Transient wave-blockage interaction in pressurized water pipelines

H.-F. Duan,*, P. Lee, M. Ghidaoui

*Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Hong Kong.

Abstract

Transient flows are commonly encountered in pressurized water pipelines and have been studied for the purpose of systems design as well as defects detection and management. The representation and analysis of the transient response in the frequency domain is an attractive approach for fault detection due to its high noise tolerance and the linearized equations allows for the clear analytical characterization of system behaviour. Recent studies have demonstrated that an extended partial constriction of the pipe flow area causes changes in the system resonant frequencies and these changes can be used for locating and sizing partial blockages in pipes. Despite the successful application of this technique under field conditions, so far there is little work on the link between the changes in the system resonant conditions and the wave-blockage interaction. This paper provides a fundamental basis for the observation that unlike localised, discrete blockages, an extended blockage creates changes in the system resonant frequencies. Analytical methods using wave perturbation analysis and transfer matrix are shown in this study, and the obtained results are compared with numerical simulations and experimental data. The analytical analyses are used to explain the blockage-induced frequency shifts and to provide insights for its practical applications in pressurized water pipelines.

© 2013 The Authors. Published by Elsevier Ltd.

Selection and peer-review under responsibility of the CCWI2013 Committee

Keywords: Transients; water pipeline; wave-blockage interaction; resonant frequency shift; transfer matrix; wave perturbation analysis

1. Introduction

Partial blockages are common in engineered as well as natural pressurized conduit systems that transport liquids and are created from a myriad of physical and chemical processes such as material deposition, tubercles (rust),

* Corresponding author. Tel.: +852-23588847; fax: +852-23581534.

E-mail address: ceduan@ust.hk
scales, plaque, bio-fouling, ice-formation in cold climate, and inadvertently throttled inline valves. Blockages in engineered systems result in the wastage of energy and financial resources, reduction in the pipe carrying capacity and the increased potential for contamination. It is therefore important detect such anomalies and deal with them quickly before they can cause severe problems and damages.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>pipe cross-sectional area</td>
</tr>
<tr>
<td>a</td>
<td>wave speed</td>
</tr>
<tr>
<td>F</td>
<td>frequency change</td>
</tr>
<tr>
<td>h</td>
<td>pressure head in the frequency domain</td>
</tr>
<tr>
<td>I</td>
<td>amplitude of incident wave</td>
</tr>
<tr>
<td>K</td>
<td>head loss coefficient of junction at blockage</td>
</tr>
<tr>
<td>k</td>
<td>wave number</td>
</tr>
<tr>
<td>L</td>
<td>total length of pipeline</td>
</tr>
<tr>
<td>l</td>
<td>length of pipe section</td>
</tr>
<tr>
<td>m</td>
<td>integer number</td>
</tr>
<tr>
<td>P</td>
<td>pressure in the time domain</td>
</tr>
<tr>
<td>Q</td>
<td>discharge in the pipeline</td>
</tr>
<tr>
<td>R_f</td>
<td>coefficient of frictional damping</td>
</tr>
<tr>
<td>R</td>
<td>amplitude of reflection wave</td>
</tr>
<tr>
<td>s</td>
<td>intermediate coefficient in Eq. (10)</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>x</td>
<td>distance from upstream reservoir</td>
</tr>
<tr>
<td>Y</td>
<td>hydraulic impedance</td>
</tr>
<tr>
<td>ΔH</td>
<td>minor head loss at junction</td>
</tr>
<tr>
<td>μ</td>
<td>coefficient of wave propagation</td>
</tr>
<tr>
<td>ω</td>
<td>angular frequency</td>
</tr>
<tr>
<td>ξ</td>
<td>restriction coefficient of blockage</td>
</tr>
<tr>
<td>J_0</td>
<td>subscript for indicating the steady value at junction</td>
</tr>
<tr>
<td>1, 2, 3</td>
<td>subscripts for indexing the pipe section</td>
</tr>
</tbody>
</table>


Brunone et al. (2008), Stephens et al. (2008) and Duan et al. (2012) proposed that blockages in pipes can be divided into two categories—discrete and extended blockages—according to their relative length to the total pipe length. Contractor (1965) showed that a discrete partial blockage, in the form of an inline orifice or valve, causes a partial reflection of waterhammer waves where the amplitude of the reflected wave is related to the constriction severity of the orifice and arrival time of the reflected wave at the measurement station provides the blockage location. The findings of Contractor (1965) have been validated in Meniconi et al. (2009 and 2011). Wang et al. (2005) showed that a discrete blockage creates a frequency dependent damping of the transient trace, which can be used for locating and sizing the blockage. Mohapatra et al. (2006), Sattar et al. (2008) and Lee et al. (2008) showed that a discrete blockage in a pipe system imposes a periodic pattern onto the amplitudes of the system resonant responses and developed a technique for locating and sizing discrete blockages on the basis of this pattern.
In reality, blockages may extend for significant distances along the pipe and impose changes onto the transient that is different to that from a discrete blockage. Duan et al. (2012) and Tuck et al. (2012) show that an extended blockage changes the position as well as the amplitude of system resonant responses. An analytical expression for the blockage-induced changes in the system resonant frequencies was derived in Duan et al. (2012) and was used for detecting extended blockages in pipelines. To determine the properties of the blockage, an optimization process using a Genetic Algorithm (GA) was used to fit the observed resonant frequencies with the theoretical expression. This method of blockage location was validated using numerical as well as experimental results in Duan et al. (2012, 2013) and Meniconi et al. (2013). It was found from these studies that the inverse calibration process is time consuming and its efficiency decreases significantly with the number of blockages in the system. A simplified form to the original analytical equations was developed in Duan et al. (2013) and the computational efficiency was increased by sacrificing the accuracy of the solution.

In the literature, field applications of transient based method of defect detection have shown the ability for the technique for detecting a variety of faults, from leakages through to discrete and extended blockages (Stephens, 2008). Recently, Meniconi et al. (2011) applied transient wave-based fault detection in Italy and arrived at the following conclusion “In our opinion, no other technique has the same performance in terms of economy and quickness of (transient) field tests for the diagnosis of real supply-pipe systems.” Despite the positive findings, many field studies also found the poorly understood dispersion, reverberation and damping processes imposed on the pressure wave by the system has significant impact on the accuracy of the technique.

This paper is aimed at understanding the modifications made to hydraulic waves by blockages with the view that such an understanding will pave the way for the development of wave-based blockage detection techniques. The investigation will be carried out using the theory of wave perturbation in conjunction with experimental and numerical data. The theoretical and experimental results will allow the extraction of much needed physical insights into the wave-blockage interaction and provide an assessment of the potential of using transient waves for detecting extended blockages.

2. Frequency response of pipeline system with blockages

Partial blockages Frequency domain transient-based methods for detecting leaks and blockages in the pipeline have been developed in a number of studies (Ferrante and Brunone 2003, Lee et al. 2006, 2008, Mohapatra et al. 2006, Sattar and Chaudhry 2008, Sattar et al. 2008, and Duan et al. 2011a, 2012, 2013). In these papers, the analytical expressions for the system responses in the frequency domain are derived using the Fourier series expansion technique as well as the transfer matrix method, where the one-dimensional (1D) waterhammer perturbation equations are linearized in the frequency domain (Chaudhry 1987, Wylie et al. 1993). The single pipe system shown as in Fig. 1 is used herein for illustrating the analytical expression for the extended blockage impact on the system frequency response. When the downstream boundary valve is slammed shut from the initial fully open state, the transient responses in the frequency domain are obtained and shown in Fig. 2 (Duan et al. 2012, and Lee et al. 2013), where clear differences in both the resonant amplitudes and frequency locations can be found between the blockage-free and blockage cases.

![Fig. 1. Pipe systems for illustration: (a) uniform pipeline without blockage; (b) pipeline with single blockage](image-url)
Considering a single extended blockage (Fig. 1b), the head frequency response per unit imposed discharge variation at the valve has been derived in Duan et al. (2011a, 2012) as,

\[
h = - \frac{iY_i \cos(\mu l_1)\cos(\mu l_2)\sin(\mu l_1) - iY_i}{Y_i} \sin(\mu l_1)\sin(\mu l_2) \sin(\mu l_1) - iY_i \cos(\mu l_1)\cos(\mu l_2)\sin(\mu l_1) \sin(\mu l_2)} \cos(\mu l_1)\cos(\mu l_2)\sin(\mu l_1) \sin(\mu l_2) - \frac{Y_i}{Y_i} \sin(\mu l_1)\cos(\mu l_2)\sin(\mu l_1) \sin(\mu l_2) \sin(\mu l_1) \sin(\mu l_2)
\]

where \( h \) = pressure head in the frequency domain; \( l \) = pipe section length; \( \mu \) = propagation coefficient and \( \mu = (\omega/a)\sqrt{1-igAR_f/\omega} \); \( Y \) = hydraulic impedance and \( Y = (-a/gA)\sqrt{1-igAR_f/\omega} \); \( A \) = pipe cross-sectional area; \( a \) = wave speed; \( R_f = fQ/\pi DA^2 \); \( \omega \) = frequency; \( i \) = imaginary unit; \( K \) = head loss coefficient of junction at blockage and \( K = 2\Delta H_0/Q_{m0} \) with \( Q_{m0} \) = steady discharge at junction and \( \Delta H_0 \) = minor head loss for steady discharge. For this illustration, \( R_f = 0 \) is assumed in Eq. (1), such that Eq. (1) can be derived further to produce two results:

1. for the case of a discrete blockage \( l_2 = 0 \), Eq. (1) becomes:

\[
\left| \frac{1}{h} \right| = \frac{K}{2Y_i} \left[ 1 + \cos \left( (2m-1)\pi \frac{l}{L_0} \right) \right],
\]

where \( m \) is an integer that denote a particular resonant peak. Note that Eq. (2) was previously obtained in Wang et al. 2005, Mohapatra et al. 2006, Sattar et al. 2008, and Lee et al. 2008, 2013. In this case, it can be seen that the discrete blockage does not cause shifting of the resonant frequencies from the original blockage-free situation, but it affects the magnitude of the resonant responses;

2. for the case of extended blockage, considering \( K_{12}K_{23}/Y_i^2 < 1 \), the resonant condition for Eq. (1) can be simplified to,

\[
\left[ (Y_1 + Y_j)(Y_2 + Y_j)\cos(\mu l_1 + \mu l_2 + \mu l_3) + (Y_2 - Y_j)(Y_2 - Y_j)\cos(\mu l_1 - \mu l_2 - \mu l_3) \right] = 0.
\]

Fig. 2. System frequency response for blockage-free and blockage pipe systems
Details of this derivation are shown in Duan et al. (2012). In this case, the resonant frequencies are shifted by the extended blockage section \((l_2, Y_2)\) and the nature of this shift can be used for determining the blockage location (Duan et al. 2012, 2013). It is also necessary to mention that Eq. (2) and Eq. (3) are also valid for real systems where \(R_f \neq 0\), as shown in Lee et al. (2008) and Duan et al. (2012, 2013). Eq. (2) and Eq. (3) demonstrated that discrete and extended blockages impose different effects on the transient responses. In particular, an extended blockage causes shifting of the resonant frequencies but discrete blockages do not impose the same effect. As yet there has been no attempt in the literature to explain why such different behaviors exist between discrete and extended blockages. In the next section in this study, a perturbation analysis will be used for this purpose.

3. Experimental evidences of frequency responses by discrete and extended blockages

The transient responses from two different pipeline systems affected by discrete blockages are retrieved from Contractor (1965) and Sattar et al. (2006) and re-plotted in Fig. 3 and Fig. 4 respectively. Both results demonstrate that discrete blockages do not cause shifting of the resonant frequencies. On the other hand, a recent extended blockage test case was conducted in the fluid mechanics laboratory at the University of Canterbury, New Zealand (Duan et al. 2013) and the comparison between the frequency responses for the blockage-free and extended blockage situations are plotted in Fig. 5, showing that the extended blockage modifies both the resonant frequencies the magnitude of the resonant response.

4. Wave perturbation analysis for wave-blockage interaction

To investigate the wave behaviors in pipeline with blockages, the 1-D wave equation for a conduit with varying pipe cross-sectional area is used (Duan et al. 2011b),

![Fig. 3](image1.png)

Fig. 3. Comparative results for experimental test pipe system with and without a discrete blockage (adapted from Sattar et al. 2006)

![Fig. 4](image2.png)

Fig. 4. Comparative results for experimental test pipe system with and without a discrete blockage (adapted from Contractor 1965)
Fig. 5. Comparative results for experimental test pipe system with and without an extended blockage (adapted from Duan et al. 2013)

\[ A \frac{\partial^2 P}{\partial t^2} = a^2 \frac{\partial}{\partial x} \left( A \frac{\partial P}{\partial x} \right), \]  

(4)

where \( P \) = pressure response in the time domain; \( x \) = longitudinal coordinate along the pipeline with \( x = 0 \) as the centre of the pipe system and \( t \) = time coordinate. Note that a frictionless pipe with a constant wave speed is considered in this analysis to highlight the interaction between the transient wave and blockage and the midpoint of the extended blockage is assumed to coincide with the midpoint of the pipeline. Furthermore, by considering an incident pressure wave with a certain frequency (\( \omega \)) impinging on the blockage from the boundary end, \( P = \hat{P}(x, \omega) e^{-i\omega t} \), where \( \hat{P}(x, \omega) \) is the amplitude of the propagating wave in the pipeline, Eq. (4) can now be simplified as,

\[ \frac{\partial}{\partial x} \left( A \frac{\partial \hat{P}}{\partial x} \right) + \frac{\omega^2}{a^2} A \hat{P} = 0, \]  

(5)

Actually Eq. (5) is the well-known Sturm-Liouville type equation (Zettl, 2005) and can usually be analyzed by the perturbation method (Mei et al. 2005).

Consider the pipeline in Fig. 1(b) where it is assumed that reflections from the either ends of the pipeline can be ignored to highlight the effect the blockage on the wave propagation. A similar derivation was carried out by Mei et al. (2005) for open channel flows. Eq. (5) is first solved by wave perturbation analysis. For each section of the pipeline, Eq. (5) is satisfied as,

\[ \frac{\partial}{\partial x} \left( A_j \frac{\partial \hat{P}_j}{\partial x} \right) + k_j^2 A_j \hat{P}_j = 0, \]  

(6)

Where \( K_j = \omega_j / a_i \) is wave number, and \( j = 1, 2, 3 \) identifies the pipe sections shown in Fig. 1(b). Under this condition, the pressure head responses for the three pipe sections from an incident wave with amplitude \( P_0 \) and frequency \( \omega_0 \) originating from \( \pm \infty \) are obtained as follows (note that \( x = 0 \) corresponds to the middle of the extended blockage in Fig. 1b):
\[ \hat{P}_1 = I_1 e^{ik(x+0.5l_2)} + R_1 e^{-ik(x+0.5l_2)}, \quad x < -0.5l_2 \]
\[ \hat{P}_2 = I_2 e^{ikx} + R_2 e^{-ikx}, \quad -0.5l_2 < x < 0.5l_2, \]
\[ \hat{P}_3 = I_3 e^{ik(x-0.5l_2)} + R_3 e^{-ik(x-0.5l_2)}, \quad x > 0.5l_2 \]

where \( l_2 \) here refers to the length of blockage section in Fig. 1(b); \( I \) and \( R \) are amplitudes of incident and reflected waves. Therefore under the conditions of reflection-free end boundaries, \( I_3 = P_0 \) since the incident wave in this pipe section is the known wave originating from the downstream boundary at \( x = +\infty \). In addition, since the reflection from the upstream boundary is ignored, \( R_1 = 0 \). This produces four remaining unknowns \((I_1, I_2, R_2, \text{and } R_3)\) in the solutions given by Eq. (7). These unknowns are evaluated from the enforcement of mass and momentum conservation at the pipe junctions (i.e., \( x = 0.5l_2 \) and \( x = -0.5l_2 \)) and leads to:

\[ \hat{P}_1 = \hat{P}_2, \quad A_1 \frac{\partial \hat{P}}{\partial x} = A_2 \frac{\partial \hat{P}_2}{\partial x} \quad \text{at } x = -0.5l_2. \]  
\[ \hat{P}_2 = \hat{P}_3, \quad A_2 \frac{\partial \hat{P}_2}{\partial x} = A_3 \frac{\partial \hat{P}_3}{\partial x} \quad \text{at } x = 0.5l_2. \]  

Combining Eq. (7) and Eq. (8) gives,

\[ R_i = \left[ \frac{(1+s_{12})(1-s_{23}) e^{2ik_2} + (1-s_{12})(1+s_{23})}{(1-s_{12})(1+s_{23}) + (1+s_{12})(1+s_{23}) e^{2ik_2}} \right] P_0, \]

where \( s_{12} = k_1 A_1/k_2 A_2 \) and \( s_{23} = k_2 A_2/k_3 A_3 \). If a single blockage is considered (i.e., \( A_1 = A_3 \) and \( a_1 = a_3 \)), which implies \( k_1 = k_3 \) and \( s_{12} = s_{23}^{-1} \). As a result, Eq. (9) becomes:

\[ R_i = -\frac{(1-e^{-ik_2})}{(1-s_{12}) e^{-2ik_2}} \xi \xi P_0, \]

where \( \xi = (1-s_{12})/(1+s_{12}) \) and \( \xi \xi \) is a measure of the radial constriction imposed by the blockage. By defining,

\[ R_i^* = \text{Amplitude} \left( \frac{R_i}{\xi P_0} \right) \quad \text{and} \quad F_i^* = \text{Angle} \left( \frac{R_i}{\xi P_0} \right), \]

where \( R_i^* \) and \( F_i^* \) are the reflection coefficient and frequency change (shift) by the blockage. It is instructive to consider a blockage with small radial constriction (i.e., \( |\xi| \ll 1 \)) which simplifies Eq. (11) to:

\[ R_i \approx -(1-e^{-ik_2}) \xi \xi P_0. \]

Substituting Eq. (12) into Eq. (7), the pressure head at downstream pipe section (i.e., for \( x > 0.5l_2 \)) is:

\[ \hat{P}_3(x) = \left[ P_0 e^{ik(x-0.5l_2)} + P_0 e^{-ik(x-0.5l_2)} \right] + \left[ \xi \xi P_0 e^{-2ik(x-0.5l_2)} - (1+\xi) P_0 e^{-ik(x-0.5l_2)} \right], \]

in which the first two terms in the first set of square brackets on the right-hand-side (RHS) represent the wave propagation in a blockage-free pipeline while the other two terms in the second set of square brackets on the RHS.
represent the effect of the extended blockage on the wave field. In particular, the presence of $e^{2ikl_2}$ in the second part in Eq. (13) clearly shows that the blockage induces a frequency shift, while the presence of $\xi_s$ shows that the blockage induces a change in wave amplitude. In addition, since $e^{2ikl_2} \rightarrow 1$ as $l_2$ tends to zero (or more precisely as $k_2l_2$ tends to zero), the expression provides a theoretical proof that a discrete blockage causes a change in the wave amplitude but no phase shift as shown in Figs. 3 and 4. This result is also consistent with the results in previous studies, such as Wang et al. (2005), Mohapatra et al. (2006), Lee et al. (2008, 2013) and Sattar et al. (2008), where a blockage of a sufficiently short length can be approximated as a lumped local loss (discrete blockage), which changes the amplitude of the system resonant responses but not their frequencies. On the other hand, the frequency shifting by the extended blockage in Eq. (13) is in line with the result in Fig. 5.

It is insightful to investigate Eq. (10) and Eq. (12) further. Since, $k_2l_2 = \omega_0l_2/a_2$ and, $\omega_0 = 2\pi a_0/L_0$, where $L_0$ and $a_0$ are the wave length and speed of the incident wave, then $k_2l_2 = 2\pi((a_0/a_2)(l_2/L_0))$. The blockage induced variations of wave amplitude and phase angle versus $\omega_0/\omega_2 = (a_0/a_2)(l_2/L_0)$ are investigated for the small blockage case of $|\xi_s| << 1$, e.g., $|\xi_s| = 0.05$, and the results based on Eq. (10) are plotted in Fig. 6.

The result in Fig. 6 demonstrates that the reflected wave amplitude and frequency varies periodically with $\omega_0/\omega_2 = (a_0/a_2)(l_2/L_0)$. As a result, it is clear that the blockage selectively reflects some waves more than others and highlights that detection methods that focus on using wave generation mechanisms of limited frequency content are only effective for blockages with certain lengths. In fact, according to Eq. (12), maximum reflection occurs if

$$2k_2l_2 = (2m-1)\pi \Rightarrow \frac{a_1}{a_2} \frac{l_2}{L_0} = \frac{\omega_0}{\omega_a} = \frac{(2m-1)}{4}. \quad (14)$$

This condition is referred to in the gravity waves literature as the resonance condition (e.g., Mei et al. 2005). The results gained above are validated by numerical applications in the next section.

5. Numerical validation and results analysis

The numerical simulation is conducted by a 1D waterhammer model using MOC. To highlight the effect of the blockage reflection and validate the analytical result, it is once again assumed that the upstream and downstream boundaries produce no reflections the system is assumed frictionless pipes are considered in this study. The wavespeed values for all sections are fixed at 1000 m/s and the blockage-free pipe diameter is 0.5 m in Fig. 1. The incident wave is the pressure oscillation based on original steady state at downstream valve end and represented by:

$$P(t) = P_s + P_0 \sin(\omega_0 t); \quad (15)$$

where $P_s$ is steady pressure and $P_s = 50$ m; $P_0$ is incident wave amplitude and $P_0 = 0.1P_s$ in this study.

![Fig. 6. Variation of wave phase and amplitude due to pipe blockage based on Eq. (11)](image-url)
To examine the dependence of the wave reflections on the relationship of $\omega_0/\omega_2$ as in Eq. (11), the incident wave frequency $\omega_0$ is fixed at $2\pi$ with an equivalent of $L_0 = 1000$ m, while the blockage length is varying from 100 m (i.e., $\omega_0/\omega_2 = 0.1$) to 1000 m ($\omega_0/\omega_2 = 1.0$) and the blockage diameter from 0.4 m ($|\xi_s| = 0.2$) to 0.48 m (i.e., $|\xi_s| = 0.05$) respectively. The numerical results of the reflection coefficient and frequency change for different cases are calculated and shown in Figs. 7 (a) and (b) respectively, with the analytical solution by Eq. (11b) also plotted in the same figure for comparison. It is worthwhile to mention that the frequency shift and reflection coefficient are extracted from the numerical results by using the “two point” wave decomposition method, which has been widely used in gravity wave problems in open channel and coastal hydraulic engineering. More details about this method can refer to the references by Huang and Ghidaoui (2007) and Goda (2005).

The results in Fig. 7 show that the analytical solution can predict both the frequency shift and reflection coefficient at acceptable accuracy under the condition of small blockage (i.e., $|\xi_s| = 0.05<<1$). However, the prediction accuracy decreases as the blockage severity increases, for example in the cases of $|\xi_s| = 0.10$ and 0.20. On the other hand, the results in Fig. 7 also reveal that the prediction accuracy from using Eq. (12) is less sensitive to the blockage severity (diameter). This is to say, the simplified Eq. (12) with regard to $\xi_s$ can be used to predict the frequency shift for both small and relatively large blockage severity cases, while it can only be valid for small blockage cases when predicting the reflection coefficient. For the prediction of reflection coefficient for relatively large blockage severity case, the original Eq. (10) should be used. It is also noted that the analytical solution in Eq. (12) and the numerical results in this study are obtained under the conditions of frictionless and reflection-free end boundaries, and thus its validity for more realistic situations needs further future work.

6. Conclusions

This paper investigates the wave-blockage interaction under unsteady flow in the pressurized pipelines. The frequency response of pipe systems with blockages (discrete and extended) is first derived by transfer matrix method and the results show that an extended blockage can change resonant frequencies and amplitudes, but a discrete blockage can only affect the resonant amplitudes. Relevant experimental data are retrieved from the literature to confirm the findings of this study. To explain the frequency shift (change) phenomenon induced by an extended blockage, a wave perturbation analysis is conducted. The results demonstrate the dependence of the frequency shift and reflection coefficient on the blockage properties (diameter and length). Moreover, the results also indicate that resonant condition of a partially blocked pipe system for different incident waves is consistent with that in gravity waves. Finally, the assumptions of frictionless and simple pipe configuration in this study meant that more future work are needed for releasing such conditions for the purpose of practical application.

Acknowledgements

This research is financially supported by the Research Grant Council (RGC) of Hang Kong under projects
number 612511 and 612910, and by the Marsden Grant project UOC-1153 from the Royal Society of New Zealand.

References


