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Kinematics of arm joint motions in basketball shooting

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Abstract

A basketball shooting arm model is used to estimate arm joint motions for a set of desired release speed, angle and backspin. The model has three rigid planar links with rotational joints imitating an upper arm, forearm and hand with shoulder, elbow and wrist joints. The kinematics of the shooting arm is solved to calculate the joint angles, and the velocity and angular velocity of links at release. There are many angular displacement and velocity combinations of shoulder, elbow and wrist joints to produce the optimal release speed, angle and backspin of the ball at release. Shoulder rotation contributes to the vertical component of release velocity of the ball and elbow and wrist actions mostly produce the horizontal component of release velocity and backspin of the ball when the forearm and hand are nearly vertical at release.

Keywords: Basketball shooting; Release conditions; Kinematics; Arm joint motion

1. Introduction

Excellent basketball shooters have a beautiful arch, input the proper backspin, and minimize the lateral deviation from the optimal shot path plane. They manipulate their shoulder, elbow and wrist to produce the optimal ball speed, angle and angular velocity at release. It is vital to analyze the kinematics of the shooting arm to understand good shooting. Knudson [1] proposed six key teaching points for jump shots based on many basketball shooting studies, and mentioned that biomechanical studies have not clearly identified the optimal coordination of human joint actions.
Some previous studies [2,3,4] measured shooters’ shoulder, elbow and wrist joint motions at release for jump shots, and reported the effect of increased shooting distance. For free throws, Tsarouchas et al. [5] represented shooting arm joint motions and studied the technique for high percentage successful shots. Their results and measurements were varied because they investigated different sets of shooters’ skill and situations. To our knowledge, no one has analyzed the kinematics of basketball shooting using a dynamic simulation model including a ball and the shooter’s arm. In wheel chair basketball, Schwark et al. [6] used a two dimensional three-segment simulation model to investigate the optimal release conditions and corresponding arm movement pattern for free throws.

We study more comprehensively the dependence of the optimal release conditions and the corresponding shooting arm motion. In this paper, we derive the kinematics of shooting arm at release and find out key actions of arm joints to produce the optimal release speed, angle and backspin [7] in short-, medium-, and long-range shots.

2. Kinematic model for basketball shots

2.1. One-hand set and jump shots

One-hand set and jump shots are most commonly used in modern basketball games. Shooters hold the ball in their shooting hand, set it near their forehead, raise and extend their elbow, and snap their wrist forward. The non-shooting arm helps support the ball and allows one to shoot the ball at the desired release position. Several previous studies suggested that shooters should keep the ball, wrist, elbow and shoulder in a vertical plane aligned with the target [1]. The ball should be released with proper backspin, a good arch and no lateral deviation from the optimal shot path plane including the target and the ball release point.

2.2. Geometry of shooting arm

We use a shooting arm model similar to the two dimensional three-segment model of Schwark et al. [6]. The shooting arm is assumed to move in the vertical plane and have three rigid links with rotational joints for an upper arm, forearm and hand with shoulder, elbow, and wrist joints. A co-ordinate system of shooting arm and basketball is shown in Fig. 1. The ball centre moves in the plane before the ball contacts either the rim or backboard, and the angular velocity vectors of the ball and the links are perpendicular to the initial shot path plane. Links U, F and H, and points S, E, W, and \( \tilde{H} \) denote the upper arm, forearm and hand, and the shoulder, elbow, wrist joints and the tip of finger, respectively. Body \( B \) and point \( B^* \) denote the basketball and the ball centre. The lengths of upper arm, forearm, hand and the ball radius are \( U_L \), \( F_L \), \( H_L \) and \( R_b \), respectively. The position of the fingertip can be written using angles: \( \Psi_S \), from the horizontal plane to the upper arm link; \( \Psi_E \), from the upper arm to forearm links; and \( \Psi_W \), from the forearm to hand links. Unit vectors \( I, J, K \) are attached to the ground with the \( JK \) plane vertical and the \( K \) vector up. The other unit vectors are attached to the links and the ball as shown in Fig. 1.

2.3. Kinematics of shooting arm

We consider position and orientation of the three-link planar arm for shots. The relationship between the set of joint angles and fingertip velocities is derived. The horizontal and vertical components of fingertip velocity relative to the shoulder joint (assumed fixed) can be expressed as

\[
\begin{align*}
|J| &= |S_L| \sin \Psi_S - (\Psi_S + \Psi_E) |F_L| \sin(\Psi_S + \Psi_E) - (\Psi_S + \Psi_E + \Psi_W) |H_L| \sin(\Psi_S + \Psi_E + \Psi_W) \\
|K| &= |S_L| \cos \Psi_S + (\Psi_S + \Psi_E) |F_L| \cos(\Psi_S + \Psi_E) + (\Psi_S + \Psi_E + \Psi_W) |H_L| \cos(\Psi_S + \Psi_E + \Psi_W)
\end{align*}
\]

The velocity of ball surface at the fingertip contact point \( \hat{B} \) can be described as

\[
|N\times B^*| = |N\times R_b\hat{B}|
\]
where $\mathbf{v}_N^B$ is the velocity of ball center, $\omega$ is the ball angular velocity, and $\mathbf{j}_B$ is the unit vector from the fingertip to the ball center. When $\mathbf{v}_N^B$ is equal to $\mathbf{v}_N^H$ at release, the ball leaves the fingertip without slipping between the fingertip and the ball surface, and the ball angular velocity can be written as $\omega \cdot \mathbf{I} = \Psi_S + \Psi_E + \Psi_H + \Psi_B$ where $\Psi_B$ is the angle between the hand link and the line including the fingertip and the ball centre.

![Fig. 1. Geometry of shooting arm with a basketball](image1.png)

![Fig. 2. Sets of shoulder, elbow and wrist joints at release.](image2.png)

### 2.4. Joint angular displacement of shooting arm at release

Some previous studies measured joint angles of shooters’ shoulder, elbow and wrist at release for jump shots [2,3,4] and free throws [5]. Elliott [2] showed jump shot actions in which the angle of upper arm from the horizontal was about $51^\circ$, the angle between the hand and forearm was about $180^\circ$ and the angle of the forearm from the horizontal was about $80^\circ$ at release. Miller and Bartlett [3] showed stick figures of shooting motion in which the forearm angle from the horizontal was about $90^\circ$ and the angle between the forearm and hand was $173^\circ$ to $201^\circ$. Okazaki and Rodacki [4] showed the range of their measured upper arm angles from the horizontal was $28^\circ$ to $38^\circ$, the average angle between the upper arm and forearm was about $135^\circ$ and the average wrist angle was about $190^\circ$ at release. Yates and Holt [8] reported that at release successful shooters demonstrated a closer alignment of the upper arm with vertical than did lower percentage shooters for jump shooting. Miller and Bartlett [9] showed a typical shooter’s motion in which the ball centre, hand and forearm were in a vertical line at release for short-, medium- and long-range jump shots. For free throws, Tsarouchas et al. [5] measured the angles of the upper arm, forearm and hand from the horizontal, which were about $40^\circ$, $82^\circ$ and $115^\circ$ to $120^\circ$ at release, respectively.

We estimate possible sets of joint angles at release based on the previous studies for shooting ball position with short (horizontal distance from the hoop centre $l = 2$ m, vertical distance from the hoop centre and $h = 0.05$ m), medium ($l = 4$ m and $h = 0.15$ m) and long ($l = 7$ m and $h = 0.30$ m) ranges. Figure 2 shows an example of sets of shoulder, elbow and wrist joint angles with $L_U = 0.3$ m, $L_E = 0.3$ m and $L_H = 0.2$ m for long-range shots when the ball release position and the shoulder position are given. The range of shoulder joint angle from the horizontal plane is from $45^\circ$ (when the forearm and wrist links are vertical) to $59^\circ$ (when the elbow angle is close to $180^\circ$). The hand link is in a straight line including the ball centre at release.
3. Results and discussion

Angular velocity combinations of shoulder, elbow and wrist joints are plotted with each set shown in Fig. 2 to produce the optimal ball release speed, angle and backspin, 4.58 m/s, 47.5°, 2π rad/s for the short-, 6.62 m/s, 46.0°, 4π rad/s for the medium-, and 9.04 m/s, 46.0°, 6π rad/s for the long-range shots [7] in Figs. 3-5 when the shoulder velocity $N^S v$ is zero and there is no slipping between the fingertip and the ball surface at release. The reasonable conditions that $\Psi_S > 0$ (the vertical component of the velocity of the elbow is usually positive) and $\Psi_S + \Psi_E + \Psi_S < \Psi_S + \Psi_E < 0$ (the angular velocities of the forearm and hand are usually negative and the magnitude of angular velocity of the hand is larger than that of the forearm) are used for calculations. Figure 3 shows the sets of angular velocities of shoulder, elbow and wrist joints for the optimal release conditions in the shoulder-elbow-wrist angular velocity space. Figures 4(a) and (b) show the top and side views of the sets of angular velocities in Fig. 3. Each straight line is a function of $\Psi_S, \Psi_S + \Psi_E$, and $\Psi_S + \Psi_E + \Psi_W$. As the angular velocity of shoulder increases, the magnitude of angular velocity of elbow increases. When the angular velocity of elbow is close to zero, the shoulder angular velocity is small but large wrist joint angular velocity is needed. When the elbow angle is close to 180° ($\Psi_S = 59°$), large wrist angular velocity is also required.

Fig. 3. Combinations of angular velocities of upper arm, forearm and hand at release for short-, medium- and long-range shots.

Fig. 4. Relationships between joint angular velocities for short-, medium- and long-range shots. (a) forearm angular velocity vs. wrist angular velocity and (b) wrist angular velocity vs. shoulder angular velocity.
For $\Psi_S + \Psi_E = 90^\circ$ and $\Psi_W = 0$ (both the forearm and the hand are vertical) at release, the vertical component of the fingertip velocity is a function of $\Psi_S$, but neither a function of $\Psi_S + \Psi_E$ nor of $\Psi_S + \Psi_E + \Psi_W$. The shoulder angular velocities can be calculated as $\Psi_S = 15.9$, 22.4 and 30.7 rad/s for short-, medium- and long-range shots. Shoulder rotation contributes to the vertical component of ball velocity and is important to a good arch of the ball trajectory.

![Graph showing combinations of angular velocities of upper arm, forearm and hand at release for medium-range shots with 1Hz, 2Hz and 3Hz backspin.](image1)

Fig. 5. Combinations of angular velocities of upper arm, forearm and hand at release for medium-range shots with 1Hz, 2Hz and 3Hz backspin.

![Graphs showing relationships between joint angular velocities for medium-range shots with 1, 2 and 3Hz backspin.](image2)

(a) forearm angular velocity vs. wrist angular velocity and (b) wrist angular velocity vs. shoulder angular velocity.

Fig. 6. Relationships between joint angular velocities for medium-range shots with 1, 2 and 3Hz backspin.

Some previous studies [3,4,5] measured angular velocities of shooters’ shoulder, elbow and wrist joints at release. Miller and Bartlett [3] represented the angular velocities of arm joints to produce three different ball release speeds. As release speed of the ball increases, the magnitude of elbow angular velocity increases but the magnitude of wrist angular velocity decreases, and there is almost no difference in the magnitude of shoulder angular velocity for the three ball release speeds. In the results of reference [4], as shot distance increases, the magnitudes of angular velocities of shoulder and elbow increase but there is no correlation between shot distance and wrist angular velocity at release. Miller and Bartlett [9] ignored the wrist action because some previous studies pointed out that there was...
no significant difference for wrist joint angular velocities and displacements in the arm action of expert shooters. Tsarouchas et al. [5] concluded that the forearm action contributed to horizontal and vertical velocity of the ball for free throws. Satern [10] measured the velocities of shoulder and elbow for several varying shooting distances and pointed out that shooters increased shoulder and elbow angular velocities at release as shooting distance increases.

Figure 5 shows angular velocity combinations for medium-range shots with different backspins; $2\pi$, $4\pi$ and $6\pi$ rad/s. For $\Psi_S + \Psi_E = 90^\circ$ and $\Psi_W = 0$ (both the forearm and the hand are vertical) at release, shoulder rotation contributes to the vertical component of ball release velocity, and elbow and wrist rotations mostly affect backspin.

4. Conclusions

We have derived a two dimensional kinematic model of the shooter’s arm including a basketball for short-, medium- and long-range shots. The kinematic model estimates the angles and angular velocities of shoulder, elbow and wrist for a set of given ball release speed, angle and backspin. A lot of combinations of shoulder, elbow and wrist angular velocities exist for each release condition. When the forearm and hand are vertical at release, the shoulder rotation contributes to the vertical component of ball release velocity, and the elbow and wrist rotations are vital for backspin.

We have focused on the kinematics of shooting arm at release. Using previous studies and our results, we have found that shooters have wide range of angular velocities of arm joints that produce their desired ball release conditions. The related problem of what is the optimal combination of joint angular velocities for high percentage shots is another significant one. Dynamics including kinetic analysis of the shooting arm are needed to solve it and suggest the best strategy for basketball shooting.

References