \frac{e^2}{8\pi^2}\left(-\frac{3}{2} + \frac{5}{2\tan^4\theta_W}\right)\log\left(\frac{m_Z^2}{m_W^2}\right) \\ &= \frac{e^2}{8\pi^2}\left[\frac{3}{2}\log\left(\frac{m_Z^2}{m_p^2}\right) + 3\bar{Q}\log\left(\frac{m_Z^2}{M^2}\right) + C^{\rm F}\right],\end{aligned} \tag{25}$$

where the terms proportional to g_A/f_V are collected in $C^{\rm F}$,

$$C^{\rm F} = \frac{g_A}{f_V}\left\{\frac{3}{2}\log\left(\frac{M^2}{m_p^2}\right) + \frac{9}{4}\right\} \tag{26}$$

and similarly for $\delta_{\rm in}^{\rm GT'}$,

$$\delta_{\rm in}^{\rm GT} = \frac{e^2}{8\pi^2}\left[\frac{3}{2}\log\left(\frac{m_Z^2}{m_p^2}\right) + 1 + 3\bar{Q}\log\left(\frac{m_Z^2}{M^2}\right) + C^{\rm GT}\right], \tag{27}$$

where $C^{\rm GT}$ is

$$C^{\rm GT} = \frac{f_V}{g_A}\left\{\frac{3}{2}\log\left(\frac{M^2}{m_p^2}\right) + \frac{5}{4}\right\}, \tag{28}$$

for point nucleons.

We observe that the M dependence (upper cutoff) that appears in the first term of (22) is cancelled by the first term in the braces in (24), which demonstrates a smooth connection from electroweak theory to effective hadronic theory for the Fermi transition. The UV divergence in the term proportional to g_A/f_V, however, fails to cancel against the divergence with the coefficient $3\bar{Q}$, unless $\bar{Q}$ has a specific (and unrealistic) value of charge. Marciano and Sirlin [10] proposed to evaluate the model-dependent long-distance divergence of the Fermi transition by rendering it softer introducing nucleon form factors, i.e., by replacing

$$\gamma^\mu \to \gamma^\mu F_1(k^2) - \frac{i}{2m_N}\sigma^{\mu\nu}k_\nu F_2(k^2), \quad m_N = \frac{1}{2}(m_p + m_n) \tag{29}$$

at the electromagnetic vertex and leave the term $3\bar{Q}\log(m_Z/M)$ as it is taking M as the mass scale of the onset of the asymptotic behaviour [10]. The same procedure was followed in Paper I for the Gamow–Teller part. It was noted, however, that the inclusion of the weak magnetism is important at the weak vertex to evaluate the long-distance integral, because the mass scale of the form factor is comparable to the proton mass and the loop integral over the weak magnetism form factor gives the same order as does the V–A contribution (in Paper I, in fact, it was found numerically that the former is larger than the latter for the Gamow–Teller part). We replace

$$W_\lambda(p_2, p_1) \to \gamma_\lambda\{f_V F_V(k^2) - g_A\gamma^5 F_A(k^2)\} - \frac{i}{2m_p}\sigma^{\mu\nu}k_\nu F_W(k^2), \tag{30}$$

in Eq. (7). From the form we observed in Eq. (21) we expect that the calculation incorporating form factors would give rise to a result summarised in the same form, while C^{F} and C^{GT} are modified exactly as in Paper I. Since we have not found an immediate proof that it should, we repeated a long calculation as we did in Paper I including the spin projection operator, and confirmed the anticipated result. In fact, we obtained C^{F} and C^{GT} exactly those that appear in the spin independent part. So we take the result of numerical integral of Paper I,

$$C^{\mathrm{F}} = 1.751 + 0.409 = 2.160, \tag{31}$$

$$C^{\mathrm{GT}} = 0.727 + 2.554 = 3.281, \tag{32}$$

where the two parts of numbers represent contributions from the (V, A) interaction and weak magnetism. The first number in C^{F} was evaluated by Marciano and Sirlin [10] and by Towner [11], and agrees with their results up to slight differences in the input parameters.

In conclusion the radiative correction to polarised neutron beta decay to order $O(\alpha)$ is summarised as

$$\begin{aligned}
&|\mathcal{M}^{(0)}|^2 + \sum_{\text{spin}}\{\mathcal{M}'\mathcal{M}^{(0)*} + \mathcal{M}'^*\mathcal{M}^{(0)}\} + \int \frac{d^3\vec{k}}{(2\pi)^3 2\omega}\sum_{\text{spin}}|\mathcal{M}^{(b)}|^2\frac{(E_0 - E - \omega)}{(E_0 - E)} \\
&\quad = 16G_V^2 m_n m_p E_\nu\Big[\{f_V^2(1 + \delta_{\text{in}}^{\mathrm{F}} + \delta_{\text{out}}) + 3g_A^2(1 + \delta_{\text{in}}^{\mathrm{GT}} + \delta_{\text{out}})\}E \\
&\qquad + 2s\Big\{f_V g_A\Big(1 + \frac{1}{2}\delta_{\text{in}}^{\mathrm{F}} + \frac{1}{2}\delta_{\text{in}}^{\mathrm{GT}} + \hat{\delta}_{\text{out}}\Big) - g_A^2(1 + \delta_{\text{in}}^{\mathrm{GT}} + \hat{\delta}_{\text{out}})\Big\}(\vec{n}\cdot\vec{\ell})\Big]
\end{aligned} \tag{33}$$

in the static nucleon approximation. The terms in the first braces are those that give the neutron beta decay rate

$$\Gamma = \frac{G_V^2}{2\pi^3}(\bar{f}_V^2 + 3\bar{g}_A^2)\int_{m_e}^{E_0} dE\, E\sqrt{E^2 - m_e^2}\, E_\nu^2\Big\{1 + \frac{\alpha}{2\pi}g(E, E_0)\Big\}F(E, Z), \tag{34}$$

where α is the fine structure constant, $F(E, Z)$ is the Fermi function for the Coulomb correction with $Z = 1$, g is the outer radiative correction defined in (16), and

$$\bar{f}_V^2 = f_V^2(1 + \delta_{\text{in}}^{\mathrm{F}}), \qquad \bar{g}_A^2 = g_A^2(1 + \delta_{\text{in}}^{\mathrm{GT}}). \tag{35}$$

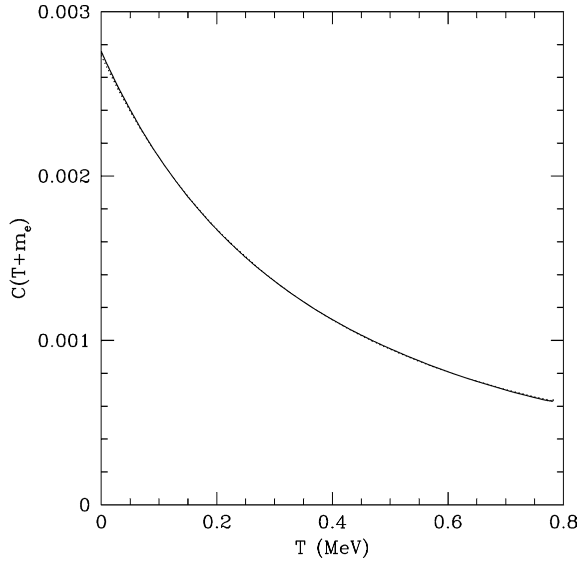

Fig. 2. Outer radiative correction $\mathcal{C}(T+m_e)$ for the asymmetry parameter as a function of the kinetic energy of electron. The solid curve is $\mathcal{C}(T+m_e)$, and the dotted curve, which overlays nearly exactly on the solid curve, is fit (38).

The asymmetry parameter is given by the ratio of the terms in the second braces to those in the first braces in (33), i.e., it is written as

$$A = 2\frac{1+\frac{\alpha}{2\pi}\hat{g}(E,E_0)}{1+\frac{\alpha}{2\pi}g(E,E_0)}\frac{\bar{f}_V\bar{g}_A-\bar{g}_A^2}{\bar{f}_V^2+3\bar{g}_A^2}. \tag{36}$$

This demonstrates that the radiative corrections that appear in the asymmetry parameter take the same factors as those that appear in the decay rate, and they may be absorbed into f_V and g_A.

The denominator of Eq. (36) is the combination that appears in the neutron decay rate (34). The energy dependent prefactor $\mathcal{C}(E)$,

$$1+\mathcal{C}(E) = \left[1+\frac{\alpha}{2\pi}\hat{g}(E,E_0)\right]\Big/\left[1+\frac{\alpha}{2\pi}g(E,E_0)\right] \tag{37}$$

is plotted in Fig. 2 as a function of the kinetic energy $T = E - m_e$. The magnitude of $(\alpha/2\pi)g(E,E_0)$ and $(\alpha/2\pi)\hat{g}(E,E_0)$ is about 2%, but the two corrections nearly cancel in $\hat{g}(E,E_0) - g(E,E_0)$, leaving the net outer correction for the asymmetry being quite small, of the order of 0.1%. For convenience we give a fit to $\mathcal{C}(E)$ for neutron beta decay with

$$\mathcal{C}(E) = -0.00163 + 0.00210/E + 0.000491E, \tag{38}$$

where E is in units of MeV. The fit, also displayed in Fig. 2, overlays nearly top on the true function of $\mathcal{C}(E)$.

The cancellation also takes place for the inner correction. After correcting for $\mathcal{C}(E)$, the axial-vector to vector coupling ratio extracted from the tree level formula is related to its tree-level value as

$$\frac{\bar{g}_A}{\bar{f}_V} = \left[1+\frac{\alpha}{4\pi}\left(1+C^{\mathrm{GT}}-C^{\mathrm{F}}\right)\right]\left(\frac{g_A}{f_V}\right) = 1.0012\frac{g_A}{f_V}. \tag{39}$$

The dominant part of the inner correction, including $\log m_Z/m_p$ cancels in $\delta_{\mathrm{in}}^{\mathrm{F}} - \delta_{\mathrm{in}}^{\mathrm{GT}}$, and the net correction is of the order of 0.1% for g_A/f_V (which is usually denoted as $\lambda \equiv G_A/G_V = -g_A/f_V$).

The Particle Data Group [13] gives a value $g_A = 1.2670 \pm 0.0030$. This is obtained by averaging 5 values reported in the literature, one of which [14] is obtained including the outer radiative correction (with the inner correction discarded), and others are results that do not include radiative corrections. The outer radiative correction reduces the value of $|g_A|$ by about 0.0007, but the scatter among the data from different authors is 0.005 (rms), so systematic errors other than the radiative correction dominate the uncertainty of g_A. As we have shown that the inner radiative corrections can be included into g_A in common irrespective of quantities measured for beta decay, it is a matter of definition whether they are included in g_A or not, in so far as we deal only with the charged current processes. If we define the tree-level axial coupling constant it is related with the value including the radiative correction by Eq. (39). This is crucial when we consider the radiative corrections for neutral current induced reactions.

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References

[1] R.T. Shann, Nuovo Cimento A 5 (1971) 591.
[2] Y. Yokoo, S. Suzuki, M. Morita, Prog. Theor. Phys. 50 (1973) 1894.
[3] A. García, M. Maya, Phys. Rev. D 17 (1978) 1376.
[4] F. Glück, K. Tóth, Phys. Rev. D 46 (1992) 2090.
[5] A. Sirlin, Nucl. Phys. B 71 (1974) 29.
[6] E.S. Abers, R.E. Norton, D.A. Dicus, Phys. Rev. Lett. 18 (1967) 676;
E.S. Abers, D.A. Dicus, R.E. Norton, H.R. Quinn, Phys. Rev. 167 (1968) 1461.
[7] A. Sirlin, Phys. Rev. 164 (1967) 1767.
[8] M. Fukugita, T. Kubota, Acta Phys. Pol. B 35 (2004) 1687.
[9] A. Sirlin, Nucl. Phys. B 196 (1982) 83.
[10] W.J. Marciano, A. Sirlin, Phys. Rev. Lett. 56 (1986) 22.
[11] I.S. Towner, Nucl. Phys. A 540 (1992) 478.
[12] T. Kinoshita, A. Sirlin, Phys. Rev. 113 (1959) 1652.
[13] Particle Data Group, K. Hagiwara, et al., Phys. Rev. D 66 (2002) 1.
[14] H. Abele, et al., Phys. Lett. B 407 (1997) 212;
H. Abele, et al., Phys. Rev. Lett. 88 (2002) 211801.