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Procedia - Social and Behavioral Sciences 180 (2015) 653 – 659
Procedia
Social and Behavioral Sciences

The 6th International Conference Edu World 2014 "Education Facing Contemporary World Issues", 7th - 9th November 2014

Developing Critical Thinking in Elementary Mathematics Education through a Suitable Selection of Content and Overall Student Performance

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Abstract

Authors of this paper draw attention to the importance of developing critical thinking in the process of education. For this purpose, we organized a research (an experiment with parallel groups) on a sample (N=246) of third-grade students of primary school, to examine if critical thinking can be developed by a better planned selection of content. The obtained results show that the selection of content can encourage and develop critical thinking of all students, and that there should be no restrictions regarding the classification of students by their overall performance at school.

Peer-review under responsibility of The Association "Education for tomorrow" / [Asociatia "Educatie pentru maine"].

Keywords: *critical thinking, task, elementary mathematics education, mathematics.*

Introduction

In the field of mathematics education, we have growing tendencies that emphasize the development of thinking skills (Jacobs et all., 2007; Sfard & Kieran, 2001; 2001; Špijunović & Maričić, 2011: 975; Felda & Cotič, 2012: 5), and particularly the development of skills of critical thinking. Thus we have emphasized opinions that critical

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Peer-review under responsibility of The Association "Education for tomorrow" / [Asociatia "Educatie pentru maine"].
doi:10.1016/j.sbspro.2015.02.174

thinking represents “an essential skill for the life in the Information Age” (Connor-Greene & Greene, 2002: 324), a prerequisite for education (Sezer, 2008), “the most important skill for problem solving, research and discovery” (Thompson, 2011: 1), “a tool for the education of the mind” (Paul & Elder, 2008: 88) etc.

The first step toward realization of the idea about the development of critical thinking in mathematics education is finding an answer to the question *What is critical thinking?* The answer to this question is not easy to give, because it is a very complex phenomenon. The sheer number of definitions and different conceptions of critical thinking is a confirmation to this (E. M. Gleser, R. H. Ennis, J. R. Yinger, J. Mc Peck, R. Paul, H. Siegel, C. Meyers, S. Brookfield, D. R. Garrison, P. M. King, K. Strohm Kitchener etc.). Most of them refer to a specific context of use, but there are only few of them that are associated with mathematical content in mathematics education for junior students of primary school. However, successful development of critical thinking implies the definition of this concept in the context of elementary mathematics education, its contents and particularities of the age of students who are a part of this type of education. Based on that, and taking into account other authors’ definitions of critical thinking, we defined critical thinking as a complex intellectual activity which emphasizes the following skills: problem formulation, problem reformulation, evaluation, problem sensitivity (Maričić, 2009: 485; Maričić, 2011a).

Each of the listed components of critical thinking was defined via narrower skills.

Thus, *the formulation of the problem* implies: a) the student's ability to perceive the mathematical problem and to formulate it on the basis of problem situation; b) the detection of mathematical symbolism and transferring that symbolism to spoken language, spotting the connections between mathematical symbols and being critical to presentation of these connections in words; c) the search for the inherent properties of the formulation of the problem, the identification of shades in the formulation of the problem and the use of precise spoken and precise mathematical language (Maričić, 2011b).

The reformulation of the problem includes: a) linguistic reformulation of a mathematical task; b) drawing conclusions based on the identification of the connections and relationships in the content of a task, explained by clear arguments; c) identifying relationships among the terms of a task and turning the cognitive way in the opposite direction.

The evaluation relates to: a) evaluation of information; b) evaluation of solutions; c) evaluation of the opinions of the authorities.

Sensitivity to the problems implies: a) assessing the reality of the situation in the problem and the resulting solution, and taking into account the circumstances in which the assignment was given; b) identification and detection of hidden and implicit information in the formulation, abstaining from fast conclusions, sensitivity to the detection of the way of problem solving; c) the ability to identify inconsistencies and contradictions in the problem formulation and problem requirements, identification of the redundant and incomplete resulting data arising from the reality of a given situation and discover the pitfalls in the formulation of the problem (Maričić, 2009: 485–486; Maričić, 2011a: 134–155).

Only on the basis of this, or some other operationalization of critical thinking can we find ways to develop it.

There are many different attitudes on finding an approach to developing critical thinking in literature. Some of them are based on finding a universal method that will lead to its development, such as the discussion method (R. Gagne, D. Berliner, Driscoll), or learning through problem solving (R. Kvaščev). Others base the development of critical thinking on teaching students to master specific skills of critical thinking (R. Paul, D. Helpern, C. Wade), ruls of correct logical reasoning (R. Ennis) and so on. It should be noted that the approach to developing critical thinking “focused on adopting rules, procedures and skills of logical thinking is not recommendable, because students of this age are not able to acquire knowledge through deduction, nor use strict logical procedures in thinking, evaluation and learning” (Maričić et al., 2013: 205). Hence the approach to developing critical thinking must take into consideration the student’s age and his abilities, but above all, it must be based on specific mathematical content. In other words, only through exercising and putting the student in the situation to think critically can we develop critical thinking (Maričić, 2009: 485), or “it is conceptually impossible to talk about critical thinking in general, because it is, like any other kind of thinking, always thinking about something” (McPeck, 1981: 3). So, without the appropriate content, “any strategy, method, form or a teaching composition on its own can hardly have a significant influence on the development of critical thinking in elementary mathematics education” (Maričić et al., 2013: 206). For those reasons, we began studying the problem of developing critical thinking from the attitude that critical thinking can be successfully developed only if it is developed in every day learning situations and on contents (tasks) the solution of which requires the skills of critical thinking.

Based on the aforementioned things, we decided to examine if the choice of content (tasks) the solution of which emphasizes the skills of critical thinking can be used to influence its development in elementary mathematics education, and if that development has anything to do with students' overall achievement. If the results would confirm this assumption, then teachers would have a more secure footing and clearer orientation in designing and organizing elementary mathematics education, they would be more successful in achieving the requirements for the development of students' critical thinking, thus making an important step toward overall improvement of mathematics education.

Research Method

The research was conducted in the school year 2012/2013 on the sample of 256 students of the third grade of primary school (aged 9.5 – 10.4 years) in the Republic of Serbia. The sample had characteristics of a random, stratified and group sample. We chose students from a school by a random selection as the experimental group (N=123), and students of another school for the control group (N=123). In the experimental group, 68.29% students had *excellent* overall achievement, 27.64% *very good*, 2.44% *good*, and 1.63% *unsatisfactory*. In the control group, 51.22% of students had *excellent* overall ahievement, 35.77% *very good*, 11.38% *good*, and 1.63% *unsatisfactory*. (Table 1).

Table 1. *Structure of the student sample, with respect to their overall achievement*

Overall achievement	Experimental group f	Experimental group %	Control group f	Control group %	Total
Excellent	84	68.29	63	51.22	147
Very good	34	27.64	44	35.77	78
Good	3	2.44	14	11.38	17
Unsatisfactory	2	1.63	2	1.63	4
Total	123		123		246

We did not perform the normalization of the experimental and the control group artifically, by transferring students from one class to another, because of the working conditions in school, but we statistically controlled the dependent variable by analyzing the covariance.

The research is based on the implementation of the experimental method. We chose an experiment with parallel groups. We first realized a pilot research on a sample of 55 students from a school which was not included in the research. On the basis of that, we verified an experimental program and designed final instrument forms, and then began the realization of the experimental program.

After we performed initial measurements of the development of critical thinking in both student groups, we introduced the experimental program into the experimental group, whereas the control group continued working in the usual manner. The experimental program was realized through 27 exercises consisting of prepared tasks, the solution of which emphasized skills of critical thinking (formulation of the problem, reformulation of the problem, evaluation and sensitivity to the problem). Within one exercise, all tasks were focused on one skill of critical thinking. We provided an equal number of exercises for each component of critical thinking, and within each exercise, an approximately equal number of tasks per skill of critical thinking.

The experimental program was realized in regular mathematics classes for the third grade of primary school, as a part of the following curriculum topics: *Square and rectangle*, *Written addition and subtraction up to 1000*; *Triangle*; *Written multiplication and division up to 1000*; *Fractions*; *Mathematical expressions*. After the realization of the experimental program, we performed final measurements of students' development of critical thinking.

We created two tests for the purpose of this research:

- TICT – test for determining the initial state of the development of student critical thinking,
- TFCT – test for determining the final state of the development of student critical thinking.

We created the tests ourselves. We made two equivalent test forms, the structure of which consisted of 12 tasks. The selection of tasks was performed by defining three tasks for each component of critical thinking, singled out by using operationalization (formulation of the problem, reformulation of the problem, evaluation and sensitivity to the problem). Within each component of critical thinking, each task referred to specific skills of that component of critical thinking. Each task was worth 5 points. The maximum number of points that could be scored on the test was 60 points.

To justify the use of the test, we defined the metric characteristics of the test. Test objectivity was provided by putting every student in approximately the same test situation, by the fact that persons in charge of the test acted according to the same instructions, and that task grading was performed uniformly, based on a key.

Test discriminativity was defined via analysis. The coefficient of the discriminative value varied between 0.12 and 0.25. (Task 1 (0,23), Task 2 (0,13), Task 3 (0,22), Task 4 (0,25), Task 5 (0,24), Task 6 (0,15), Task 7 (0,12), Task 8 (0,15), Task 9 (0,25), Task 10 (0,12), Task 11 (0,12), Task 12 (0,17)).

Test reliability was determined by the retest procedure of calculating the Pearson coefficient of correlation between the results scored by the examinees on equivalent forms of the test for critical thinking. The obtained correlation coefficient was 0.81.

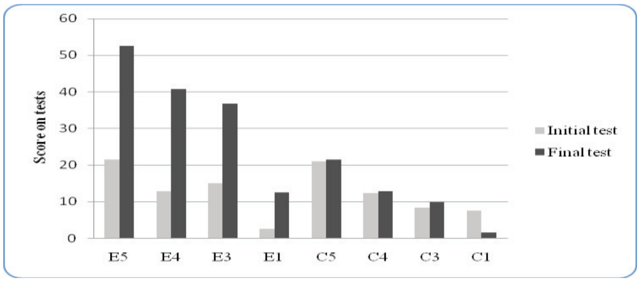
The experimental program was realized by the authors of this paper. In the realization of this program, individual student work was dominant, with constant help of the experimenters and giving instructions for work, with frontal and group discussions within a class, and frontal and group feedback information for each taks within an exercise.

Statistical data analysis was based on the use of the software package IBM SPSS20. From the statistical procedures, we used *covariance analysis* and the *t-test.* Covariance analysis was used for statistical normalization of groups and longitudinal monitoring of the experimental program's effects. The *t-test* was used to determine statistical significance of the difference in the development of student critical thinking, influenced by effects of the experimental program between the subgroups of the experimental and the control group, formed with respect to overall achievement the student scored in mathematics.

Research Results

To examine if a more purposeful choice of content can help in the development of critical thinking in elementary mathematics education, and the influence of the independent variable overall student achievement in learning on its development, we created eight student groups: four groups in the experimental (E5 – *excellent* achievement, E4 – *very good*, E3 – *good*, E1 – *unsatisfactory*) and four groups in the control group (C5 – *excellent*, C4 – *very good*, C3 – *good*, C1 – *unsatisfactory*). We created groups in accordance with the overall achievement the students had at the end of the first semester of the third grade of primary school. We did not have any students whose overall achievement was *satisfactory,* and no students with this achievement had *unsatisfactory* (1) rating in mathematics.

Students with *excellent* achievement had the best results on the initial measurement, both in the experimental and in the control group (M_{E5}=21.49; M_{C5} =20.98). Students with *very good* (M_{E4}=12.88; M_{C4}=12.25), *good* (M_{E3}=15.00; M_{C3}=8.27) and *unsatisfactory* achievement (M_{E1}=2.5; M_{C1} = 7.5) scored approximately the same results on the initial measurement. After the effects of the experimental program, all subgroups of the experimental group scored a larger number of points on average on the final measurement (M_{E5}=52.50; M_{E4}=40.74; M_{E3}=36.67; M_{E1}= 12.50), whereas students of the control group scored approximately the same result as on the initial measurement (Graph 1).



Graph 1. *Average number of points scored by the subgroups of the experimental and control group created regardless of the overal achievement at school*

The variance between groups of students with different overall achievement on the initial measurement (F_X=10.956, p< .001) shows that there are statistically significant differences in the development of critical thinking between the existing groups. These differences occur as a consequence of differences in the overall student achievement, because it is natural to expect that students with excellent achievement at school should score better results on the test for measuring the development of critical thinking. The variance between groups on the final (F_Y=151.067, p< .001) is significantly greater than the variance on the initial measurement, which indicates the effects of the experimental program (Table 2). The confirmation of the significance of differences between the groups also comes from the covariance (F_{YX}= 167.773, p < .001), which discards the suspicion that the differences between the groups are random and a consequence of a lack of balance among the groups, but confirms that they occurred due to the effects of the experimental program.

Table 2. *Analysis of variance and covariance*

Source of Variation	df	F_X	p	F_Y	p	F_{YX}	p
Between group (*b*)	7	10.956*	.001	151.067*	.001	167.773	0.001
In groups (*w*)	238	–	–	–	–	6.963	–
Total (T)	245						

To determine between which groups of the experimental and control group, formed on the basis of overall student achievement, there are statistically significant differences in the development of critical thinking, we used the *t*- test (Table 3).

Table 3. *t-test between subgroups of the sample, formed with regard to overall student achievement*

Groups	$dM_{Y'}$	SE_d	t	P
E1, E3	18.1559	6.5012	2.7927	0.01
E1, E4	23.2428	5.1818	4.4855	0.01
E1, C5	30.6431	5.0954	6.0139	0.01
E1, C1	13.4043	7.1217	1.8822	n.sg.
E1, C3	5.4964	5.3835	1.0210	n.sg.
E1, C4	4.3475	5.1490	0.8443	n.sg.
E1, C5	0.0561	5.1151	0.0110	n.sg.
E3, C4	5.0869	4.2893	1.1860	n.sg.
E3, E5	12.4873	4.1845	2.9842	0.01
E3, C1	31.5602	6.5012	4.8545	0.01
E3, C3	23.6523	4.5309	5.2203	0.01
E3, C4	22.5034	4.2496	5.2955	0.01
E3, C5	18.0998	4.2085	4.3008	0.01
E4, E5	7.4003	1.4476	5.1122	0.01
E4, C1	36.6471	5.1818	7.0723	0.01
E4, C3	28.7392	2.2615	12.7080	0.01
E4, C4	27.5903	1.6262	16.9665	0.01
E4, C5	23.1867	1.5155	15.2996	0.01
E5, C1	44.0474	5.0954	8.6446	0.01
E5, C3	36.1396	2.0558	17.5789	0.01
E5, C4	34.9906	1.3253	26.4017	0.01
E5, C5	30.5870	1.1869	25.7695	0.01
C1, C3	7.9079	5.3835	1.4689	n.sg.
C1, C4	9.0568	5.1490	1.7590	n.sg.
C1, C5	13.4604	5.1151	2.6315	0.05
C3, C4	1.1489	2.1853	0.5258	n.sg.
C3, C5	5.5525	2.1042	2.6387	0.05
C4, C5	4.4036	1.3992	3.1473	0.01

The greatest progress in the development of critical thinking, influenced by the experimental program, was achieved by the experimental group of students whose achievement is *excellent* (E5). Compared to all other groups of the control group, formed in accordance with the overal student achievement, this group achieved a significant difference, and it is particularly significant that this difference is notable in comparison to students with *excellent* (E5, C5: t=25.7695, p< .01) and *very good* achievement (E5, C4: t=26.4017, p< .01). In addition, these students made a statistically significant difference in the development of critical thinking compared to other students from the experimental group formed by this criterion, even compared to students with *very good* achievement (E5, E4: t=5.1122, p< .01).

The group of students of the experimental group with *very good* achievement also scored a statistically significant difference in the development of critical thinking, influenced by the content of the experimental program.

The difference this group scored compared to students of the control group with the same overall achievement (E4, C4: t=16,9665, p< .01) is particularly significant, as well as the difference compared to students of the control group *excellent* achievement (E4, C5: t=15.2996, p< .01). The improvement of this student group is not statistically significant in comparison to students of the experimental group whose achievement is *good* (E3, E4: t=1.1860).

Students of the experimental group with *good* overall achievement also scored a statistically significant difference in the development of critical thinking compared to all groups of the control group formed on the basis of overall student achievement. This difference is greatest between these students and students of the control group with identical achievement (E3, C3: t=5.2203, p< .01), then *excellent* (E3, C5: t = 4,3008, p<0,01), *unsatisfactory* (E3, C1: t = 4,8545, p<0,01) and *very good* (E3, C4: t=2.9955, p< .01).

Only the students of the experimental group, whose overall achievement is *unsatisfactory,* did not score a statistically significant difference in the development of critical thinking compared to other subgroups of the control group, formed on the basis of overall student achievement.

Differences between arithmetic means are not statistically significant between *unsatisfactory* students of the control group and *good* and *very good* students of the same group, not between *good* and *very good* students of the control group.

Obtained results show that the selection of content can significantly contribute to the development of students' skills of critical thinking in elementary mathematics education, that it is possible to achieve this with all students, regardless of their overall achievement in school, and that the best results can be expected from students with the best overall achievement in school. Research results show that there is a correlation between the grade the student was awarded in mathematics education and the development of critical thinking, where the influence of content selection is more pronouced in students who have better achievement in (Maričić *et al*, 2013: 210).

Conclusion

Despite the fact that developing critical thinking represents an important task of mathematics education, there are numerous open questions when it comes to its realization. One part of these problems stems from its generalization and indeterminancy, another from the lack of a clear strategy for its development. In this paper, we attempted to operationalize the concept of critical thinking in the context of elementary mathematics education, through specific skills that are emphasized during work with mathematical content and to examine if a more purposeful selection of content can influence the development of critical thinking.

Obtained results indicate that students' critical thinking can be successfully encouraged and developed in elementary mathematics education by the selection of content (tasks), the solution of which emphasizes skills of critical thinking in all students, and that we should not make any limitations regarding the classification of studnets by their overall achievement in school. In addition, the greatest progress in the development of critical thinking was achieved with students who had the best overall achievement, but it should be noted, that this is possible with all student groups. The results should offer a clearer orientation to all involved in mathematics education in the process of selecting the content by which they want to achieve certain objectives in teaching. We hop we encouraged other authors to study the issue of developing critical thinking in elementary mathematics education, but also teachers and textbook authors to pay more attention to selection of the content for elementary mathematics education.

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