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Effective gluon interactions in the colour superconductive phase of two flavor QCD

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Abstract

The gluon self-energies and dispersion laws in the color superconducting phase of QCD with two massless flavors are calculated using the effective theory near the Fermi surface. These quantities are calculated at zero temperature for all the eight gluons, those of the remaining $SU(2)$ color group and those corresponding to the broken generators. The construction of the effective interaction is completed with the one loop calculation of the three- and four-point gluon interactions.

1. Introduction

Color superconductivity in QCD at large densities is a rather ancient idea [1] that has recently received a new attention in a series of papers [2,3] (for recent review, see [4]). Both three-flavor (Color Flavor Locking model = CFL) and two-flavor cases (2SC model) have been studied. In this Letter we consider the 2SC model, i.e., two massless quarks in the color superconducting phase, whose main features are as follows. At zero density the theory is invariant under the group $SU(3)_C \times SU(2)_L \times SU(2)_R$, but at high density Cooper pairing of two quarks is energetically favored [2]. The condensation from single-gluon exchange between two quarks takes place in the color antitriplet channel. The condensate breaks $SU(3)_C$ down to an $SU(2)_C$ subgroup and therefore the quarks with non-trivial $SU(2)_C$ charges acquire a gap Δ; moreover, five gluons acquire mass by the Higgs mechanism. The chiral symmetry remains unbroken, which implies that there are no Goldstone bosons. We will use here an effective theory near the Fermi surface [5], which has been recently applied to the CFL phase [6] and to the crystalline phase [7] (for a discussion of the superconductive crystalline phase, see [8]). The main results of this approach are summarized in Section 2. In Section 3 we compute the gluon self-energies; we confirm the results obtained by other authors [9,10] for the masses of the five gluons associated to the broken generators and for the dispersion laws of the unbroken gluons; moreover, we extend these results to the dispersion laws of all the gluons. In Section 4 we present the one-loop corrections to the three and four gluon vertices in the 2SC phase for all the eight gluons, which allows for a complete effective description

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of the gluon degrees of freedom of this phase at zero temperature.

2. Effective theory

To start with let us recall some results valid for the 2SC model. In Ref. [5,6] an effective two dimensional field theory for the CFL phase of QCD in terms of velocity dependent fields was developed; in particular, in [6] it was applied to the computation of the gluon dispersion laws. In [7] this theory was extended to the 2SC model (in the crystalline phase). The main ideas are as follows. To describe excitations near the Fermi surface one writes the momentum p of the quarks as

$$p^\nu = \mu v^\nu + l^\nu, \tag{1}$$

where μ is the quarks chemical potential and $v^\nu = (0, \vec{v}_F)$, with $\vec{v}_F$ Fermi velocity ($|\vec{v}_F| = 1$). Only positive energy states ψ_+ contribute to the Lagrangian, whereas negative energy states decouple and can be expressed in terms of the positive energy states. If we define

$$\gamma_\perp^\mu = \frac{1}{2}\gamma_\nu\left(2g^{\mu\nu} - V^\mu \tilde{V}^\nu - \tilde{V}^\mu V^\nu\right), \tag{2}$$

$$V^\mu = (1, \vec{v}_F), \qquad \tilde{V}^\mu = (1, -\vec{v}_F), \tag{3}$$

we can write the negative energy states as

$$\psi_- = -\frac{1}{2\mu}\gamma_0 \not{\partial}_T \psi_+, \tag{4}$$

showing the decoupling of ψ_- in the $\mu \to \infty$ limit. Expressing ψ_- in terms of ψ_+ results in an effective theory which at the next to leading order in the inverse of μ is described by the Lagrangian

$$\mathcal{L} = \sum_{\vec{v}_F}\left[\psi_+^\dagger i V \cdot D \psi_+ - \frac{1}{2\mu}\psi_+^\dagger (\not{D}_\perp)^2 \psi_+\right], \tag{5}$$

where D_μ is the covariant derivative with respect to the color group. At this stage it is useful to use a different basis for the fermion fields. We introduce the six fields φ_+^A $(A = 0, \ldots, 5)$ by the formulae

$$\psi_{+,i\alpha} = \sum_{A=0}^{3} \frac{(\sigma_A)_{i\alpha}}{\sqrt{2}}\varphi_+^A \quad (i, \alpha = 1, 2),$$
$$\psi_{+,13} = \varphi_+^4, \qquad \psi_{+,23} = \varphi_+^5, \tag{6}$$

where σ_A are the Pauli matrices for $A = 1, 2, 3$ and $\sigma_0 = 1$. The Greek indices α, β are color indices and the Latin indices i, j are for the two flavors $1, 2$. Here clearly φ_+^A are positive energy, velocity-dependent fields:

$$\varphi_+^A \equiv \varphi_{+,\vec{v}}^A. \tag{7}$$

We also introduce the positive energy fields with opposite velocity:

$$\varphi_-^A \equiv \varphi_{+,-\vec{v}}^A. \tag{8}$$

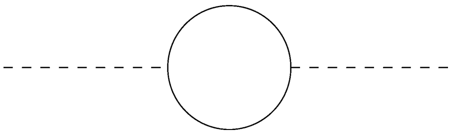
By defining

$$\chi^A = \begin{pmatrix} \varphi_+^A \\ C\varphi_-^{A*} \end{pmatrix}, \tag{9}$$

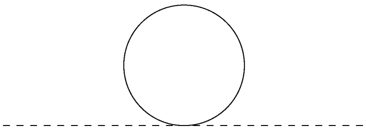
the Lagrangian can be written as follows ($F_{\mu\nu}^a$ are the eight gluon fields):

$$\mathcal{L} = \frac{1}{2}\sum_{\vec{v}_F}\sum_{A=0}^{5}\chi_A^\dagger \times \begin{bmatrix} iV\cdot D - \frac{1}{2\mu}\not{D}_\perp^2 & \Delta^A \\ \Delta^A & i\tilde{V}\cdot D^* - \frac{1}{2\mu}\not{D}_\perp^2 \end{bmatrix} - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a, \tag{10}$$

which describes, at the lowest order, the effective theory. This Lagrangian allows for the evaluation of the two diagrams in Fig. 1 which give the one-



(a)



(b)

Fig. 1. Gluon self energy diagrams; in both (a) and (b) dotted lines represent gluon fields and full lines are the fermion propagators; (b) is the tadpole diagram.

loop contributions to the polarization tensor $\Pi_{ab}^{\mu\nu}(p)$ (a, b are color $SU(3)$ indices, $a, b = 1, \ldots, 8$). As discussed in [6] the integration over the loop variable ℓ is 2-dimensional (ℓ_0, $\ell_\|$), while the integration over directions perpendicular to the Fermi velocity gives a factor of μ^2/π. The diagrams in Fig. 1 are the only diagrams not suppressed in the $\mu \to \infty$ limit. We write

$$\Pi_{ab}^{\mu\nu}(p) = \Pi_{ab}^{\mu\nu}(0) + \delta\Pi_{ab}^{\mu\nu}(p), \tag{11}$$

with

$$\Pi_{ab}^{\mu\nu}(0) = \frac{\mu^2 g^2}{2\pi^2}\int \frac{d\vec{v}_F}{4\pi}\Sigma_{ab}^{0,\mu\nu}, \tag{12}$$

and

$$\delta\Pi_{ab}^{\mu\nu}(p) = \frac{\mu^2 g^2}{2\pi^2}\int \frac{d\vec{v}_F}{4\pi}\Sigma_{ab}^{\mu\nu}(p). \tag{13}$$

To start with we consider the diagram 1(a). Let us introduce the following notations. We will denote the indices a, b by the letters i, j for $a, b = 1, 2, 3$, while for $a, b = 4, 5, 6, 7$ we use the Greek letters α, β. The results of our calculation are, for $a, b = i, j$ ($= 1, 2, 3$):

$$\begin{aligned}\Pi_{ij}^{\mu\nu} &= i\delta_{ij}\frac{g^2\mu^2}{4\pi^3}\int\frac{d\vec{v}_F}{4\pi}\int d^2\ell \\ &\quad\times\left(\frac{V^\mu V^\nu \widetilde{V}\cdot\ell\widetilde{V}\cdot(\ell+p) + \widetilde{V}^\mu\widetilde{V}^\nu V\cdot\ell V\cdot(\ell+p)}{D_1(\ell+p)D_1(\ell)}\right.\\ &\qquad\left. + \Delta^2\frac{V^\mu\widetilde{V}^\nu + V^\nu\widetilde{V}^\mu}{D_1(\ell+p)D_1(\ell)}\right).\end{aligned} \tag{14}$$

For $a, b = \alpha, \beta$ the polarization tensor is

$$\begin{aligned}\Pi_{\alpha\beta}^{\mu\nu} &= i\delta_{\alpha\beta}\frac{g^2\mu^2}{8\pi^3}\int\frac{d\vec{v}_F}{4\pi}\int d^2\ell \\ &\quad\times\big(V^\mu V^\nu\widetilde{V}\cdot\ell\widetilde{V}\cdot(\ell+p) \\ &\qquad + \widetilde{V}^\mu\widetilde{V}^\nu V\cdot\ell V\cdot(\ell+p)\big) \\ &\quad\times\left(\frac{1}{D_1(\ell+p)D_2(\ell)} + \frac{1}{D_2(\ell+p)D_1(\ell)}\right).\end{aligned} \tag{15}$$

Finally, for the gluon 8 we have:

$$\begin{aligned}\Pi_{88}^{\mu\nu} &= i\frac{g^2\mu^2}{12\pi^3}\int\frac{d\vec{v}_F}{4\pi}\int d^2\ell \\ &\quad\times\Big[\big(V^\mu V^\nu\widetilde{V}\cdot\ell\widetilde{V}\cdot(\ell+p) \\ &\qquad + \widetilde{V}^\mu\widetilde{V}^\nu V\cdot\ell V\cdot(\ell+p)\big) \\ &\qquad\times\left(\frac{1}{D_1(\ell+p)D_1(\ell)} + \frac{2}{D_2(\ell+p)D_2(\ell)}\right) \\ &\qquad - \Delta^2\frac{V^\mu\widetilde{V}^\nu + V^\nu\widetilde{V}^\mu}{D_1(\ell+p)D_1(\ell)}\Big].\end{aligned} \tag{16}$$

In the low momentum limit we can expand the polarization tensor for $a, b = i, j$ ($= 1, 2, 3$) in the following way:

$$\Sigma_{ij}^{0,\mu\nu} = \delta_{ij}\left(\frac{\widetilde{V}^\mu\widetilde{V}^\nu + V^\mu V^\nu}{2} - \frac{\widetilde{V}^\mu V^\nu + \widetilde{V}^\nu V^\mu}{2}\right), \tag{17}$$

and

$$\begin{aligned}\Sigma_{ij}^{\mu\nu}(p) = \delta_{ij}\Bigg(&\frac{V^\mu V^\nu(\widetilde{V}\cdot p)^2 + \widetilde{V}^\mu\widetilde{V}^\nu(V\cdot p)^2}{12\Delta^2} \\ &- \frac{V^\mu\widetilde{V}^\nu + \widetilde{V}^\mu V^\nu}{12\Delta^2}\big(V\cdot p\,\widetilde{V}\cdot p\big)\Bigg).\end{aligned} \tag{18}$$

The validity of this approximation will be discussed below.
It follows from Eqs. (12)–(18) that

$$\begin{aligned}\Pi_{ij}^{00}(p) &= \Pi_{ij}^{00}(0) + \delta\Pi_{ij}^{00}(p) = \delta\Pi_{ij}^{00}(p) \\ &= \delta_{ij}\frac{\mu^2 g^2}{18\pi^2\Delta^2}|\vec{p}\,|^2,\end{aligned} \tag{19}$$

$$\begin{aligned}\Pi_{ij}^{kl}(p) &= \Pi_{ij}^{kl}(0) + \delta\Pi_{ij}^{kl}(p) \\ &= \delta_{ij}\delta^{kl}\frac{\mu^2 g^2}{3\pi^2}\left(1 + \frac{p_0^2}{6\Delta^2}\right),\end{aligned} \tag{20}$$

and

$$\Pi_{ij}^{0k}(p) = \delta\Pi_{ij}^{0k}(p) = \delta_{ij}\frac{\mu^2 g^2}{18\pi^2\Delta^2}p^0 p^k. \tag{21}$$

These results agree with the outcomes of [9] and [10].

For $a, b = \alpha, \beta$ ($\alpha, \beta = 4, 5, 6, 7$) we find:

$$\Sigma_{\alpha\beta}^{0,\mu\nu} = \delta_{\alpha\beta}\left(\frac{V^\mu V^\nu + \widetilde{V}^\mu\widetilde{V}^\nu}{2}\right) \tag{22}$$

and

$$\Sigma_{\alpha\beta}^{\mu\nu}(p) = \delta_{\alpha\beta}\left(V^\mu V^\nu\frac{(\widetilde{V}\cdot p)^2}{4\Delta^2} + \widetilde{V}^\mu\widetilde{V}^\nu\frac{(V\cdot p)^2}{4\Delta^2}\right). \tag{23}$$

After integrating over the Fermi velocities we obtain

$$\Pi^{00}_{\alpha\beta}(p) = \Pi^{00}_{\alpha\beta}(0) + \delta\Pi^{00}_{\alpha\beta}(p) = \delta_{\alpha\beta}\frac{\mu^2 g^2}{2\pi^2}\left(1 + \frac{p_0^2 + |\vec{p}\,|^2/3}{2\Delta^2}\right), \tag{24}$$

$$\Pi^{0i}_{\alpha\beta}(p) = \delta\Pi^{0i}_{\alpha\beta}(p) = \delta_{\alpha\beta}\frac{\mu^2 g^2}{6\pi^2\Delta^2}p^0 p^i, \tag{25}$$

and

$$\Pi^{ij}_{\alpha\beta}(p) = \Pi^{ij}_{\alpha\beta}(0) + \delta\Pi^{ij}_{\alpha\beta}(p) = \delta_{\alpha\beta}\frac{\mu^2 g^2}{6\pi^2}\left(\delta^{ij} + \frac{\delta^{ij} p_0^2}{2\Delta^2} + \frac{\delta^{ij}\vec{p}^{\,2} + 2p^i p^j}{10\Delta^2}\right). \tag{26}$$

For $a, b = 8$ we get

$$\Sigma^{0,\mu\nu}_{88} = \left(\frac{5}{3}\frac{V^\mu V^\nu + \widetilde{V}^\mu \widetilde{V}^\nu}{2} + \frac{V^\mu \widetilde{V}^\nu + \widetilde{V}^\mu V^\nu}{6}\right), \tag{27}$$

$$\Sigma^{\mu\nu}_{88}(p) = \frac{1}{2}\left(\frac{V^\mu V^\nu (\widetilde{V}\cdot p)^2 + \widetilde{V}^\mu \widetilde{V}^\nu (V\cdot p)^2}{9\Delta^2} + \frac{V^\mu \widetilde{V}^\nu + \widetilde{V}^\mu V^\nu}{9\Delta^2}(V\cdot p\ \widetilde{V}\cdot p)\right), \tag{28}$$

therefore, we obtain

$$\Pi^{00}_{88}(p) = \Pi^{00}_{88}(0) + \delta\Pi^{00}_{88}(p) = \frac{\mu^2 g^2}{\pi^2}\left(1 + \frac{p_0^2}{18\Delta^2}\right), \tag{29}$$

and

$$\Pi^{0i}_{88}(p) = \delta\Pi^{0i}_{88}(p) = \frac{\mu^2 g^2}{54\pi^2\Delta^2}p^0 p^i, \tag{30}$$

$$\Pi^{ij}_{88}(p) = \Pi^{ij}_{88}(0) + \delta\Pi^{ij}_{88}(p) = \frac{\mu^2 g^2}{18\pi^2}\left(4\delta^{ij} + \frac{\delta^{ij}\vec{p}^{\,2} + 2p^i p^j}{15\Delta^2}\right). \tag{31}$$

These results complete the analysis of Fig. 1(a). Now we consider the diagram in Fig. 1(b). We note that this diagram is independent of the external momentum p, therefore it can only contribute to the gluon masses. We also note that the diagrams of Fig. 1 present infrared divergences (in ℓ_0). To control these divergences one considers the system in a heat bath and substitutes the energy euclidean integration $\ell_4 = -i\ell_0$ with a sum over the Matsubara frequencies $\ell_4 \to \omega_n = 2\pi(n+\frac{1}{2})\beta$; eventually one performs the limit $T = \frac{1}{\beta} \to 0$. In this way one finds for the contribution of the diagram 1(b) to $\Pi^{\mu\nu}_{ab}(0)$ the result ($a, b = 1, \ldots, 8$):

$$\Pi^{\mu\nu}_{ab}(0) = \frac{\mu^2 g^2}{4\pi^2}\delta_{ab}\int\frac{d\vec{v}_F}{4\pi}\gamma^\mu_T\gamma^\nu_T. \tag{32}$$

Therefore,

$$\Pi^{00}_{ab}(0) = 0, \tag{33}$$

showing that there is no contribution from this diagram to the Debye screening, while one gets

$$\Pi^{ij}_{ab}(0) = -\delta^{ij}\delta_{ab}\frac{\mu^2 g^2}{3\pi^2}. \tag{34}$$

In Table 1 we summarize the results for the Debye and Meissner masses obtained by the calculations of the two diagrams in Fig. 1, where a is the gluon color and $m_g^2 = \frac{\mu^2 g^2}{3\pi^2}$ is the squared gluon mass. Our results are in agreement with a calculation performed by [9] with a different method.

Table 1
Debye and Meissner masses for the gluons in the 2SC phase

a	$\Pi^{00}(0)$	$-\Pi^{ij}(0)$
1–3	0	0
4–7	$\frac{3}{2}m_g^2$	$\frac{1}{2}m_g^2$
8	$3m_g^2$	$\frac{1}{3}m_g^2$

3. Dispersion law for the gluons

In this section we will compute the dispersion laws for the gluons. We begin our discussion by considering the unbroken colors $a, b = i, j$ $(= 1, 2, 3)$.

3.1. Gluons 1, 2, 3

In this case we reobtain, by the present method, the results already found in [10] by a different approach, i.e.,

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}_i F^i_{\mu\nu} + \frac{1}{2}\Pi^{\mu\nu}_{ij}A^i_\mu A^j_\nu, \tag{35}$$

with $\Pi_{ij}^{\mu\nu}$ discussed above. Introducing the fields $E_i^a \equiv F_{0i}^a$ and $B_i^a \equiv i\varepsilon_{ijk}F_{jk}^a$, and using (19), (20) and (21) these results can be written as follows

$$\mathcal{L} = \frac{1}{2}\left(E_i^a E_i^a - B_i^a B_i^a\right) + \frac{k}{2}E_i^a E_i^a, \tag{36}$$

with

$$k = \frac{g^2\mu^2}{18\pi^2\Delta^2}. \tag{37}$$

As discussed in [10] this means that the medium has a very high *dielectric constant* $\epsilon = k+1$ and a *magnetic permeability* $\lambda = 1$. The gluon speed in this medium is now

$$v = \frac{1}{\sqrt{\epsilon\lambda}} \propto \frac{\Delta}{g\mu} \tag{38}$$

and in the high density limit it tends to zero. As shown in [10] the one loop Lagrangian (36) assumes the gauge invariant expression

$$\mathcal{L} = -\frac{1}{4}F_j^{\mu\nu}F_{\mu\nu}^j \quad (j=1,2,3), \tag{39}$$

provided the following rescaling is used

$$A_0^j \to A_0^{j\prime} = k^{3/4}A_0^j, \tag{40}$$

$$A_i^j \to A_i^{j\prime} = k^{1/4}A_i^j, \tag{41}$$

$$x_0 \to x_0' = k^{-1/2}x_0, \tag{42}$$

$$g \to g' = k^{-1/4}g. \tag{43}$$

3.2. Gluons 4–8

Let us now consider the equations of motion in momentum space for the gluon field A_μ^b, $b = 4,5,6,7,8$:

$$\left[\delta_{ab}\left(-g^{\mu\nu}p^2 + p^\mu p^\nu\right) + \Pi_{ab}^{\mu\nu}\right]A_\nu^b = 0. \tag{44}$$

We define the invariant quantities Π_0, Π_1, Π_2 and Π_3 by means of the following equations,

$$\Pi^{\mu\nu}(p_0,\vec p) = \begin{cases} \Pi^{00} = \Pi_0(p_0,\vec p), \\ \Pi^{0i} = \Pi^{i0} = \Pi_1(p_0,\vec p)n^i, \\ \Pi^{ij} = \Pi_2(p_0,\vec p)\delta^{ij} \\ \qquad + \Pi_3(p_0,\vec p)n^i n^j, \end{cases} \tag{45}$$

where we have suppressed the color indices, and $\vec n = \vec p/p$.

For the broken degrees of freedom it is useful to consider the scalar gluon field A_0^a and the longitudinal and transverse gluon fields defined by

$$A_{i\,L}^a = (\vec n\cdot\vec A^a)n_i, \qquad A_{i\,T}^a = A_i^a - A_{i\,L}^a. \tag{46}$$

By the equation

$$p_\nu \Pi_{ab}^{\nu\mu}A_\mu^b = 0, \tag{47}$$

one obtains the relationship

$$\left(p_0\Pi_0 - |\vec p\,|\Pi_1\right)A_0 = \vec n\cdot\vec A\left(p_0\Pi_1 - |\vec p\,|(\Pi_2+\Pi_3)\right), \tag{48}$$

between the scalar and the longitudinal component of the gluon fields. The dispersion laws for the scalar, longitudinal and transverse gluons are respectively

$$\begin{aligned} &\left(\Pi_2+\Pi_3+p_0^2\right)\left(|\vec p\,|^2+\Pi_0\right) = p_0|\vec p\,|\left(2\Pi_1 + p_0|\vec p\,|\right), \\ &\left(\Pi_2+\Pi_3+p_0^2\right)\left(|\vec p\,|p_0+\Pi_0\right) \\ &\quad = p_0|\vec p\,|\left(2\Pi_1+p_0^2\right)+\Pi_1^2, \\ &p_0^2 - |\vec p\,|^2 + \Pi_2 = 0. \end{aligned} \tag{49}$$

Expanding (48) at first order in p we find for $b = \beta = 4,5,6,7$

$$p^0 A_0^\beta = \frac{1}{3}\vec p\cdot\vec A^\beta, \tag{50}$$

while for $b = 8$

$$p^0 A_0^8 = \frac{1}{9}\vec p\cdot\vec A^8. \tag{51}$$

In any case we define two masses, the rest mass:

$$m^R = \left|p_0\left(|\vec p\,| = 0\right)\right|, \tag{52}$$

and the effective mass m^*, by the formula:

$$\vec p = m^*\frac{\partial E}{\partial\vec p}, \tag{53}$$

in the low momentum limit. Using Eqs. (24)–(26) we observe that the effect of the kinetic terms in (44) is negligible at the order $g^2\mu^2/\Delta^2$ for $\beta = 4,5,6,7$ in the high density limit. The dispersion law for the time-like component is:

$$\left(m_D^2 + \frac{m_D^2}{2\Delta^2}\left(p_0^2 + \frac{1}{3}|\vec p\,|^2\right) - \frac{m_D^2}{\Delta^2}p_0^2\right)A_0^\beta = 0, \tag{54}$$

where, from Table 1, we get $m_D^2 = \Pi^{00}(0) = \frac{\mu^2 g^2}{2\pi^2}$; therefore,

$$p_0 = \pm E_0, \quad E_0 = \frac{1}{\sqrt3}\sqrt{|\vec p\,|^2 + 6\Delta^2}. \tag{55}$$

The rest mass for these gluons, in the gradient expansion approximation, is given by

$$m^R_{A_0} = \sqrt{2}\,\Delta. \tag{56}$$

This result shows that the rest mass is of of the order of Δ and one could therefore wonder if the result (56) is significant, since it is obtained in the limit $|p/\Delta| \ll 1$. To estimate the validity of this approximation we use the exact result, which can be obtained by Eq. (49). Since $\Pi_3(p_0, 0) = 0$, the rest mass of the three species A_0, A_L, A_T is given by

$$m^2 + \Pi_2\big(m^2, 0\big) = 0. \tag{57}$$

To obtain $\Pi_2(m^2, 0)$ we integrate Eq. (15), with $|\vec{p}\,| = 0$, and we get

$$\Pi_2\big(m^2, 0\big) = \frac{\mu^2 g^2}{3\pi^2}\left[-1 + \int_0^{+\infty} dx \times \frac{x + \sqrt{x^2+1}}{(x+\sqrt{x^2+1}\,)^2 - (m/\Delta)^2}\left(1 - \frac{x}{\sqrt{x^2+1}}\right)\right]. \tag{58}$$

The numerical result of (57) is

$$m \equiv m_R = 0.894\Delta. \tag{59}$$

A comparison with (56) shows that the difference is of the order of 40–50% and this is also the estimated difference for the dispersion law at small $\vec{p}$. We notice that also in the three flavour case the gradient expansion approximation tends to overestimate the correct result [11]. For the effective mass, we get in the gradient expansion approximation

$$m^*_{A_0} = \sqrt{18}\,\Delta. \tag{60}$$

For the longitudinal and transverse cases we get respectively

$$E_L^2 + \frac{7}{15}|\vec{p}\,|^2 = 2\Delta^2, \tag{61}$$

$$E_T^2 + \frac{1}{5}|\vec{p}\,|^2 = 2\Delta^2, \tag{62}$$

therefore, the rest masses are given by

$$m^R_{A_L} = m^R_{A_T} = m^R_{A_0} = \sqrt{2}\,\Delta. \tag{63}$$

The equality of the three rest masses is an exact result, as we have stressed already. On the other hand the effective longitudinal and transverse masses are both negative and their values are:

$$m^*_{A_L} = -\frac{15\sqrt{2}}{7}\Delta, \tag{64}$$

$$m^*_{A_T} = -5\sqrt{2}\Delta. \tag{65}$$

We interpret this result as follows: the spectrum of the quasi-particles associated to these gluon modes has a maximum for $|\vec{p}\,| = 0$, which means that at very small temperatures, which is the limit in which we work, these quasi-particles are unlikely to be produced.

For the time-like component of the gluon 8 we have the dispersion law

$$E_0 = \sqrt{9\Delta^2 + 9|\vec{p}\,|^2\frac{\Delta^2}{m_D^2}}, \tag{66}$$

where, in this case $m_D^2 = \Pi^{00}(0) = \mu^2 g^2/\pi^2$; therefore,

$$m^R_{A_0} = 3\Delta, \tag{67}$$

$$m^*_{A_0} = \frac{m_D^2}{3\Delta}. \tag{68}$$

For the longitudinal and transverse modes we have, respectively,

$$E_L = \sqrt{\frac{4}{270}\frac{m_M^2}{\Delta^2}|\vec{p}\,|^2 + m_M^2}, \tag{69}$$

$$E_T = \sqrt{-\frac{1}{30}\frac{m_M^2}{\Delta^2}|\vec{p}\,|^2 + m_M^2}, \tag{70}$$

where the Meissner mass is obtained from Table 1: $m_M^2 = \frac{\mu^2 g^2}{9\pi^2}$. From these equations we see that

$$m^R_{A_L} = m^R_{A_T} = m_M, \tag{71}$$

whereas the effective masses are as follows

$$m^*_{A_L} = \frac{270}{4}\frac{\Delta^2}{m_M}, \tag{72}$$

$$m^*_{A_T} = -30\frac{\Delta^2}{m_M}. \tag{73}$$

We note the peculiar feature of the spatial modes of the 8th gluon that has a very large rest mass, see Eq. (71). This is an exact result that is not obtained by the gradient expansion approximation. In fact integrating

Eq. (16), with $|\vec{p}\,| = 0$, we get

$$\Pi_2\big(m^2, 0\big) = -\frac{\mu^2 g^2}{9\pi^2} = -m_M^2, \tag{74}$$

and, therefore, from Eq. (57) one gets the result (71).

The longitudinal and transverse gluons with color 8 get nonetheless a vanishingly small effective mass, see Eqs. (72) and (73), due to the very large coefficient of $|\vec{p}\,|^2$ in (69) and (70). These results should be contrasted with those obtained for the gluons 4, 5, 6, 7, and for the temporal mode of the gluon 8 (in all these cases the rest mass is of order Δ). Since our effective description is limited to energies $< \Delta$ these results mean that the longitudinal and transverse modes of the 8th gluon are decoupled from the low-energy physics.

4. Three and four point gluon vertices

To complete the effective Lagrangian for the eight gluons we compute the one loop corrections to the gluon vertices Γ_3 (3 gluons) and Γ_4 (4 gluons). We shall not consider in the sequel a possible light glueball which is discussed in [12]. Therefore we write the full gluon Lagrangian as

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \frac{1}{2} \Pi^{\mu\nu}_{ab} A^a_\mu A^b_\nu + \mathcal{L}^1_{(3)} + \mathcal{L}^1_{(4)}, \tag{75}$$

where $\mathcal{L}^1_{(3)}$ and $\mathcal{L}^1_{(4)}$ are the one loop Lagrangian terms for the three and four point gluon vertices, respectively. For the three point gluon vertex we have two different diagrams with a quark loop. The first one is depicted in Fig. 2; the second one arises from the $1/\mu$ term in Eq. (5) and is suppressed together with diagrams not containing quark loops. For the four point gluon vertex we have three different diagrams with a quark loop, but only the one depicted in Fig. 3 survives in the $\mu \to \infty$ limit.

To start with, let us consider the three-point function. At the tree level ($\mathcal{L} = \mathcal{L}^0$) the contribution to the Lagrangian can be written in the form:

$$\mathcal{L}^0_{(3)} = -g f_{abc} A^\mu_a A^\nu_b \partial_\mu A_{c,\nu}. \tag{76}$$

At one loop ($\mathcal{L} = \mathcal{L}^1$) we have to distinguish between the contribution of the diagram involving the gluons in the unbroken $SU(2)$ gauge group, which we call $\mathcal{L}^1_{(3),1}$, and those involving gluons corresponding to

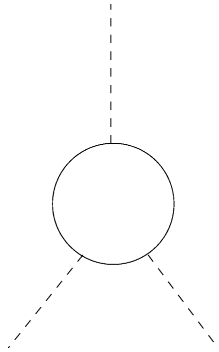


Fig. 2. Three gluon vertex. Dotted lines represent gluon fields; full lines are fermion propagators.

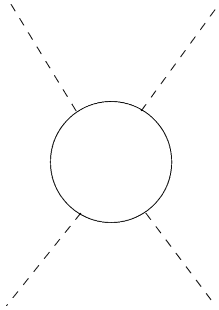


Fig. 3. Four gluon vertex. Dotted lines represent gluon fields; full lines are fermion propagators.

broken generators, which we call $\mathcal{L}^1_{(3),2}$. So the one-loop correction at the three gluon vertex may be written as follows:

$$\mathcal{L}^1_{(3)} = \mathcal{L}^1_{(3),1} + \mathcal{L}^1_{(3),2}. \tag{77}$$

For the $SU(2)$ contribution our results are as follows

$$\mathcal{L}^1_{(3),1} = -gk f_{abc} A^\mu_a A^\nu_b \partial^\mu A^\nu_c [\delta_{\mu 0}\delta_{\nu i} + \delta_{\mu i}\delta_{\nu 0}], \tag{78}$$

with $a, b, c = i, j, l \in \{1, 2, 3\}$ and $k = \frac{g^2\mu^2}{18\pi^2\Delta^2}$. This term can be obtained in a simpler manner, by requiring gauge invariance for the $SU(2)$ gluons. On the other hand for $a, b, c = i, \alpha, \beta$ $(i = 1, 2, 3;\ \alpha, \beta = 4, \ldots, 7)$ or $a, b, c = 8, \alpha, \beta$ we have

$$\mathcal{L}^1_{(3),2} = -g\frac{3k}{2}\big(I^1_{\mu\nu\rho\sigma} C^{abc}_1 + I^1_{\sigma\mu\nu\rho} C^{abc}_2\big) A^\mu_a A^\nu_b \partial^\sigma A^\rho_c, \tag{79}$$

where

$$I^1_{\mu\nu\rho\sigma} = \int \frac{d\vec{v}_F}{4\pi} \widetilde{V}_\mu V_\nu V_\rho V_\sigma \tag{80}$$

Table 2
$I^1_{\mu\nu\rho\sigma}$ non-vanishing elements; $i, j = 1, 2, 3$

$\mu\nu\rho\sigma$	0000	$00ii$	$0i0i$	$0ii0$	$ii00$
$I^1_{\mu\nu\rho\sigma}$	1	1/3	1/3	1/3	−1/3
$\mu\nu\rho\sigma$	$i0i0$	$i00i$	$iijj$	$ijji$	$ijij$
$I^1_{\mu\nu\rho\sigma}$	−1/3	−1/3	1/15	1/15	1/15

Table 3
C_1^{abc} and C_2^{abc} non-vanishing elements

abc	C_1^{abc}	C_2^{abc}
$\alpha 8\beta$	$\frac{1}{3} f_{\alpha 8\beta}$	$\frac{2}{3} f_{\alpha 8\beta}$
$\alpha i\beta$	$f_{i\alpha\beta}$	0

is the matrix which expresses the breaking of Lorentz invariance. Its non-vanishing elements are listed in Table 2.

On the other hand C_1^{abc} and C_2^{abc} are the matrices which express the breaking of $SU(3)$ color and whose non-vanishing elements are given in the Table 3, where f_{abc} are the $SU(3)$ structure constants. Since the coupling of the $SU(2)$ gluons with gluons of color $\alpha = 4, 5, 6, 7$ is completely fixed by the gauge invariance, we have written down the corresponding term $C^{i\alpha\beta}$ only for a check. They are exactly the $SU(3)$ structure constants.

Let us consider the four-gluon vertex whose tree contribution is

$$\mathcal{L}^0_{(4)} = -\frac{g^2}{4} f_{abe} f_{cde} A^\mu_a A^\nu_b A_{c\mu} A_{d\nu}. \tag{81}$$

The one-loop correction may be written as follows:

$$\mathcal{L}^1_{(4)} = \mathcal{L}^1_{(4),1} + \mathcal{L}^1_{(4),2} + \mathcal{L}^1_{(4),3} + \mathcal{L}^1_{(4),4}. \tag{82}$$

We have four contributions to $\mathcal{L}^1_{(4)}$. The first term is

$$\mathcal{L}^1_{(4),1} = -k\frac{g^2}{4} f_{abe} f_{cde} A^\mu_a A^\nu_b A^\mu_c A^\nu_d [\delta_{\mu 0}\delta_{\nu i} + \delta_{\mu i}\delta_{\nu 0}], \tag{83}$$

with $k = \frac{g^2\mu^2}{18\pi^2\Delta^2}$. It comes from diagrams where all the gluons are in the $SU(2)$ group, therefore the indices a, b, c, d take the values i, j, l, m. The second term comes from diagrams with gluons of colors $a, b, c, d = \alpha, \beta, 8, 8$. The contribution to the Lagrangian is as follows:

$$\mathcal{L}^1_{(4),2} = -\frac{g^2}{4}\left(\frac{9k}{2}\right) D_1^{abcd} I^2_{\mu\nu\rho\sigma} A^\mu_a A^\nu_b A^\rho_c A^\sigma_d. \tag{84}$$

The non-vanishing elements of the tensor D_1^{abcd} are in Table 4 and

$$I^2_{\mu\nu\rho\sigma} = \int \frac{d\vec{v}_F}{4\pi} V_\mu V_\nu V_\rho V_\sigma. \tag{85}$$

The third contribution is from diagrams with gluons of colors $a, b, c, d = i, \alpha, \beta, 8$, with the gluon 8 connected to two ungapped quarks. The contribution is

$$\mathcal{L}^1_{(4),3} = -\frac{g^2}{4}(9k) D_2^{abcd} I^1_{\mu\nu\rho\sigma} A^\mu_a A^\nu_b A^\rho_c A^\sigma_d, \tag{86}$$

where $I^1_{\mu\nu\rho\sigma}$ is the same tensor we defined in Eq. (80). The relevant D_2^{abcd} values are in Table 4. The last contribution to the four-gluon vertex is

$$\mathcal{L}^1_{(4),4} = -\frac{g^2}{4}\left(\frac{9k}{2}\right) D_3^{abcd} I^3_{\mu\nu\rho\sigma} A^\mu_a A^\nu_b A^\rho_c A^\sigma_d, \tag{87}$$

where

$$I^3_{\mu\nu\rho\sigma} = \int \frac{d\vec{v}_F}{4\pi} \widetilde{V}_\mu \widetilde{V}_\nu V_\rho V_\sigma \tag{88}$$

and the non-vanishing D_3^{abcd} are in Table 4.

As we wrote above, some of these terms are completely fixed by gauge invariance, once we know the renormalization properties of the $SU(2)$ fields.

To evaluate the effective Lagrangian terms $\mathcal{L}_{(3)}$ and $\mathcal{L}_{(4)}$ one might redefine the fields A^a_μ ($a = 1, 2, 3$) and the coordinates according to Eqs. (40)–(43); also the

Table 4
D^{abcd} non-vanishing elements

$abcd$	D_1^{abcd}
$\alpha 88\alpha$	$\frac{4}{9} f_{\alpha 8I} f_{8\alpha I}$
$abcd$	D_2^{abcd}
$i\alpha 8\beta$	$\frac{2}{3} f_{i\alpha I} f_{8\beta I}$
$abcd$	D_3^{abcd}
$i\alpha\beta j$	$f_{i\alpha I} f_{\beta j I}$
$8\alpha\beta i$	$\frac{1}{3} f_{8\alpha I} f_{\beta i I}$
$\alpha\beta\gamma\delta$	$f_{\alpha\beta I} f_{\gamma\delta I} 2(1 + \log 4)$

other gluon fields can be redefined to include wave function renormalization constants. We do not make this exercise since, for the gluons corresponding to the broken colors, we gain little or no physical insight from this procedure, as the corresponding Lagrangian cannot be put in the form (39).

5. Conclusions

We have used the effective theory near the Fermi surface to calculate the gluon self-energies and dispersion laws in the color superconducting phase of QCD with two massless flavors (2SC). The results confirm, within a different formalism, and extend, results already obtained, notably by Rischke, Son, and Stephanov. The three gluons of the unbroken $SU(2)$ color have no Debye screening and Meissner effect, but they are affected in their dynamics by the medium polarizability. The remaining gluons show Debye screening and Meissner effect. The Debye and Meissner masses can be read from the Table 1. For the gluons $1, 2, 3$ one easily finds the known result that they have no rest mass and no effective mass and that their velocity is that relevant to a polarizable medium of unit magnetic permeability and a dielectric constant depending in a known way on the theory parameters. The gluons $4, 5, 6$ and 7 get masses of order of the gap, showing a behaviour quite similar by the one exhibited by the gluons in the CFL phase [6]. The behaviour of the longitudinal gluon 8 is not quite the same since there is no renormalization in the part of the Lagrangian involving the time derivative. As a consequence the rest mass of this gluon is not of the order Δ (the gap), but rather of order $g\mu$ (the Meissner mass). This makes these particles to behave in-medium in a rather peculiar way. Very difficult to be produced relatively to the other modes, because of their large rest mass, but once produced they move as particles with a vanishingly small effective mass, of order $\Delta^2/g\mu$. The gluons $4, 5, 6, 7$ have large (with respect to the gap parameter) negative longitudinal and transverse effective masses, so that they are unlikely to be produced at small temperatures. To complete the construction of the effective interaction we have calculated to one loop the three and four point gluon interactions. Gauge invariance can be used directly for the couplings involving gluons of the unbroken $SU(2)$ color. For the remaining three and four point gluon selfcouplings one has to evaluate explicitly the relevant loop diagrams. The results are given in Eqs. (76)–(86).

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