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Energy Procedia 61 (2014) 2662 – 2665

Energy Procedia

The 6th International Conference on Applied Energy – ICAE2014

A Nonlinear Non-dimensional Dynamic Model for Free Piston Thermal-lag Stirling Engine

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Abstract

This study is aimed to develop a theoretical model to determine criterion of instability and predict performance of a free-piston thermal-lag Stirling engine (FPTLSE). Experiments are conducted to verify the theoretical predictions partly. In the present study, a nonlinear model consisting of non-dimensional equation of motion and energy equation are derived. These governing equations are solved simultaneously by employing a multi-scale method, in which zero-order approximate solutions representing piston motion and temperature variation are obtained. Results show that the FPTLSE can be a self-started engine that may be able to start automatically after heating. In addition, the predictions by the theoretical model are found to agree closely with the experimental data for the oscillation of the piston. The present model is capable of predicting the dynamic behavior of the engine.

Peer-review under responsibility of the Organizing Committee of ICAE2014

Keywords: Free piston ; Thermal-lag ; Stirling engine ; Instability ; Nonlinear model

1. Introduction

Different from the conventional Stirling engines, the thermal-lag Stirling engine is equipped with only a moving part (piston) and a fixed part (regenerative heater) in engine's cylinder, the number of moving parts is reduced. Furthermore, if the piston of thermal-lag Stirling engine is suspended by a spring, the engine can be called as a free-piston thermal-lag Stirling engine (FPTLSE). In a FPTLSE, the mechanism can be greatly simplified without the flywheel and the link, and hence, it is regarded as a potential type of Stirling engines that featuring low cost in manufacturing and maintenance as well. In addition, the piston is typically placed in low temperature region so that thermal expansion with the piston does not pose a severe problem. Besides, the engine may be capable of starting automatically. These advantages make this engine competitive and suitable for working with a concentrating solar power (CSP) system, especially in the mountains or other remote areas where regular maintenance is difficult.

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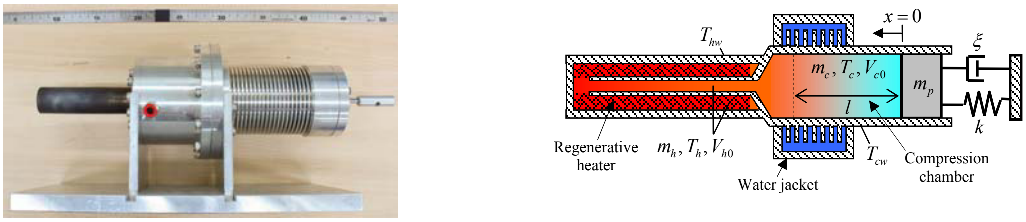
Peer-review under responsibility of the Organizing Committee of ICAE2014
doi:10.1016/j.egypro.2014.12.270



(a) Photo (b) Schematic

Fig. 1. A FPTLSE prototype engine developed by Power Engine and Clean Energy (PEACE) Lab., NCKU.

The thermal-lag Stirling engine was first built and named by Tailer [1]. Although the structure of FPTLSE is similar with the standing wave engine in Ref. [2], however, the operating principle is essentially different. Therefore, Cheng and Yang [3] revisited the problems with the thermal-lag Stirling engine, and presented a thermodynamic model to evaluate dependence of indicated power and thermal efficiency on engine speed. Moreover, they combined the dynamic and the thermodynamic models and firstly performed a dynamic simulation of modes of the start-up process of the thermal-lag Stirling engine [4].

So far, there have been a limited number of existing reports [1, 3-4] relevant to the thermal-lag Stirling engine; however, the study of the FPTLSE has not been reported. Therefore, this study is aimed to development of a theoretical model to determine criterion of instability and predict performance of the FPTLSE. Experiments are conducted to verify the theoretical predictions partly.

2. Theoretical model

A FPTLSE prototype engine and its schematic are shown in Fig. 1(a) and 1(b), respectively. The prototype engine uses air of 1-atm initial pressure as the working gas. Bore size and weight of the piston are designed to be 0.045 m and 96 g respectively. Length of compression chamber is 0.05 m. The spring in Fig. 1(b) may be a planar spring or gas spring, and the damping is caused by external loading or friction. The space of the cylinder is divided into two spaces: (1) compression chamber and (2) regenerative heater, which are separated by a dash line as shown in Fig. 1(b). While the piston is moving toward the regenerative heater, the cold gas in the compressed chamber is being pushed into the regenerative heater for heating. As the temperature of the gas is elevated, the hot gas expands and pushes the piston back to its initial position. While the piston is being pushed back, the expansion work of the gas is output to the external loading and in the same time a part of the hot gas is pulled out from the regenerative heater and gets into the compression chamber. The gas in the compressed chamber is cooled down again and completes one thermodynamic cycle. In the photo of the prototype engine, the piston is not connected to any loading or spring, the stiffness of the spring is simply set to be zero.

The working gas is treated as an ideal gas. The pressure in the whole cylinder is uniform instantaneously while varied with time. On the other hand, the temperatures of the gas in the compression chamber and the regenerative heater are different (denoted by T_{cv} and T_{hv}, respectively) but they are uniform in respective spaces while varied with time. The gas temperature at the separation line between the two spaces is assumed to be the average temperature between T_{cv} and T_{hv}. Equation of motion for the piston can be written in non-dimensional form as:

$$\bar{x}'' + 2\Gamma\bar{x}' + \bar{x} = \varepsilon\left[\frac{\tau}{\kappa}\sum_{k=1}^{\infty}\bar{T}_{hv}^{\ k} - (1-\bar{x})\sum_{k=1}^{\infty}(-1)^{k-1}\bar{T}_{cv}^{\ k}\right] \qquad (1)$$

where $\bar{x}(=x/l)$, $\bar{T}_{cv}(=-1+T_c/T_{cw})$, $\bar{T}_{hv}(=1-T_h/T_{hw})$, $\tau(=T_{cw}/T_{hw})$, $\kappa(=V_{c0}/V_{h0})$, $\Gamma(=\xi/2m_p\sqrt{\omega_k^2+\omega_g^2})$ and $\varepsilon(=\omega_g^2/[\omega_k^2+\omega_g^2])$ representing dimensionless displacement of piston, dimensionless temperature in the compression chamber, dimensionless temperature in the regenerative heater, wall temperature ratio, volume ratio of compression chamber to regenerative heater, damping ratio and frequency ratio, respectively. Meanwhile, $\omega_k(=\sqrt{k/m_p})$ and $\omega_g(=\sqrt{\kappa p_0 A/m_p l(\kappa+\tau)})$, in rad/s, are natural frequencies caused by the spring and the working gas, respectively, p_0 the initial pressure of the gas and A cross section area of the piston. The term in the right-hand side of Eq. (1) is the forces produced by working gas, which is a function of piston displacement and gas temperature and is obtained by expanding the pressure function. The temperature in Eq. (1) can be determined from the dimensionless energy equations as:

$$\bar{m}_c\bar{T}_{cv}' = -H\bar{T}_{cv} + \gamma(1+\tau+\tau\bar{T}_{cv}-\bar{T}_{hv})\bar{m}_c'/2\tau - (1+\bar{T}_{cv})\bar{m}_c' + (\gamma-1)\bar{m}_c(1+\bar{T}_{cv})\bar{x}'/(1-\bar{x}) \quad (2)$$

$$(1-\bar{m}_c)\bar{T}_{hv}' = \delta H\bar{T}_{cv} - \gamma(1+\tau+\tau\bar{T}_{cv}-\bar{T}_{hv})\bar{m}_c'/2 + (1+\bar{T}_{hv})\bar{m}_c' \quad (3)$$

where $\delta(=R_c/R_h)$, $H(=1/m\omega c_v R_c)$ and $\bar{m}_c(=m_c/m)$ are ratio of thermal resistance of compression chamber to regenerative heater, heat transfer ratio, and dimensionless mass of working gas in the compression chamber, respectively, in which m is total mass of the working gas. Eqs. (1) to (3) are solved simultaneously by a multi-scale method in this study to obtain the zero-order approximate solutions for the displacement of piston and gas temperatures. Indicated work under different design parameters can be calculated by $W_i = 2\pi p_0 V_{c0}\kappa a^2\Gamma/\varepsilon(\kappa+\tau)$ in J. Where a is the amplitude of the displacement of piston in steady-oscillation regime, which can be obtained from Eq.(1). Furthermore, in the instability analysis, the neutral curve can be determined by solving the equation at $a = 0$ as Fig.2 shows.

3. Results and Discussion

For validation, the displacement of piston is measured by laser displacement meter and compared with theoretical solutions. Fig. 3 shows the variation in the displacement of the piston with dimensionless time $(\omega_g t)$ at $\tau = 0.33$, $\kappa = 1.0$, $\varepsilon = 1.0$, $\delta = 1000$, $H = 0.4$, and $\Gamma = 0.01077$. Transient oscillation of the

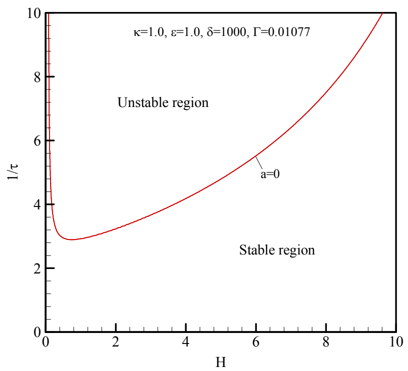


Fig. 2. Neutral curve in $1/\tau$ - H domain.

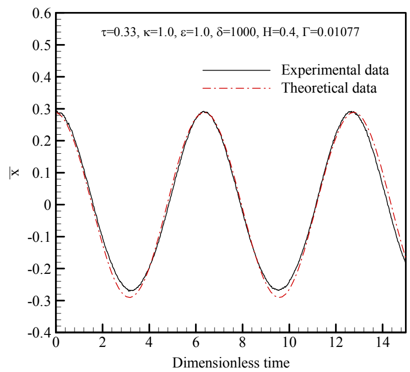


Fig. 3. Comparison between theoretical and experimental data for piston oscillation in steady-oscillation regime.

piston from initial start to the steady-oscillation regime is observed in experiment. It is found that the initial dimensionless amplitude of the oscillation is only about 0.08 and it gradually grows to 0.27 in the steady-oscillation regime. It implies that FPTLSE could be a self-started engine that can be started easily by giving just a small disturbance from heating. In addition, the periodic amplitude variation is verified by the experiments. Fig. 3 shows the comparison between the experimental and the theoretical data in the steady-oscillation regime. It is observed that the two curves closely agree with each other. The small difference at the troughs of the curves probably is caused by a shift of equilibrium point of the piston which has been observed in the experiments. Relative error in the amplitude between the two set of data is approximately less than 3.58 %. On the other hand, the theoretical prediction of the frequency of the oscillation of the piston is calculated to be 24.78 Hz. Compared to the experimental reading of 25.27 Hz, the relative error of the theoretical solution is less than 2 %. The theoretical prediction of the indicated work of the engine can be further determined, and then the engine output power is equal to the product of indicated work and the frequency. For the present engine, the output power is estimated to be 0.87 W.

4. Conclusion

In this study, a nonlinear model for the FPTLSE is presented and solved. The theoretical predictions by the model are found to agree closely with the experimental data for the oscillation of the piston.

Acknowledgements

Financial support from National Science Council, Taiwan, R.O.C., under grant NSC 102-2221-E-006-088, is highly appreciated.

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