



ELSEVIER

SCIENCE @ DIRECT®

PHYSICS LETTERS B

Physics Letters B 568 (2003) 281–290

www.elsevier.com/locate/npe

Deconstructed $U(1)$ and supersymmetry breaking

Emilian Dudas^{a,b}, Adam Falkowski^c, Stefan Pokorski^c^a *LPT¹, Bât. 210, Université Paris-Sud, F-91405 Orsay cedex, France*^b *Centre de Physique Théorique, Ecole Polytechnique, F-91128 Palaiseau cedex, France*^c *Institute of Theoretical Physics, Warsaw University, Hoża 69, 00-681 Warsaw, Poland*

Received 4 April 2003; received in revised form 27 May 2003; accepted 5 June 2003

Editor: G.F. Giudice

Abstract

We discuss supersymmetry breaking induced by simultaneous presence of a Wilson-line type superpotential and boundary-localized Fayet–Iliopoulos terms in a four-dimensional theory based on deconstruction of five-dimensional Abelian gauge theories on orbifolds. Large hierarchy between the scale of supersymmetry breaking and the fundamental scale can be generated dynamically. The model has several potentially interesting phenomenological applications. We also discuss the conditions that are necessary for interpreting our $U(1)^N$ model as an ultraviolet completion of some 5d theory. In particular, the corresponding 5d theory contains Chern–Simons couplings.

© 2003 Published by Elsevier B.V. Open access under [CC BY license](https://creativecommons.org/licenses/by/4.0/).

1. Introduction

Deconstructed higher-dimensional gauge theories [1] can be viewed as an ultraviolet (UV) completion of gauge theories [2] in more than 4 dimensions or as a new tool for building models in four dimensions [3]. In the latter case one does not have to insist on exact higher-dimensional correspondence but one just explores the possibilities offered by the basic structure of such theories which is a product gauge symmetry containing bi-fundamental matter. Both views on deconstructed gauge theories may provide new theoretical insight: completing 5d theories in the UV by their deconstructed versions may give us more rigorous calculational tools for non-renormalizable gauge theories and may help to understand better their structure, whereas studying 4d gauge theories with product gauge group in their own sake may give us the benefits usually attributed to extra-dimensional theories but in simple 4d setting.

Deconstruction as a UV completion of 5d super-Yang–Mills theories is interesting from the point of view of recovering $\mathcal{N} = 2$ supersymmetry in four dimensions [4]. Deconstruction as a model building tool, not necessarily

E-mail addresses: emilian.dudas@th.u-psud.fr (E. Dudas), adam.falkowski@fuw.edu.pl (A. Falkowski), pokorski@fuw.edu.pl (S. Pokorski).

¹ Unité Mixte de Recherche du CNRS (UMR 8627).

with exact correspondence to a 5d theory, is interesting as (among other reasons) it provides a mechanism for naturally generating hierarchical (dimensionless and dimensionful) physical parameters [5].

In this Letter we consider deconstructed supersymmetric $U(1)$ gauge theories. Several aspects of such theories with unbroken supersymmetry are discussed in Ref. [6]. Here we propose a novel mechanism of supersymmetry breaking based on the simultaneous presence of a Wilson-line type superpotential and boundary-localized Fayet–Iliopoulos (FI) terms. In the present Letter we explore both above-described aspects of deconstruction. We begin with a simple product $U(1)$ supersymmetric model that is a self-consistent theory in 4d. A very important role in the construction of the model is played by the condition of mixed gauge anomaly cancellation. The construction of the model is presented in Section 2. From the purely four-dimensional point of view, the model presents an interesting new mechanism of supersymmetry breaking, where its dominant source is an expectation value of the D-terms, similarly as in the scenario of supersymmetry breaking with a single anomalous $U(1)$ [7,8].

However, as we discuss in Section 3, the simple model of Section 2 has no 5d correspondence: its continuum limit violates 5d Lorentz invariance. It turns out that very interesting conclusions follow from insisting on the correct 5d correspondence, i.e., on 5d Lorentz invariance and $\mathcal{N} = 2$ supersymmetry. The cancellation of mixed gauge anomalies plays again a crucial role in that discussion. We show that the necessary extension of the simple model of Section 2 is highly constrained. In particular, the 5d continuum theory includes the Chern–Simons term.

In Section 4 we return to the simple model of Section 2 which, as we said, is sufficient as a model of supersymmetry breaking and computationally simpler than the one of Section 3. Thus, in Section 4 we minimize the scalar potential and show that supersymmetry is indeed spontaneously broken, with the scale of supersymmetry breaking suppressed with respect to the fundamental scale by the factor ϵ^N , where N is the number of $U(1)$ gauge groups and $\epsilon \sim 0.1$. In the present case the scale of supersymmetry breaking is dynamically determined by the model. Furthermore, if we embed the model into a locally supersymmetric version, supersymmetry breaking is of a hybrid type, with both D-term and F-term breaking but with D-term dominating. Finally, in Section 5 we briefly discuss potential phenomenological consequences of such a scenario for fermion mass generation and for a solution to the supersymmetric flavour problem.

2. A simple four-dimensional model with product $U(1)$

We consider supersymmetric theories with a product $U(1)$ gauge group. The setup involves the product gauge group $U(1)_1 \times \cdots \times U(1)_N \equiv U(1)^N$ and $N - 1$ chiral multiplets (links) Φ_p charged under the neighboring groups. The quiver diagram for our model is given in Fig. 1. The link Φ_p has charge Q under $U(1)_p$ and $-Q$ under $U(1)_{p+1}$. The first and the N th group are not linked. Furthermore, at the boundaries of the group product space we add chiral multiplets: X_L with charge $-Q$ under $U(1)_1$ and X_R with charge Q under $U(1)_N$. In the following we normalize $Q = 1$. We shall also assume that some matter chiral multiplets (including the MSSM matter) live at the boundaries, that is they transform under $U(1)_1$ or $U(1)_N$. In general, we consider the situation when $\text{Tr } Q_1 \neq 0$, $\text{Tr } Q_N \neq 0$.

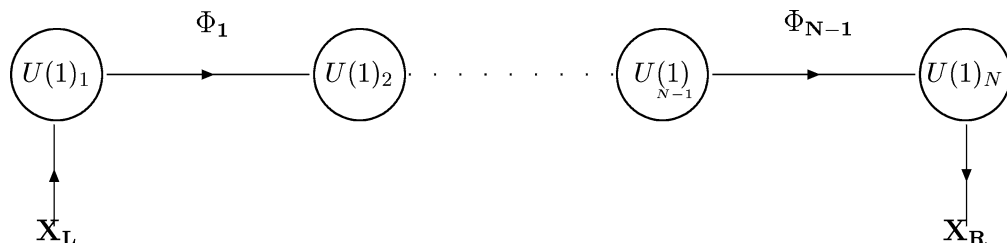


Fig. 1. The quiver diagram of the model.

Note that, for N large enough, we cannot write any renormalizable potential for the links. Still, the symmetries of the theory allow for a non-renormalizable superpotential of the Wilson-line type,

$$W = M_P X_L \left(\frac{\Phi_1 \cdots \Phi_{N-1}}{M_P^{N-1}} \right) X_R, \tag{1}$$

where we identified the fundamental scale with the Planck scale. This superpotential will be the source of supersymmetry breaking.

The model as it stands is inconsistent as the $U(1)$ gauge symmetries are anomalous. There are two kinds of anomalies. The first are mixed anomalies of the neighboring groups, that are produced by the presence of the links. The second are the anomalies of the first and the N th group produced by the matter multiplets living at the boundaries. It is well known (see, e.g., Ref. [9]) that the structure of anomalies depends on the renormalization conditions imposed on the divergences of the currents. Let us define the correlator of three gauge currents $\Gamma_{pqr}^{\mu\nu\rho}(x, y, z) = \langle 0|T j_p^\mu(x) j_q^\nu(y) j_r^\rho(z)|0 \rangle$, where j_p^μ is the chiral current coupling to the p th gauge field. The only non-vanishing divergences relevant for the mixed anomalies are those involving $\Gamma_{p,p+1,p+1}^{\mu\nu\rho}$ and $\Gamma_{p+1,p,p}^{\mu\nu\rho}$. One possible regularization is such that the anomaly is placed in the current appearing only once in the correlator. In such regularization the gauge variation of the model is given by

$$\delta \mathcal{L}_{an} = -\frac{i}{4\pi^2} \sum_p \int d^2\theta \Lambda_p (W_{p+1}^\alpha W_{\alpha,p+1} - W_{p-1}^\alpha W_{\alpha,p-1}) + \text{h.c.} \tag{2}$$

Here Λ_p are the infinitesimal parameters of the $U(1)^N$ gauge transformations written in the superfield formalism. These anomalies can be canceled by the Green–Schwarz mechanism. To this end the theory should contain superfields M_k coupling to the gauge fields via the gauge kinetic functions, $f_p = \frac{1}{g_p^2} + \sum_k s_{pk} M_k$. Under the gauge transformations, the fields M_k must transform non-linearly $V_p \rightarrow V_p + i(\bar{\Lambda}_p - \Lambda_p)$, $M_k \rightarrow M_k + 2i \sum_k \epsilon_{kp} \Lambda_p$. To cancel the anomalies, the Green–Schwarz condition must be satisfied,

$$C_{pq} = \frac{1}{2} \sum_k s_{pk} \epsilon_{kq}, \tag{3}$$

where the $N \times N$ anomaly matrix C , defined as $C_{pq} \equiv \frac{1}{4\pi^2} \text{Tr}(Q_p Q_q^2)$, reads:

$$C_{pq} = \frac{1}{4\pi^2} (\delta_{p,q-1} - \delta_{p,q+1} + \text{Tr}(Q_1)^3 \delta_{p,1} \delta_{q,1} + \text{Tr}(Q_N)^3 \delta_{p,N} \delta_{q,N}). \tag{4}$$

In the setup at hand, there is a natural solution to these constraints. Note that the role of the moduli M_k can be played by the links. In the superspace language, the links transform as $\Phi_p \rightarrow e^{2i\Lambda_p} \Phi_p e^{-2i\Lambda_{p+1}}$. Hence we can define an object that transforms non-linearly,

$$\log(\Phi_{p-1} \Phi_p) \rightarrow \log(\Phi_{p-1} \Phi_p) + 2i(\Lambda_{p-1} - \Lambda_{p+1}). \tag{5}$$

If the model were described by a periodic quiver diagram, only mixed anomalies would be present and these can be canceled by using the links only. However, in the present case the group product space has boundaries where anomalies can appear. Therefore, we introduce new superfields, S_L and S_R , transforming as

$$S_L \rightarrow S_L + i M_P (\text{Tr}(Q_1^3) - 1) \Lambda_1, \quad S_R \rightarrow S_R + i M_P (\text{Tr}(Q_N^3) + 1) \Lambda_N. \tag{6}$$

Then anomalies (2) are canceled provided the gauge kinetic function are chosen as

$$f_1 = \frac{1}{g^2} - \frac{1}{2\pi^2} \log\left(\frac{\Phi_1}{v}\right) + \frac{1}{\pi^2 M_P} S_L \delta_{p,1},$$

$$\begin{aligned}
f_p &= \frac{1}{g^2} - \frac{1}{2\pi^2} \log\left(\frac{\Phi_{p-1}\Phi_p}{v^2}\right), & p = 2, \dots, N-1 \\
f_N &= \frac{1}{g^2} - \frac{1}{2\pi^2} \log\left(\frac{\Phi_{N-1}}{v}\right) + \frac{1}{\pi^2 M_P} S_R \delta_{p,N}, & (7)
\end{aligned}$$

where v is an arbitrary scale which, for convenience, is chosen equal to the links vevs. Also, we have set all the $U(1)$ gauge couplings to be equal for simplicity. An interesting feature of this theory is that all the gauge couplings vanish in the unbroken phase $\langle \Phi_p \rangle = 0$. Therefore, the model in the UV is a free theory.

We assume the Kahler potential of the form:

$$\begin{aligned}
\mathcal{K} &= \sum_{p=1}^{N-1} |\Phi_p|^2 e^{-2V_p + 2V_{p+1}} + |X_L|^2 e^{2V_1} + |X_R|^2 e^{-2V_N} \\
&\quad + \frac{1}{2} [S_L + \bar{S}_L + M_P (\text{Tr}(Q_1)^3 - 1) V_1]^2 + \frac{1}{2} [S_R + \bar{S}_R + M_P (\text{Tr}(Q_N)^3 + 1) V_N]^2. & (8)
\end{aligned}$$

The links have a minimal kinetic term. The presence of vector multiplets in the kinetic term for S_L and S_R makes the Kahler potential gauge invariant. It also generates FI terms at the boundaries of the group product space, that are proportional to the vacuum expectation values of S_L and S_R ,

$$\xi_1 = M_P (\text{Tr}(Q_1)^3 - 1) (S_L + \bar{S}_L), \quad \xi_N = M_P (\text{Tr}(Q_N)^3 + 1) (S_R + \bar{S}_R). \quad (9)$$

We will not construct an explicit superpotential that gives vevs to S_L and S_R . Instead we simply assume that their vevs are such that $\xi_1 \sim \xi_N \sim \epsilon M_P$, with $\epsilon \sim 0.1$ and in the following we will ignore their dynamics.

In Section 4 we show that the model we have constructed provides an interesting new mechanism of supersymmetry breaking. However, first, in Section 3 we discuss the model from the point of view of the correspondence with 5d gauge theories.

3. Anomalies and consistent five-dimensional limit

An interesting point we want to discuss is the construction of a model with a consistent 5d limit and the role of anomaly cancellation in the 4d deconstructed version. In order to cancel the mixed anomalies we used the Green–Schwarz mechanism, and a vital role was played by the links coupled to the gauge fields via the gauge kinetic function. Recall [4] that the links contain the degrees of freedom that are translated to the fifth component of the 5d gauge field. More precisely, the links can be represented as $\Phi_p = \frac{v}{\sqrt{2}} e^{(\Sigma_p + iG_p)/v}$ and G_p is what matches A_5 (while Σ_p matches the scalar singlet of the 5d vector multiplet). Thus a natural candidate for a 5d match to our Green–Schwarz mechanism is the 5d Chern–Simons term,

$$\mathcal{L}_{\text{CS}} = \frac{1}{24\pi^2} \epsilon_{\alpha\beta\gamma\delta\epsilon} [A_\alpha \partial_\beta A_\gamma \partial_\delta A_\epsilon] = \frac{1}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} [3A_5 \partial_\mu A_\nu \partial_\rho A_\sigma - 2A_\mu \partial_\nu A_\rho \partial_5 A_\sigma]. \quad (10)$$

One can check that the terms we have proposed in Section 2 do not have a 5d invariant continuum limit and therefore cannot correspond to the 5d Chern–Simons term. 5d Lorentz invariance in the continuum limit must be imposed as an additional constraint. The remainder of this section is devoted to finding a 4d action that can cancel the mixed anomalies and, at the same time, match the Chern–Simons term in the continuum limit.

Let us consider for simplicity the simpler case with a closed (periodic) quiver diagram. The model has only mixed anomalies and, using the same anomaly renormalization scheme as in Section 2, Eqs. (4) and (7) simplify to $C_{pq} = \frac{1}{4\pi^2} (\delta_{p,q+1} - \delta_{p,q-1})$ and

$$f_p = \frac{1}{g^2} + \frac{1}{2\pi^2} \chi_p, \quad (11)$$

where $\chi_p = -\ln(\Phi_{p-1}\Phi_p/v^2)$. In the infrared we want to recover a 5d $\mathcal{N} = 1$ supersymmetric theory compactified on a circle. From a four-dimensional viewpoint, this should be a theory with two supersymmetries $\mathcal{N} = 2$. It is known that in 5d that the gauge couplings can be functions of scalars Σ_p in the vector multiplets, which is indeed consistent with our identification of the gauge kinetic function. Moreover, the couplings of the vector multiplet are completely specified by a real function, the prepotential $\mathcal{F}(\Sigma_p)$, which is a polynomial function at most trilinear in the scalar fields Σ_p (for a review of 5d Abelian supersymmetric theories, see, e.g., [10]). For example, the gauge couplings $\tau_{pq}(X)F_{MN}^p F^{MN,q}$ are provided by $\tau_{pq} = \partial_p \partial_q \mathcal{F}(\Sigma) \equiv \mathcal{F}_{pq}$. In four dimensions (for a review, see, e.g., [11]), the gauge kinetic function becomes a holomorphic function of the superfields χ^p and we need to substitute $\Sigma_p \rightarrow \frac{v}{4}(\chi_p + \bar{\chi}_p)$. The effective Lagrangian of vector multiplets is, by using $\mathcal{N} = 1$ language, given as

$$\mathcal{L} = \frac{1}{2} \int d^2\theta \sum_{pq} \mathcal{F}_{pq} W^{\alpha,p} W_\alpha^q + \text{h.c.} + \int d^4\theta \mathcal{K}(\chi^p, \bar{\chi}_q), \tag{12}$$

where the Kahler potential is given in terms of the prepotential by

$$\mathcal{K}(\chi^p, \bar{\chi}_p) = \sum_p (\bar{\mathcal{F}}_p \chi^p + \mathcal{F}^p \bar{\chi}_p). \tag{13}$$

By matching with Eq. (11) we find that the 5d prepotential and the derived 4d Kahler potential of this theory are

$$\begin{aligned} \mathcal{F}(\Sigma_p) &= \frac{1}{2g^2} \sum_p \Sigma_p^2 - \frac{1}{6\pi^2} \sum_p \Sigma_p^3, \\ \mathcal{K}(\chi^p, \bar{\chi}_p) &= \frac{v^2}{16g^2} \sum_p (\chi_p + \bar{\chi}_p + 2V_{p-1} - 2V_{p+1})^2 + \frac{v^2}{384\pi^2} \sum_p (\chi_p + \bar{\chi}_p + 2V_{p-1} - 2V_{p+1})^3. \end{aligned} \tag{14}$$

However the action is not 5d invariant in the continuum limit, as Eq. (14) clearly does not yield the last term of the 5d Chern–Simons couplings (10), containing $\partial_5 A_\mu$.

Interestingly enough, there is another consistent regularization of gauge anomalies, which is compatible with 5d Lorentz invariance. In this regularization, the anomalous divergences are placed symmetrically in each current in $\Gamma_{p,p,p+1}^{\mu\nu\rho}$ and $\Gamma_{p+1,p,p}^{\mu\nu\rho}$. The anomalous variation of the action is then equal to

$$\delta\mathcal{L}_{an} = -\frac{i}{12\pi^2} \sum_p \int d^2\theta \Lambda_p (W_{p+1}^\alpha W_{\alpha,p+1} - W_{p-1}^\alpha W_{\alpha,p-1} - 2W_p^\alpha W_{\alpha,p+1} + 2W_p^\alpha W_{\alpha,p-1}) + \text{h.c.}, \tag{15}$$

whereas the anomalous couplings present in the Lagrangian (14) can account only for the first two terms in Eq. (15).

In this symmetric regularization, a Wess–Zumino term is needed and it is naturally selected to be

$$\mathcal{L}_{WZ} = -\frac{1}{12\pi^2} \int d^4\theta [(V_{p+1} - V_{p-1})D_\alpha V_p - V_p(D^\alpha V_{p+1} - D^\alpha V_{p-1})] W_{\alpha,p} + \text{h.c.}, \tag{16}$$

whose variation under gauge transformations is

$$\delta\mathcal{L}_{WZ} = -\frac{i}{6\pi^2} \sum_p \int d^2\theta \Lambda_p (W_{p-1}^\alpha W_{\alpha,p-1} - W_{p+1}^\alpha W_{\alpha,p+1} - W_p^\alpha W_{\alpha,p+1} + W_{p-1}^\alpha W_{\alpha,p}) + \text{h.c.} \tag{17}$$

and which, combined with the gauge variation coming from Eq. (11), exactly cancel the anomalous one-loop variation Eq. (15).

Using the dictionary $A_{\mu,p} \rightarrow A_\mu(y_p)$, $(A_{\mu,p+1} - A_{\mu,p})/\Delta y \rightarrow \partial_5 A_\mu(y_p)$, $G_p \rightarrow A_5(y_p)$ with the lattice spacing $\Delta y = (v)^{-1}$, it is straightforward to check that the full Kahler potential (14) supplemented by (16) is actually the deconstructed version of the Chern–Simons one discussed in [12] and therefore in the continuum limit, we indeed recover in the IR a 5d supersymmetric theory. The manifestly supersymmetric massless and massive vector multiplets are in $\mathcal{N} = 1$ language (V_p, χ_p) . On the circle, the Chern–Simons term does not play any role

in anomaly cancellation as its variation is a total derivative. However, interestingly enough, in our deconstruction model this term (in its supersymmetric form) is present in order to cancel mixed gauged anomalies.

4. Supersymmetry breaking

In this section we discuss supersymmetry breaking triggered by the Wilson-type superpotential (1). The model with a consistent 5d limit considered in Section 3, on the orbifold, is hard to analyse. We return, therefore, to the simple model of Section 2 and study supersymmetry breaking. One may worry that supersymmetry breaking could affect the cancellation of gauge anomalies as the WZ term could undergo renormalization. But since, in our setup, supersymmetry breaking is of spontaneous nature, all loop induced terms can be parametrized by supersymmetric operators with F- and D-term vacuum expectation values inserted. It is possible to show that neither $AA\tilde{F}$ nor $\log(\phi_p)F\tilde{F}$ terms (that are important for anomaly cancellation) are not induced at the loop level.

The D-term potential in the model of Section 2 takes the form:

$$V_D = \frac{1}{2} [(\text{Re } f_1)^{-1} (|\Phi_1|^2 - |X_L|^2 + \xi_1)^2 + (\text{Re } f_2)^{-1} (|\Phi_2|^2 - |\Phi_1|^2)^2 + \dots \\ + (\text{Re } f_{N-1})^{-1} (|\Phi_{N-1}|^2 - |\Phi_{N-2}|^2)^2 + (\text{Re } f_N)^{-1} (|\Phi_{N-1}|^2 + |X_R|^2 + \xi_N)^2]. \quad (18)$$

Since the F-term potential is suppressed by powers of M_P , the D-term dominates the scalar potential and, in the zeroth order approximation, the vacuum adjusts itself to minimize it. Depending on values and signs of the FI terms various patterns of gauge symmetry and supersymmetry breaking may occur. Here we are interested in the situation when the product group is entirely broken, which happens for

$$\xi_N > 0, \quad \xi_1 > -\xi_N, \quad (19)$$

which we assume from now on.

In the zeroth order approximation, ignoring the contributions from the F-term potential and from the non-trivial gauge kinetic term, the D-term potential possesses a vacuum solution with a flat direction parametrized by the vev of X_R ,

$$\langle |\Phi_p|^2 \rangle = \xi_N + \langle |X_R|^2 \rangle, \quad \langle |X_L|^2 \rangle = \xi_N + \xi_1 + \langle |X_R|^2 \rangle. \quad (20)$$

In this background the product gauge symmetry is entirely broken. There is one massless chiral multiplet (for $\langle |X_R|^2 \rangle = 0$ it is just X_R). The remaining degrees of freedom form a tower of gauge multiplets with masses starting at $m^2 \sim \xi/N$. Supersymmetry is unbroken at this order.

Now we include the effects of the F-term potential. One can easily see that its addition lifts the flat direction and (for a globally supersymmetric scalar potential) sets the minimum at $\langle X_R^2 \rangle = 0$ (up to corrections suppressed by $(1/M_P)^N$). In such case, effectively, the scalar potential is augmented only by $|\frac{\partial W}{\partial X_R}|^2$. But since this operator originates from a non-renormalizable superpotential it is only a small perturbation to the zeroth order supersymmetric solution. The vacuum shift is suppressed by the small parameter κ defined as

$$\kappa^2 = \frac{1}{g^2} \left(\frac{\xi_N}{M_P^2} \right)^{N-2} \approx \epsilon^{2N-4}. \quad (21)$$

We shall solve the equations of motion to the lowest non-trivial order in κ^2 . We expand the links around the zeroth-order vacuum solution,

$$\langle |\Phi_p|^2 \rangle = \xi_N + a_p, \quad \langle |X_L|^2 \rangle = \xi_1 + \xi_N + a_0, \quad \langle |X_R|^2 \rangle = a_N, \quad (22)$$

where a_p are of order κ^2 . One can check that effects of the non-trivial gauge kinetic function appear only at order κ^4 . To order κ^2 the equations of motion read:

$$\begin{aligned} a_0 - a_1 + \kappa^2 \xi_N &= 0, \\ -a_{p-1} + 2a_p - a_{p+1} + \kappa^2(\xi_N + \xi_1) &= 0, \quad p = 1, \dots, N - 1. \end{aligned} \tag{23}$$

We encounter a difference equation of the form: $-a_{p-1} + 2a_p - a_{p+1} + X = 0$, with $X = \kappa^2(\xi_1 + \xi_N)$. The general solution is given by $a_p = A + Bp + \frac{1}{2}Xp^2$, where A and B are arbitrary constants. The first of Eq. (23) acts as a ‘boundary condition’ for the difference equation, $a_{-1} = a_0 + \kappa^2 \xi_1$. This allows to determine the constant $B = X/2 - \kappa^2 \xi_1$. The constant A is not determined and so the flat direction persists at the order κ^2 . However, the value of A is not important in what follows, in particular, the supersymmetry breaking parameters do not depend on A . Hence, we find that the vacuum solution to first order in κ^2 is given by

$$\begin{aligned} \langle |\Phi_p|^2 \rangle &= \xi_N + A + \frac{1}{2}\kappa^2(p(\xi_N - \xi_1) + p^2(\xi_N + \xi_1)), \\ \langle |X_L|^2 \rangle &= \xi_1 + \xi_N + A, \\ \langle |X_R|^2 \rangle &= A + \frac{1}{2}\kappa^2(N(\xi_N - \xi_1) + N^2(\xi_N + \xi_1)). \end{aligned} \tag{24}$$

In this shifted vacuum supersymmetry is broken and the expectation values of the D-terms are:

$$D_p = \kappa^2[p(\xi_1 + \xi_N) - \xi_1]. \tag{25}$$

There is also an F-term acquiring vev:

$$F_{X_R} = \frac{\partial W}{\partial X_R} = \kappa \sqrt{\frac{\xi_N(\xi_1 + \xi_N)}{a}}. \tag{26}$$

Note that all the D-terms are positive. The consequence of this fact is that the matter we assumed to be present at the boundaries has to have non-negative $U(1)_1$ or $U(1)_N$ charges. Otherwise, the scalars of a negatively charged multiplet would get a tachyonic mass and render the model unstable.

The most interesting point in this construction is that, in a natural way, the supersymmetry breaking scale is suppressed with respect to the fundamental scale M_P . Recall that $\kappa = \epsilon^{N-2}$, $\xi = \epsilon^2 M_P^2$. Defining the supersymmetry breaking scale as the scale of the D-term of the first group, $M_{\text{SUSY}}^2 = D_1$, we get:

$$M_{\text{SUSY}} = \epsilon^{N-1} M_P. \tag{27}$$

For $\epsilon \sim 0.1$, even for a moderate number of replications, say $N \sim 10$, it is easy to generate a huge hierarchy between the fundamental and the supersymmetry breaking scale. Hence the ‘desert’ above the TeV scale can simply be a consequence of the existence of a product $U(1)$ group at some high energy scale. The origin of the hierarchy is the fact that supersymmetry breaking is triggered by a non-local, Wilson type object—the superpotential of Eq. (1). Thus we expect the hierarchy is not particularly sensitive to the technical assumptions we have made.

This picture is slightly modified when the model is embedded in supergravity [13]. The superpotential we assume here has the form $W_{\text{SUGRA}} = \widehat{W}(S_L, S_R) + W(\Phi_p)$, where $W(\Phi_p)$ is the same as in the globally supersymmetric case, see Eq. (1). We do not specify the precise form of \widehat{W} but simply assume that it stabilizes S_L, S_R (in the following denoted collectively as S) at the value close to the fundamental scale, $\langle S + S^\dagger \rangle \approx M_P$, and that $\langle M_P \frac{\partial \widehat{W}}{\partial S} \rangle \approx \langle \widehat{W} \rangle$. Then, to the leading order in the $|\xi_N/M_P^2|$ expansion the scalar potential takes the form

$$V \approx V_{\text{GLOBAL}} + (N - 2) \frac{\langle \widehat{W} \rangle}{M_P^2} (W + W^\dagger) - 2 \frac{\langle \widehat{W} \rangle^2}{M_P^2}. \tag{28}$$

To avoid a large cosmological constant, $\langle \widehat{W} \rangle$ should cancel the positive vacuum energy generated by the globally supersymmetric part of the potential V_{GLOBAL} . The latter is dominated by the F-term F_{X_R} , thus we need $F_S \approx F_{X_R}$. The gravitino mass can be estimated to be $m_{3/2} = \langle \widehat{W} \rangle / M_P^2 \approx F_{X_R} / M_P$.

It is interesting to investigate what is the higher-dimensional theory interpretation of the model of supersymmetry breaking we presented. Let us consider a 5d supersymmetric $U(1)$ gauge theory compactified on the orbifold S_1/Z_2 [4,6]. The chiral multiplets X_L and X_R reside at the two different fixed points—respectively, at $x_5 = 0$ and $x_5 = \pi R$. Moreover, there are brane localized FI terms, $S_5 \sim \xi_1 \delta(x_5) + \xi_N \delta(x_5 - \pi R)$. When the fifth dimension is integrated out one finds a broken 4d $\mathcal{N} = 1$ $U(1)$ theory with two chiral multiplets X_L and X_R . At the tree-level X_L and X_R do not couple to each other as they originate from two sequestered branes. However, integrating out heavy gauge bosons with masses of the order of the cutoff Λ will induce a tiny coupling and one expects $W \sim \Lambda e^{-\pi R \Lambda} X_L X_R$. We could view this extra-dimensional model as a construction justifying the smallness of the holomorphic mass term for the $X_{L,R}$ fields. Rewriting the cutoff as $\Lambda = N/R$, where N is the number of the KK modes below the cutoff we get $M_{\text{SUSY}} \sim \Lambda e^{-\pi N}$. This should be confronted with $M_{\text{SUSY}} \sim M_{P\kappa}$ where $\kappa \sim \epsilon^N$. In both cases the supersymmetry breaking is controlled by a moderately small parameter raised to the power N —number of heavy modes in the theory.

5. Phenomenological consequences

In Section 4 we showed that the scenario with replicated $U(1)$ s and a Wilson-type superpotential leads to supersymmetry breaking, whose scale is naturally much lower than the fundamental scale. Both D-terms and F-terms expectation values are non-vanishing. In this section we discuss the consequences of such hybrid scenario for low energy phenomenology.

Assuming that both FI terms are of the same order ξ all the scales of the low-energy Lagrangian are determined in terms of M_P , κ and $\epsilon = \sqrt{|\xi|/M_P^2}$. We define the supersymmetry breaking scale M_{SUSY}^2 as the magnitude of the D-term of the first group. Orders of magnitude of parameters relevant for low-energy phenomenology are

$$M_{\text{SUSY}} \approx M_{P\kappa\epsilon}, \quad D_p \approx p M_{\text{SUSY}}^2, \quad F_{X_R}, F_S \approx M_{\text{SUSY}} M_P \epsilon, \quad \langle X_L \rangle, \langle \Phi_p \rangle \approx M_P \epsilon. \quad (29)$$

The pattern of soft masses depends on how MSSM fields are embedded in the model. A matter multiplet P charged under $U_p(1)$ receives soft scalar mass terms from the appropriate D term,

$$m_Q^2 = q_p D_p \approx q_p p M_{\text{SUSY}}^2, \quad (30)$$

where q_p is the charge of Q under $U(1)_p$. For multiplets neutral under $U(1)$ the supersymmetry breaking is transmitted via Kahler potential, e.g.,

$$\int d^4\theta \frac{X_R^\dagger X_R}{M_P^2} Q^\dagger Q \rightarrow m_Q^2 \sim \frac{F_{X_R}^2}{M_P^2} Q^\dagger Q \approx M_{\text{SUSY}}^2 \epsilon^2 \quad (31)$$

and these soft masses pick up an additional factor ϵ .

Soft Majorana gaugino masses are mediated by the F-term of S and are of the same order of magnitude as neutral scalars,

$$\int d^2\theta \frac{S}{M_P} W_\alpha W_\alpha \rightarrow m_\lambda \approx \frac{F_S}{M_P} \approx M_{\text{SUSY}} \epsilon. \quad (32)$$

This is also the order of magnitude of the gravitino mass.

Thus we see that the MSSM spectroscopy could exhibit two different scale, M_{SUSY} and ϵM_{SUSY} , that differ by an order of magnitude. The possibility of such a splitting among superpartner masses is very advantageous from the phenomenological point of view. If the first two generation squark and leptons are much heavier than those of the

third generation, then one can reconcile the naturalness bounds with the constraints arising from flavour changing neutral current processes.

An interesting thing about supersymmetry breaking by anomalous $U(1)$ is that all the necessary ingredients to implement the Froggatt–Nielsen mechanism [14] are already at hand. The MSSM matter fields need to have positive $U(1)$ charges (otherwise they would acquire negative mass squares). In Yukawa interactions the excess charge has to be compensated by coupling to the appropriate power of the negatively charged field X_L . If $U(1)$ acts differently on the three generations, various Yukawa interactions are suppressed by powers of the parameter $\epsilon = \sqrt{\xi/M_P^2}$. Note that ϵ has generically the order of magnitude of the Cabbibo angle. In the following we shall assume $\epsilon \approx 0.2$.

Froggatt–Nielsen mechanism works also in the product $U(1)$ case. Proceeding along the lines of [15,16] one assumes that all quarks are charged under the first group, $U(1)_1$. All quark masses come from supersymmetric interactions (in general non-renormalizable ones) with X_L :

$$W = \lambda_{ij}^U H_u Q_i U_j \left(\frac{X_L}{M_P} \right)^{h_u+q_i+u_j} + \lambda_{ij}^D H_d Q_i D_j \left(\frac{X_L}{M_P} \right)^{h_d+q_i+d_j}, \quad (33)$$

where we denote the $U(1)_1$ charges of the higgses, left-handed quarks and right-handed quarks by h_u, h_d, q_i, u_i, d_i , respectively. Various charge assignments leading to acceptable mass and mixing patterns are summarized in Ref. [16].

The mechanism of fermion mass generation can control also the squark mass pattern. In the flavour basis the off-diagonal entries in the squark mass matrix originate from the Kahler potential and are expected to be of order $(m_F^2)_{ij} \sim M_{\text{SUSY}}^2 \epsilon^{2+|f_i-f_j|}$. However, in the fermion mass eigenstate basis the non-diagonal contributions in the squark mass matrix can be generated also from a splitting of diagonal entries in the flavour basis. Thus, to solve the supersymmetric flavour problem one has to control also the diagonal entries [17]. The dominance of the D-term breaking considered in Ref. [8] and in the present Letter offers such a mechanism.

6. Conclusions

In this Letter we have studied supersymmetry breaking in theories with a product $U(1)$ group and bi-fundamental matter. Fayet–Iliopoulos terms at the boundary of the group product space together with the Wilson-line type superpotential trigger supersymmetry breaking. A very interesting thing about this setup is that the scale of supersymmetry breaking hierarchically lower than the fundamental scale can be generated dynamically.

Furthermore, we have shown that interesting conclusions follow from insisting on a 5d Lorentz invariant continuum limit of such theories. Cancellation of mixed anomalies requires introducing Wess–Zumino terms in the 4d theory which, in the continuum limit, match the 5d Chern–Simons terms. Hence a consistent UV completion of our model is a 5d supersymmetric $U(1)$ theory that contains the Chern–Simons couplings. Similar conclusions are expected to hold for $U(n)$.

Acknowledgements

We wish to thank Z. Lalak, J. Mourad and S. Theisen for useful discussions. E.D., A.F. and S.P. were supported in part by the RTN European Program HPRN-CT-2000-00148. S.P. was partially supported Polish KBN grant 2 P03B 129 24 for years 2003–2004. A.F. was partially supported Polish KBN grant 2 P03B 070 23 for years 2002–2004.

References

- [1] N. Arkani-Hamed, A.G. Cohen, H. Georgi, Phys. Rev. Lett. 86 (2001) 4757, hep-th/0104005; C.T. Hill, S. Pokorski, J. Wang, Phys. Rev. D 64 (2001) 105005, hep-th/0104035.
- [2] N. Arkani-Hamed, A.G. Cohen, H. Georgi, Phys. Lett. B 513 (2001) 232, hep-ph/0105239; H.C. Cheng, C.T. Hill, J. Wang, Phys. Rev. D 64 (2001) 095003, hep-ph/0105323; C. Csaki, G.D. Kribs, J. Terning, Phys. Rev. D 65 (2002) 015004, hep-ph/0107266; H.C. Cheng, K.T. Matchev, J. Wang, Phys. Lett. B 521 (2001) 308, hep-ph/0107268; P.H. Chankowski, A. Falkowski, S. Pokorski, JHEP 0208 (2002) 003, hep-ph/0109272; A. Falkowski, C. Grojean, S. Pokorski, Phys. Lett. B 535 (2002) 258, hep-ph/0203033; A. Falkowski, H.D. Kim, JHEP 0208 (2002) 052, hep-ph/0208058; L. Randall, Y. Shadmi, N. Weiner, hep-th/0208120.
- [3] N. Arkani-Hamed, A.G. Cohen, H. Georgi, JHEP 0207 (2002) 020, hep-th/0109082; N. Arkani-Hamed, A.G. Cohen, H. Georgi, hep-th/0108089; P. Brax, A. Falkowski, Z. Lalak, S. Pokorski, Phys. Lett. B 538 (2002) 426, hep-th/0204195; N. Arkani-Hamed, A.G. Cohen, E. Katz, A.E. Nelson, T. Gregoire, J.G. Wacker, JHEP 0208 (2002) 021, hep-ph/0206020; N. Arkani-Hamed, A.G. Cohen, E. Katz, A.E. Nelson, JHEP 0207 (2002) 034, hep-ph/0206021; T. Gregoire, J.G. Wacker, JHEP 0208 (2002) 019, hep-ph/0206023.
- [4] C. Csaki, J. Erlich, C. Grojean, G.D. Kribs, Phys. Rev. D 65 (2002) 015003, hep-ph/0106044.
- [5] W. Skiba, D. Smith, Phys. Rev. D 65 (2002) 095002, hep-ph/0201056; C.T. Hill, A.K. Leibovich, Phys. Rev. D 66 (2002) 075010, hep-ph/0205237; N. Arkani-Hamed, H.C. Cheng, P. Creminelli, L. Randall, hep-th/0301218; H. Abe, T. Kobayashi, N. Maru, K. Yoshioka, hep-ph/0205344.
- [6] A. Falkowski, H.P. Nilles, M. Olechowski, S. Pokorski, hep-th/0212206.
- [7] P. Fayet, Nucl. Phys. B 90 (1975) 104.
- [8] P. Binetruy, E. Dudas, Phys. Lett. B 389 (1996) 503, hep-th/9607172; G.R. Dvali, A. Pomarol, Phys. Rev. Lett. 77 (1996) 3728, hep-ph/9607383.
- [9] S. Pokorski, Gauge Field Theories, Second Edition, Cambridge Univ. Press., Cambridge, 2000, Section 13.1; S. Weinberg, The Quantum Theory of Fields. Vol. 2: Modern Applications, Cambridge Univ. Press, Cambridge, 1996, Section 22.3.
- [10] K.A. Intriligator, D.R. Morrison, N. Seiberg, Nucl. Phys. B 497 (1997) 56, hep-th/9702198.
- [11] J.D. Lykken, hep-th/9612114; A. Van Proeyen, math.dg/0002122.
- [12] N. Arkani-Hamed, T. Gregoire, J. Wacker, JHEP 0203 (2002) 055, hep-th/0101233; N. Marcus, A. Sagnotti, W. Siegel, Nucl. Phys. B 224 (1983) 159.
- [13] T. Barreiro, B. de Carlos, J.A. Casas, J.M. Moreno, Phys. Lett. B 445 (1998) 82, hep-ph/9808244.
- [14] C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B 147 (1979) 277.
- [15] L.E. Ibanez, G.G. Ross, Phys. Lett. B 332 (1994) 100, hep-ph/9403338; P. Binetruy, P. Ramond, Phys. Lett. B 350 (1995) 49, hep-ph/9412385.
- [16] E. Dudas, S. Pokorski, C.A. Savoy, Phys. Lett. B 356 (1995) 45, hep-ph/9504292.
- [17] E. Dudas, S. Pokorski, C.A. Savoy, Phys. Lett. B 369 (1996) 255, hep-ph/9509410; A.E. Nelson, D. Wright, Phys. Rev. D 56 (1997) 1598, hep-ph/9702359.