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www.elsevier.com/locate/physletbLocalization of q -form fields on AdS_{p+1} branesChun-E Fu^a, Yu-Xiao Liu^{b,*}, Heng Guo^c, Feng-Wei Chen^b, Sheng-Li Zhang^a^a School of Science, Xi'an Jiaotong University, Xi'an 710049, PR China^b Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, PR China^c School of Science, Xidian University, Xi'an 710071, PR China

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ABSTRACT

In this paper, we investigate localization of a free massless q -form bulk field on thin and thick AdS_{p+1} branes with codimension one. It is found that the zero mode of the q -form field with $q > (p+2)/2$ can be localized on the thin negative tension brane, which is different from the flat brane case given in Fu et al. (2012) [57]. For the thick AdS_{p+1} branes, the q -form field with $q > (p+2)/2$ also has a localized zero mode under some conditions. Furthermore, we find that there are massive bound KK modes of the q -form field, which are localized on this type p -branes.

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1. Introduction

Extra dimension and brane-world theories have received more and more attention since the last century, especially when the Arkani-Hamed–Dimopoulos–Dvali (ADD) [1] and Randall–Sundrum (RS) [2,3] brane-world models were brought up, as they opened a new avenue to solve the long-standing hierarchy problem and the cosmology problem [4–11]. Then many brane-world models were set up [10–28].

In this paper, we will consider an interesting brane-world model, the AdS_{p+1} branes with codimension one, which have p spacial dimensions and an effective non-zero cosmological constant. These branes could have some realistic applications if the branes had 3 infinite large dimensions (which are those we can feel) and $p-3$ finite small enough dimensions with topology $S^1 \times S^1 \times \dots \times S^1 = T^{p-3}$ (thus cannot be seen. According to string/M theory, there are 6 or 7 extra dimensions). It is known that the AdS space has many unusual properties [29], and plays a central role in the AdS/CFT correspondence [30,31].

In the brane-world theory, investigating the KK modes of various fields is an important and interesting work [32–49], because the information of extra dimensions is encoded in these KK modes. And the most important thing is that the massless modes (zero modes) are in fact the fields which have existed on the brane. Thus, the zero modes of ordinary fields (gravity, scalar, vector,

fermion, etc.) localized on the brane can rebuild the effective gravity and standard model in our world [35].

It is known that, in the string/M theory, the q -form fields are related to closed strings, which play an important role in the D - or M -brane [50,51]. As the brane-world theory is motivated by the string/M theory, we naturally ask what about the q -form fields on the AdS_{p+1} branes. On the other hand, although the q -form fields with $q > 1$ are dual to the scalar field in four-dimensional spacetime, they denote new types of particles in higher-dimensional spacetime. Thus, we investigate in this paper the KK modes of the q -form fields on the AdS_{p+1} branes.

In fact, the q -form fields have been investigated in some literatures. For example, in Refs. [52–56], the KK modes of the 2-form and 3-form fields were discussed in the RS brane model, and it was found that the zero modes of these fields cannot be localized on the brane, unless they are coupled with a dilaton field. In Ref. [57], some authors of the current work considered any form (q -form) field on the flat p -branes with p the spatial dimensions of the brane, and found that only the lower form fields (with $q < (p-1)/2$ or $q < p/2$) can be localized on the flat p -branes.

In this work we will find that the higher form fields (with $q > (p+2)/2$) can be localized on the thin negative tension AdS_{p+1} brane and the thick AdS_{p+1} brane, which is an interesting result. For example, on the thin AdS_4 brane with negative tension and the thick one, there is a 3-form field that can be localized. This may be useful for some unknown problems such as the cosmological constant problem or dark energy problem [58]. In fact, in Ref. [7], the authors had considered a 3-form field with a nonstandard action to solve the cosmological constant problem in the five-dimensional RS model.

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This paper is organized as follows. We first simply review the localization mechanism for the KK modes of the q -form fields in Section 2. Then we try to find the solutions of AdS_{p+1} branes in Section 3, and investigate the localization of the KK modes on these branes. Finally we give a brief conclusion.

2. q -form fields and their KK modes

In Ref. [57], some authors of the current paper proposed a localization mechanism for a free massless q -form field, which can be applied to any RS-like braneworld model. Here we briefly review the localization mechanism.

The action for a free massless q -form field $X_{M_1 M_2 \dots M_q}$, which is completely antisymmetric, in a curved D -dimensional spacetime is

$$S_q = \int d^D x \sqrt{-g} Y_{M_1 M_2 \dots M_{q+1}} Y^{M_1 M_2 \dots M_{q+1}}, \quad (1)$$

where $Y_{M_1 M_2 \dots M_{q+1}} = \partial_{[M_1} X_{M_2 \dots M_{q+1}]}$ is the field strength. The q -form field may have some localized KK modes on lower dimensional branes (such as the p -brane with $D = p + 2$) embedded in the D -dimensional spacetime with the metric $ds^2 = e^{2A(w)} (\hat{g}_{\mu\nu}(x^\lambda) dx^\mu dx^\nu + dw^2)$. This can be found by the KK decomposition and KK reduction.

There exists a gauge freedom $\delta X_{M_1 \dots M_q} = \partial_{[M_1} \Lambda_{M_2 \dots M_q]}$ for the q -form field. So we can choose a simple gauge $X_{\mu_1 \dots \mu_{q-1} w} = 0$. Decomposing the remained components of the q -form field as

$$X_{\mu_1 \mu_2 \dots \mu_q}(x^\mu, w) = \sum_n \hat{X}_{\mu_1 \mu_2 \dots \mu_q}^{(n)}(x^\mu) U^{(n)}(w) e^{(2q-p)A/2}, \quad (2)$$

where $\hat{X}_{\mu_1 \mu_2 \dots \mu_q}^{(n)}(x^\mu)$ is only relative to the brane coordinates and $U^{(n)}(w)$ is the function of the extra dimension, and substituting the decomposition into the equation of motion for the q -form field, one will find that the KK mode $U^{(n)}(w)$ satisfies the following Schrödinger-like equation:

$$[-\partial_w^2 + V_q(w)] U^{(n)}(w) = m_n^2 U^{(n)}(w), \quad (3)$$

where m_n is the mass of the KK mode, and the effective potential $V_q(w)$ takes the form

$$V_q(w) = \frac{(p-2q)^2}{4} (\partial_w A(w))^2 + \frac{p-2q}{2} \partial_w^2 A(w). \quad (4)$$

For the above Schrödinger-like equation (3), we need an orthonormality condition to seek the exact solution for $U^{(n)}$ and m_n . As our aim is to find the KK modes which can be localized on the brane, the orthonormality condition can be obtained by the KK reduction for the action (1):

$$\begin{aligned} S = & \sum_{n,m} \int d^{p+1} x \sqrt{-\hat{g}} \left(\hat{Y}_{\mu_1 \mu_2 \dots \mu_{q+1}}^{(n)}(x^\mu) \hat{Y}^{(m)\mu_1 \mu_2 \dots \mu_{q+1}}(x^\mu) \right. \\ & + \frac{1}{q+1} m_n^2 \hat{X}_{\mu_1 \mu_2 \dots \mu_q}^{(n)}(x^\mu) \hat{X}^{(m)\mu_1 \mu_2 \dots \mu_q}(x^\mu) \left. \right) \\ & \times \int dw U^{(m)}(w) U^{(n)}(w). \end{aligned} \quad (5)$$

If $U^{(m)}(w)$ satisfy the following orthonormality condition:

$$\int U^{(m)}(w) U^{(n)}(w) dw = \delta_{mn}, \quad (6)$$

we will get the effective action for a series of q -form fields on the brane:

$$\begin{aligned} S_{\text{eff}} = & \sum_n \int d^{p+1} x \sqrt{-\hat{g}} \left(\hat{Y}_{\mu_1 \mu_2 \dots \mu_{q+1}}^{(n)} \hat{Y}^{(n)\mu_1 \mu_2 \dots \mu_{q+1}} \right. \\ & + \frac{m_n^2}{q+1} \hat{X}_{\mu_1 \mu_2 \dots \mu_q}^{(n)} \hat{X}^{(n)\mu_1 \mu_2 \dots \mu_q} \left. \right). \end{aligned} \quad (7)$$

This effective action is just for the KK modes localized on the brane, and the localization condition is the orthonormality condition (6).

Thus with the Schrödinger-like equation (3) and the orthonormality condition (6), we can investigate the KK modes of a q -form field on branes. There are usually two kinds of KK modes localized on the brane, i.e., the massless (zero mode) and the massive bound KK modes. The zero mode is actually irrelative to the extra dimension (from $U_0 \propto e^{(p-2q)A/2}$, we have $X_{\mu_1 \mu_2 \dots \mu_q}^{(0)}(x^\mu, w) = \hat{X}_{\mu_1 \mu_2 \dots \mu_q}^{(0)}(x^\mu)$), and so it can be regarded as the matter that has existed on the brane. While the massive bound modes are relative to the extra dimension, and the mass spectrum carries the information of the extra dimension. Thus, we should find the mass spectrum of the KK modes. This requires to discuss the effective potential V_q (4), which is decided by the geometry of the background spacetime. In this paper, we will consider AdS_{p+1} branes and investigate the KK modes of q -form fields on them.

We will first find solutions of the thin and thick AdS_{p+1} branes with codimension one. For the thin brane, we can get analytic solutions. For the thick ones, the solutions are depended on the special background scalar potential $V(\phi)$. In this paper, we will consider the simple but typical ϕ^4 potential, and use the numerical method to obtain the solutions with the initial and boundary conditions. However, we will not give the particular results in this paper, because the initial and boundary conditions are enough for us to analyze the behavior of the effective potentials for the KK modes.

3. AdS_{p+1} branes

We assume that the line element for the $D = p + 2$ dimensional spacetime is

$$ds^2 = e^{2A(r)} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2, \quad (8)$$

where r denotes the physical extra dimension perpendicular to the p -brane, and $\hat{g}_{\mu\nu}$ is the induced metric on the brane. In this work, we will focus on the AdS_{p+1} branes, for which the metric $\hat{g}_{\mu\nu}$ is given by

$$\hat{g}_{\mu\nu} dx^\mu dx^\nu = e^{2Hx_p} (-dt^2 + dx_1^2 + \dots + dx_{p-1}^2) + dx_p^2. \quad (9)$$

Here, H is a parameter associated with the effective cosmology constant Λ_p on the p -brane by $\Lambda_p = -pH^2$ [59]. Then the nonvanishing components of the Einstein tensor are turned out to be

$$G_{\mu\nu} = \frac{1}{2} p [2A'' + (p+1)A'^2 + (p-1)e^{-2A} H^2] \hat{g}_{\mu\nu}. \quad (10)$$

In this paper, the prime denotes the derivative with respect to the physical extra dimension coordinate r .

Then with the matter field we can get the Einstein equations. In the following, we will consider the thin and thick branes, in which the ‘‘matters’’ are made up by the brane tension σ (the energy density per unit area) and a scalar field ϕ , respectively.

3.1. Thin AdS_{p+1} branes

We first consider the thin brane case, for which the action is

$$S = \frac{1}{2} \int d^D x \sqrt{-g} (R - p\Lambda) + \int d^{D-1} x \sqrt{-g^{(b)}} (-\sigma), \quad (11)$$

where Λ is a negative cosmology constant and $g_{\mu\nu}^{(b)}$ is the induced metric on the brane. From the action (11) we obtain the Einstein equations:

$$G_{MN} + \frac{1}{2}p\Lambda g_{MN} = -\frac{1}{\sqrt{g_{rr}}}\delta(r)\sigma\delta_M^\mu\delta_N^\nu g_{\mu\nu}, \quad (12)$$

i.e.,

$$G_{00} = -\left[\frac{1}{\sqrt{g_{rr}}}\sigma\delta(r) + \frac{1}{2}p\Lambda\right]g_{00},$$

$$G_{rr} = -\frac{1}{2}p\Lambda g_{rr}. \quad (13)$$

Considering the expressions of the Einstein tensor for the AdS_{p+1} brane, the above Einstein equations are rewritten as:

$$A'^2 = b^2 - H^2 e^{-2A}, \quad (14)$$

$$A'' = -\frac{\sigma}{p}\delta(r) + H^2 e^{-2A}, \quad (15)$$

where $b = \sqrt{-\Lambda/(p+1)}$. From the above equations, we can obtain the solution of the AdS_{p+1} brane, which will be shown below.

Our aim is to investigate the KK modes of the q -form field; to this end we need to discuss the effective potential V_q for the KK modes. With the coordinate transformation $dw = e^{-A}dr$, the potential V_q in Eq. (4) can be expressed as the function of r :

$$V_q(w(r)) = \frac{p-2q}{4}e^{2A(r)}[(p-2q+2)A'(r)^2 + 2A''(r)]. \quad (16)$$

It is clear that with the Einstein equations (14), the behavior of the effective potential $V_q(w(r))$ at $r=0$ and $r \rightarrow r_{\text{bou}}$ is

$$V_q(r \rightarrow 0) = -\frac{(p-2q)\sigma}{2p}\delta(0), \quad (17)$$

$$V_q(r \rightarrow r_{\text{bou}}) = \frac{p-2q}{4}[(p-2q+2)b^2 e^{2A(r \rightarrow r_{\text{bou}})} - (p-2q)H^2], \quad (18)$$

where r_{bou} is the boundary of the extra dimension. If $V_q(r \rightarrow 0) < 0$ and $V_q(r \rightarrow r_{\text{bou}}) > 0$, there will exist a zero KK mode $U_0 \propto e^{(p-2q)A/2}$. We then can check whether it can be localized on the brane by the orthonormality condition (6):

$$\int U_0^2 dw = \int U_0^2 e^{-A} dr \propto \int e^{(p-2q-1)A} dr \rightarrow \text{cons}. \quad (19)$$

In order to check the above condition, we will look for the solution of the warp factor by solving the Einstein equations. And from (17), we see that for the AdS_{p+1} branes with negative tension σ , $V_q(r \rightarrow 0) < 0$ can be satisfied for $q > p/2$, which means that there may exist the q -form fields with $q > p/2$ on the branes, and this is different with that on the flat p -branes [57]. While for branes with positive tension, the q -form fields on the branes are the ones with $q < p/2$, which is similar to that on the flat p -branes. So in the following we only focus on the thin AdS_{p+1} with $\sigma < 0$, and discuss the existence and localization conditions for the zero mode of the q -form fields in detail.

For the AdS_{p+1} brane with negative tension p -brane, the solution is

$$A(r) = \ln\left(\frac{H}{b} \cosh(b|r|+c)\right), \quad (20)$$

$$\sigma = -2p\sqrt{b^2 - H^2}. \quad (21)$$

Here $c = \text{arccosh}(b/H)$, and the brane cosmology constant $\Lambda_{\text{brane}}^{AdS_{p+1}}$ is given by

$$\Lambda_{\text{brane}}^{AdS_{p+1}} = -pH^2 = p\left(\frac{\Lambda}{p+1} + \frac{\sigma^2}{4p^2}\right), \quad (22)$$

where the bulk cosmology constant should satisfy the following condition:

$$\Lambda < -\frac{(p+1)}{4p^2}\sigma^2, \quad \text{or} \quad \Lambda < -(p+1)H^2. \quad (23)$$

From the warp factor, it can be seen that the physical extra dimension for AdS_{p+1} is infinite, i.e., $r_{\text{bou}} \rightarrow \infty$, and the behavior of the warp factors at infinity is $A(r \rightarrow \infty) \rightarrow e^{br}$.

According to Eqs. (17) and (18), the values of the effective potential at $r \rightarrow 0$ and at infinity are obtained:

$$V_q(r \rightarrow 0) = -\frac{(p-2q)\sigma}{2p}\delta(0), \quad (24)$$

$$V_q(r \rightarrow \infty) \rightarrow \frac{b^2}{4}(p-2q)(p-2q+2)e^{2br}, \quad (25)$$

which determines whether a localized zero mode exists. If there is a zero mode, we can check whether it can be localized on the brane:

$$\int_{-\infty}^{\infty} U_0^2 dw = \int_{-\infty}^{\infty} e^{(p-2q-1)A} dr \rightarrow 2 \int_0^{\infty} e^{b(p-2q-1)r} dr < \infty. \quad (26)$$

Then it is clear that only for

$$q > \frac{p-1}{2} \quad (27)$$

the above integral is finite, and the zero mode can be localized on the brane. With the localization condition (27), we seek for the existence condition for the zero mode, i.e., $V_q(r \rightarrow 0) < 0$ and $V_q(r \rightarrow \infty) > 0$, which requires that

$$q > \frac{p}{2} + 1. \quad (28)$$

Remember that only the q -form fields with $q < p+1$ play a role in the p -brane. Thus finally the conditions for the zero mode to be existence and localized are:

$$\frac{p}{2} + 1 < q < p+1. \quad (29)$$

Thus for the thin AdS_{p+1} brane with negative tension, there exists a localized zero mode with the condition (29). For example, on the thin AdS_4 brane, a 3-form field can be localized. This result is different with that in the flat thin p -brane [57].

3.2. Thick AdS_{p+1} branes

In this subsection, we consider the thick AdS_{p+1} branes generated by a scalar field with scalar potential, which are more realistic. The action of such a system is

$$S = \int d^D x \sqrt{-g} \left(\frac{1}{2}R - \frac{1}{2}g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right), \quad (30)$$

where $\phi = \phi(w)$ is the scalar field, and $V(\phi)$ is a potential which generates the brane. We can obtain the Einstein equations using the metric ansatz (8):

$$R_{MN} - \frac{1}{2}g_{MN}R = g_{MN} \left(-\frac{1}{2}\phi'^2 - V \right) + \partial_M \phi \partial_N \phi, \quad (31)$$

which read as

$$G_{00} = g_{00} \left(-\frac{1}{2} \phi'^2 - V \right), \quad (32)$$

$$G_{rr} = \frac{1}{2} \phi'^2 - V. \quad (33)$$

With the expressions of the Einstein tensor (10), we get

$$A'^2 = -e^{-2A} H^2 + \frac{1}{p(p+1)} (\phi'^2 - 2V), \quad (34)$$

$$A'' = -pe^{-2A} H^2 - (p+1)A'^2 - 2V/p. \quad (35)$$

The field equation for the scalar field is

$$\frac{dV}{d\phi} = (p+1)A'\phi' + \phi''. \quad (36)$$

In order to find the solutions of these Eqs. (34)–(36), we use the numerical method. Because the three equations are not independent, we suppose the form of the background scalar potential is

$$V(\phi) = v_0 + g_1 \phi^2 + g_2 \phi^4, \quad (37)$$

where v_0 , g_1 , and g_2 are constants. As the equations of motion are second order, we need four initial or boundary conditions. We choose the conditions as

$$A(0) = 0, \quad A'(0) = 0, \quad \phi(0) = 0, \quad \phi'(r_{\text{bou}}) = 0. \quad (38)$$

Here the r_{bou} is the boundary of the extra dimension, which may be infinite or finite.

What should be noted from Eq. (34) is that with the initial conditions $A(0) = 0$ and $A'(0) = 0$, there is a constraint between the parameters v_0 , H , and the spacial dimension p :

$$H^2 > -\frac{2v_0}{p(p+1)}, \quad (39)$$

where the constant v_0 can be negative or positive for AdS_{p+1} brane.

Also with the initial and boundary conditions, we can analyze the behavior of the effective potential for the KK modes of the q -form field, $V_q(w(r))$, at $r=0$ and at $r \rightarrow r_{\text{bou}}$ from the equations of motion of the background spacetime (34)–(36). As the warp factor is $A(0) = 0$, $A'(0) = 0$, and $A''(0) = (-pH^2 - 2v_0)/p$ at $r=0$, the value of the effective potential at $r=0$ is found to be

$$V_q(r \rightarrow 0) = \frac{p-2q}{2} \xi, \quad (40)$$

with $\xi = (-pH^2 - \frac{2v_0}{p})$. At $r \rightarrow r_{\text{bou}}$, as $\phi'(r \rightarrow r_{\text{bou}}) = 0$, it is easy to get

$$V_q(r \rightarrow r_{\text{bou}}) = \frac{(p-2q)}{4} [k(p-2q+2)e^{2A} - (p-2q)H^2], \quad (41)$$

where

$$k = -\frac{2V(\phi(r \rightarrow r_{\text{bou}}))}{p(p+1)}. \quad (42)$$

From (40), it can be seen that for $\xi > 0$ and $q > p/2$, $V_q(r \rightarrow 0) < 0$, which suggests that there may be higher form fields with $q > p/2$ on the branes, and this is new result different with that on the thick flat p -branes [57]. With the conditions (39), we find that for the AdS_{p+1} brane, the value of ξ can be positive with $v_0 < 0$.

For the AdS_{p+1} case, as $r \rightarrow r_{\text{bou}}$ Eq. (34) can be reduced to $A'^2(\phi(r \rightarrow r_{\text{bou}})) = -e^{-2A(r_{\text{bou}})} H^2 + k$, from which it is seen that k must be positive. The behavior of the warp factor at r_{bou} is

$$A(r \rightarrow r_{\text{bou}}) = \log(\cosh \sqrt{k}(r - c_1)) \rightarrow \sqrt{k}r. \quad (43)$$

We see that the physical extra dimension is infinite.

For a zero mode of the q -form field, we can check whether the localization condition (19) is satisfied. As the integral is

$$\int_{-\infty}^{\infty} U_0^2 dw = \int_{-\infty}^{\infty} e^{(p-2q-1)A} dr \rightarrow 2 \int_0^{\infty} e^{\sqrt{k}(p-2q-1)r} dr, \quad (44)$$

it will be finite for

$$q > \frac{p-1}{2}. \quad (45)$$

Under the localization condition (45), we find the existence condition, which requires $V_q(r=0) < 0$ and $V_q(r \rightarrow +\infty) > 0$, i.e.,

$$H^2 < -\frac{2v_0}{p^2} \quad \text{and} \quad q > \frac{p}{2} + 1, \quad (46)$$

or

$$H^2 > -\frac{2v_0}{p^2} \quad \text{and} \quad q < \frac{p}{2}. \quad (47)$$

Considering the constrains (39) and (45), we finally get the following conclusion: under the condition for $H^2: -\frac{2v_0}{p(p+1)} < H^2 < -\frac{2v_0}{p^2}$, the zero mode for the q -form field with $q > p/2 + 1$ can be localized on the AdS_{p+1} brane; while under another condition: $H^2 > -\frac{2v_0}{p^2}$, the zero mode for the q -form field with $(p-1)/2 < q < p/2$ can also be localized on the brane. Because there is no integer solution of q for $(p-1)/2 < q < p/2$, we cannot get a localized zero mode under the condition $H^2 > -\frac{2v_0}{p^2}$.

On the other hand, because the effective potential tends to infinite at $r \rightarrow \infty$, there are infinite massive bound KK modes.

4. Conclusions

In this work, we investigated the KK modes of the q -form fields on the AdS_{p+1} branes with codimension one, including in the thin and thick AdS_{p+1} ones. Through the KK decomposition and KK reduction, we found that the KK modes satisfy a Schrödinger-like equation. By analyzing the equation under the orthonormality condition, we finally obtained the mass spectrum of the KK modes on these branes.

It was found that for the thin negative tension AdS_{p+1} brane, there are localized zero modes on the brane for the q -form fields with $p/2 + 1 < q < p + 1$. For example, there exists a localized massless 3-form field on the thin AdS_4 brane.

While for the thick AdS_{p+1} branes, under some conditions between the parameters H , v_0 and p , the q -form fields with $p/2 + 1 < q < p + 1$ have localized zero modes. And there are massive bound KK modes, which are also localized on this type p -branes.

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