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ORIGINAL ARTICLE

Complement and Isomorphism on Bipolar Fuzzy Graphs

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Abstract In this paper, we discuss some properties of the self complement and self weak complement bipolar fuzzy graphs, and get a sufficient condition for a bipolar fuzzy graph to be the self weak complement bipolar fuzzy graph. Also we investigate relations between operations union, join, and complement on bipolar fuzzy graphs.

Keywords Isomorphism · Complement · Self complement · Bipolar fuzzy graph

1. Introduction

Presently, science and technology is featured with complex processes and phenomena for which complete information is not always available. For such cases, mathematical models are developed to handle various types of systems containing elements of uncertainty. A large number of these models is based on an extension of the ordinary set theory, namely, fuzzy sets. Graph theory has numerous applications to problems in computer science, electrical engineering, system analysis, operations research, economics, networking routing, and transportation. The graph isomorphic transformations are reduced to redefinition of vertices and edges. This redefinition does not

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change properties of the graph determined by an adjacent and an incidence of its vertices and edges. Fuzzy independent sets, domination fuzzy sets, and fuzzy chromatic sets are invariants concerning the isomorphism transformations of the fuzzy graphs and fuzzy hyper graph and allow theirs structural analysis [7]. In 1975, Zadeh [27] introduced the notion of fuzzy sets as a method for representing uncertainty. Since then, the theory of fuzzy sets has become a vigorous area of research in different disciplines including logic, topology, algebra, analysis, information theory, artificial intelligence, operations research, neural networks and planning etc. [6, 10]. The fuzzy graphs theory as a generalization of Eulers graph theory was first introduced by Rosenfeld [14] in 1975. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs obtaining analogs of several graphs theoretical concepts. Later, Bhattacharya [5] gave some remarks on fuzzy graphs, and Mordeson and Peng [9] introduced some operation of fuzzy graphs. In 1994, Zhang [28, 29] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is $[-1, 1]$. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree $(0, 1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1, 0)$ of an element indicates that the element somewhat satisfies the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several directions. For instance, when we assess the position of an object in space, we may have positive information expressed as a set of possible places and negative information expressed as a set of impossible places. This corresponds to the idea that the union of positive and negative information does not cover the whole space.

In 2011, Akram [2] defined bipolar fuzzy graphs. Akram and Davvaz [3] investigated strong intuitionistic fuzzy graphs. Then Akram and Karunambigai [4] defined length, distance, eccentricity, radius and diameter of a bipolar fuzzy graph and introduced the concept of self centered bipolar fuzzy graphs. Akram and Dudek [1] defined interval-valued fuzzy graphs. Samanta and Pal defined fuzzy tolerance graphs [19], fuzzy threshold graphs [20], irregular bipolar fuzzy graphs [21], fuzzy k-competition graphs and p-competition fuzzy graphs [22] and some results on bipolar fuzzy sets and bipolar fuzzy intersection graphs [23]. Talebi and Rashmanlou [24] studied properties of isomorphism and complement on interval-valued fuzzy graphs. Likewise, they defined isomorphism on vague graphs [25]. Irregular interval-valued fuzzy graphs were studied by Pal and Rashmanlou [13]. Recently, Rashmanlou and Pal defined antipodal interval-valued fuzzy graphs [15], balanced interval-valued fuzzy graphs [16], some properties of highly irregular interval-valued fuzzy graphs [18] and isometry on interval-valued fuzzy graphs [17]. Nagoorgani and Malarvizhi [11, 12] investigated isomorphism properties on fuzzy graphs. Also they defined the self complementary fuzzy graphs. Bhutani [6] introduced the concept of weak isomorphism and isomorphism between fuzzy graphs.

In this paper, we defined the self complement and self weak complement bipolar fuzzy graphs with some properties of its discussed. We also study some properties of isomorphism and complement on bipolar fuzzy graphs.

2. Preliminaries

A fuzzy graph with V, a non-empty finite set as the underlying set is a pair $G = (\sigma, \mu)$ where $\sigma : V \to (0, 1)$ is a fuzzy subset of V, $\mu : V \times V \to (0, 1)$ is a symmetric fuzzy relation on the fuzzy subset σ such that $\mu(x, y) \leq \min(\sigma(x), \sigma(y))$ for all $x, y \in V$. A fuzzy relation μ is symmetric if $\mu(x, y) = \mu(y, x)$ for all $x, y \in V$. The underlying crisp graph of the fuzzy graph $G = (\sigma, \mu)$ is denoted as $G^* = (\sigma^*, \mu^*)$ where $\sigma^* = \{x \in V \mid \sigma(x) > 0\}$ and $\mu^* = \{(x, y) \in V \times V \mid \mu(x, y) > 0\}$. If $\mu(x, y) > 0$, then x and y are called neighbors. For simplicity, an edge (x, y) will be denoted by xy.

Let X be a non-empty set. A bipolar fuzzy set B in X is an object having the form $B = \{(x, \mu^p(x), \mu^N(x)) \mid x \in X\}$, where $\mu^p : X \to (0, 1)$ and $\mu^N : X \to (-1, 0)$ are mappings.

Let X be a non-empty set. Then we call a mapping $A = (\mu_A^P, \mu_A^N) : X{\times}X \to (-1, 1){\times}(-1, 1)$ a bipolar fuzzy relation on X such that $\mu_A^P(x, y) \in [0, 1]$ and $\mu_A^N(x, y) \in (-1, 0)$. Let $A = (\mu_A^P, \mu_A^N)$ be a bipolar fuzzy relation on a set X and $B = (\mu_B^P, \mu_B^N)$ be bipolar fuzzy set on X. Then, $A = (\mu_A^P, \mu_A^N)$ is called a bipolar fuzzy relation on $B = (\mu_B^P, \mu_B^N)$ if $\mu_A^P(x, y) \leq \min(\mu_B^P(x), \mu_B^P(y))$ and $\mu_A^N(x, y) \geq \max(\mu_B^N(x), \mu_B^N(y))$ for all $x, y \in X$.

Let $G = (\sigma, \mu)$ be a fuzzy graph on underlying set V. The complement of G is defined as $\overline{G} = (\sigma, \overline{\mu})$ where $\overline{\mu}(x, y) = \min(\sigma_{(x)}, \sigma_{(y)}) - \mu(x, y)$ for all $x, y \in V$. When $G = (\sigma, \mu)$ is a fuzzy graph, $\overline{G} = (\sigma, \overline{\mu})$ is also a fuzzy graph. Let $G = (\sigma, \mu)$ and $G' = (\sigma', \mu')$ be two fuzzy graphs with underlying sets V and V', respectively. A homomorphism from G to G' is a map $h : V \to V'$ which satisfies $\sigma(x) \leq \sigma'(h(x))$ for all $x \in V$ and $\mu(x, y) \leq \mu'(h(x), h(y))$ for all $x, y \in V$.

An isomorphism h from G to G' is a bijective homomorphism satisfing $\sigma(x) = \sigma'(h(x))$ for all $x \in V$ and $\mu(x, y) = \mu(h(x), h(y))$ for all $x, y \in V$.

A weak isomorphism h from G to G' is a bijective homomorphism that satisfies $\sigma(x) = \sigma'(h(x))$ for all $x \in V$.

A co-weak isomorphism h from G to G' is a bijective homomorphism that satisfies $\mu(x, y) = \mu'(h(x), h(y))$ for all $x, y \in V$. Series system models are developed and work out under the following notations.

Definition 1 [2] *A bipolar fuzzy graph with an underlying set V is defined to be a pair $G = (A, B)$ where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy set in $E \subseteq V^2$ where $V^2 = \{\{x, y\} | x, y \in V, x \neq y\}$ such that $\mu_B^P(x, y) \leq \min(\mu_A^P(x), \mu_A^P(y))$, $\mu_B^N(x, y) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all $\{x, y\} \in E$. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively. We use the notation xy for an element of E. Thus, $G = (A, B)$ is a bipolar fuzzy graph of $G^* = (V, E)$ if $\mu_B^P(xy) \leq \min(\mu_A^P(x), \mu_A^P(y))$, $\mu_B^N(xy) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all $xy \in E$.*

Definition 2 *The complement of a bipolar fuzzy graph $G = (A, B)$ of a graph $G^* = (V, E)$ is a bipolar fuzzy graph $\overline{G} = (\overline{A}, \overline{B})$ of $\overline{G^*} = (V, V^2)$, where $\overline{A} = A = (\mu_A^P, \mu_A^N)$*

and $\overline{B} = (\overline{\mu_B^P}, \overline{\mu_B^N})$ *is defined by* $\overline{\mu_B^P}(xy) = \min(\mu_A^P(x), \mu_A^P(y)) - \mu_B^P(xy)$, $\overline{\mu_B^N}(xy) = \max(\mu_A^N(x), \mu_A^N(y)) - \mu_B^N(xy)$ *for all* $x, y \in V$.

Throughout this paper, $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ are taken to be the bipolar fuzzy graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively.

Example 1 Consider a graph $G^* = (V, E)$ with $V = \{a, b, c, d\}$, $E = \{ab, ad, bd, bc\}$. Let $A = (\mu_A^P, \mu_A^N)$ be a bipolar fuzzy subset of V and let $B = (\mu_B^P, \mu_B^N)$ be a bipolar fuzzy subset of $E \subset V \times V$ defined by

Table 1: Weight of vertices.

	a	b	c	d
μ_A^P	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{5}$
μ_A^N	-1	-1	-1	-1

Table 2: Weight of edges.

	ab	ad	bd	bc
μ_B^P	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{4}$
μ_B^N	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$

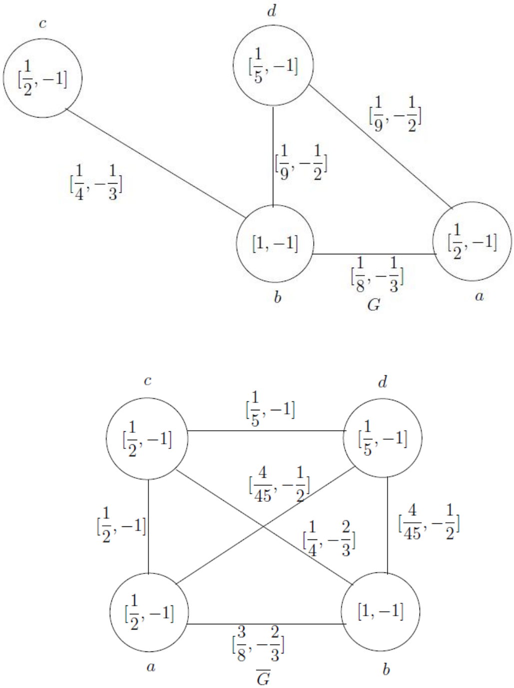


Fig. 1 Bipolar fuzzy graphs G and $\overline{G}$

It is not difficult to show that $\overline{G}$ is a bipolar fuzzy graph.

Definition 3 [2] *Let G_1 and G_2 be the bipolar fuzzy graphs. A homomorphism f from G_1 to G_2 is a mapping $f : V_1 \to V_2$ which satisfies the following conditions:*

(a) $\mu^P_{A_1}(x_1) \leq \mu^P_{A_2}(f(x_1)), \mu^N_{A_1}(x_1) \geq \mu^N_{A_2}(f(x_1))$ *for all* $x_1 \in V_1$.

(b) $\mu^P_{B_1}(x_1y_1) \leq \mu^P_{B_2}(f(x_1)f(y_1)), \mu^N_{B_1}(x_1y_1) \geq \mu^N_{B_2}(f(x_1)f(y_1))$ *for all* $x_1y_1 \in E_1$.

Definition 4 [2] *Let G_1 and G_2 be bipolar fuzzy graphs. An isomorphism f from G_1 to G_2 is a bijective mapping $f : V_1 \to V_2$ which satisfies the following conditions:*

(a) $\mu^P_{A_1}(x_1) = \mu^P_{A_2}(f(x_1)), \mu^N_{A_1}(x_1) = \mu^N_{A_2}(f(x_1))$ *for all* $x_1 \in V_1$.

(b) $\mu^P_{B_1}(x_1y_1) = \mu^P_{B_2}(f(x_1)f(y_1)), \mu^N_{B_1}(x_1y_1) = \mu^N_{B_2}(f(x_1)f(y_1))$ *for all* $x_1y_1 \in E_1$.

We denote $G_1 \cong G_2$ if there is an isomorphism from G_1 to G_2.

Definition 5 [2] *Let G_1 and G_2 be the bipolar fuzzy graphs. Then, a weak isomorphism f from G_1 to G_2 is a bijective mapping $f : V_1 \to V_2$ which satisfies the following conditions:*

(a) *f is homomorphism,*

(b) $\mu^P_{A_1}(x_1) = \mu^P_{A_2}(f(x_1)), \mu^N_{A_1}(x_1) = \mu^N_{A_2}(f(x_1))$ *for all* $x_1 \in V_1$.

Definition 6 [2] *Let G_1 and G_2 be the bipolar fuzzy graphs. A co-weak isomorphism f from G_1 to G_2 is a bijective mapping $f : V_1 \to V_2$ which satisfies,*

(a) *f is homomorphism,*

(b) $\mu^P_{B_1}(x_1y_1) = \mu^P_{B_2}(f(x_1)f(y_1)),\ \mu^N_{B_1}(x_1y_1) = \mu^N_{B_2}(f(x_1)f(y_1))$ *for all* $x_1y_1 \in E_1$.

Definition 7 [2] *The union $G_1 \cup G_2 = (A_1 \cup A_2, B_1 \cup B_2)$ of two bipolar fuzzy graphs G_1 and G_2 is defined as follows:*

$$\text{(A)}\ \begin{cases} (\mu^P_{A_1} \cup \mu^P_{A_2})(x) = \mu^P_{A_1}(x), & if\ x \in V_1, x \notin V_2, \\ (\mu^P_{A_1} \cup \mu^P_{A_2})(x) = \mu^P_{A_2}(x), & if\ x \in V_2, x \notin V_1, \\ (\mu^P_{A_1} \cup \mu^P_{A_2})(x) = \max(\mu^P_{A_1}(x), \mu^P_{A_2}(x)), & if\ x \in V_1 \cap V_2; \end{cases}$$

$$\text{(B)}\ \begin{cases} (\mu^N_{A_1} \cup \mu^N_{A_2})(x) = \mu^N_{A_1}(x), & if\ x \in V_1, x \notin V_2, \\ (\mu^N_{A_1} \cup \mu^N_{A_2})(x) = \mu^N_{A_2}(x), & if\ x \in V_2, x \notin V_1, \\ (\mu^N_{A_1} \cup \mu^N_{A_2})(x) = \min(\mu^N_{A_1}(x), \mu^N_{A_2}(x)), & if\ x \in V_1 \cap V_2; \end{cases}$$

$$\text{(C)}\ \begin{cases} (\mu^P_{B_1} \cup \mu^P_{B_2})(xy) = \mu^P_{B_1}(xy), & if\ xy \in E_1, xy \notin E_2, \\ (\mu^P_{B_1} \cup \mu^P_{B_2})(xy) = \mu^P_{B_2}(xy), & if\ xy \in E_2, xy \notin E_1, \\ (\mu^P_{B_1} \cup \mu^P_{B_2})(xy) = \max(\mu^P_{B_1}(xy), \mu^P_{B_2}(xy)), & if\ xy \in E_1 \cap E_2; \end{cases}$$

$$\text{(D)}\ \begin{cases} (\mu^N_{B_1} \cup \mu^N_{B_2})(xy) = \mu^N_{B_1}(xy), & if\ xy \in E_1, xy \notin E_2, \\ (\mu^N_{B_1} \cup \mu^N_{B_2})(xy) = \mu^P_{B_2}(xy), & if\ xy \in E_2, xy \notin E_1, \\ (\mu^N_{B_1} \cup \mu^N_{B_2})(xy) = \min(\mu^N_{B_1}(xy), \mu^N_{B_2}(xy)), & if\ xy \in E_1 \cap E_2. \end{cases}$$

Definition 8 [2] *The join $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$ of two bipolar fuzzy graphs G_1 and G_2 is defined as follows:*

$$\text{(A)} \quad \begin{cases} (\mu^P_{A_1} + \mu^P_{A_2})(x) = (\mu^P_{A_1} \cup \mu^P_{A_2})(x), & for\ all\ x \in V_1 \cup V_2, \\ (\mu^N_{A_1} + \mu^N_{A_2})(x) = (\mu^N_{A_1} \cup \mu^N_{A_2})(x); \end{cases}$$

$$\text{(B)} \quad \begin{cases} (\mu^P_{B_1} + \mu^P_{B_2})(xy) = (\mu^P_{B_1} \cup \mu^P_{B_2})(xy), \\ (\mu^N_{B_1} + \mu^N_{B_2})(xy) = (\mu^N_{B_1} \cup \mu^N_{B_2})(xy), & if\ x, y \in V_1 \cup V_2; \end{cases}$$

$$\text{(C)} \quad \begin{cases} (\mu^P_{B_1} + \mu^P_{B_2})(xy) = \min(\mu^P_{A_1}(x), \mu^P_{A_2}(y)), & if\ xy \in E', \\ (\mu^N_{B_1} + \mu^N_{B_2})(xy) = \max(\mu^N_{A_1}(x), \mu^N_{A_2}(y)); \end{cases}$$

where E' is the set of all edges joining the nodes of V_1 and V_2.

3. Self Complement and Self Weak Complement Bipolar Fuzzy Graphs

In 1965, Zadeh [24] first introduced fuzzy sets as a mathematical way of representing impreciseness or vagueness in everyday life.

Definition 9 *A bipolar fuzzy graph $G = (A, B)$ of a graph $G^* = (V, E)$ is said to be self weak complement if G is weak isomorphism with its complement $\overline{G}$, i.e., there exist a bijective homomorphism f from G to $\overline{G}$ such that for all $x, y \in V$,*

$$\mu^P_A(x) = \overline{\mu^P_A}(f(x)), \mu^N_A(x) = \overline{\mu^N_A}(f(x))$$

and

$$\mu^P_B(xy) \leq \overline{\mu^P_B}(f(x)f(y)), \mu^N_B(xy) \geq \overline{\mu^N_B}(f(x)f(y)).$$

Definition 10 *A bipolar fuzzy graph G is said to be self complement if $G \cong \overline{G}$.*

Example 2 Consider a graph $G^* = (V, E)$ such that $V = \{a, b, c\}, E = \{ab, bc\}$. Then a bipolar fuzzy graph $G = (A, B)$, where

$$A = \left\langle (\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}), (\frac{a}{-0.4}, \frac{b}{-0.5}, \frac{c}{-0.4}) \right\rangle,$$
$$B = \left\langle (\frac{ab}{0.1}, \frac{bc}{0.15}), (\frac{ab}{-0.2}, \frac{bc}{-0.2}) \right\rangle$$

is self weak complement. In fact, identity bijective mapping is an weak isomorphism from G to $\overline{G}$.

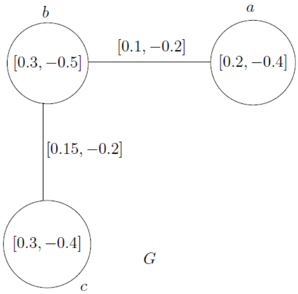


Fig. 2 Self weak complement bipolar fuzzy graph G

Example 3 Consider a graph $G^* = (V, E)$ with $V = \{a, b, c\}, E = \{ab, bc\}$. Then a bipolar fuzzy graph $G = (A, B)$, where

$$A = \Big\langle (\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}), (\frac{a}{-0.4}, \frac{b}{-0.5}, \frac{c}{-0.5}) \Big\rangle,$$
$$B = \Big\langle (\frac{ab}{0.2}, \frac{bc}{0.15}), (\frac{ab}{-0.4}, \frac{bc}{-0.25}) \Big\rangle$$

is self complementary. In fact, bijective mapping f from G to $\overline{G}$ defined by $a \to a$, $b \to c$, $c \to b$ is an isomorphism.

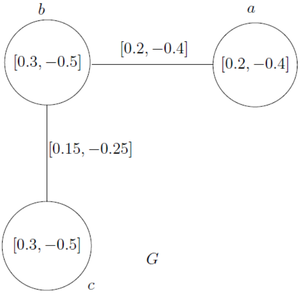


Fig. 3 Self complementary bipolar fuzzy graph G

Theorem 1 *Let $G = (A, B)$ be a self complement bipolar fuzzy graph of a graph $G^* = (V, E)$. Then*

$$\sum_{x \neq y} \mu_B^P(xy) = \frac{1}{2} \sum_{x \neq y} \min(\mu_A^P(x), \mu_A^P(y))$$

and

$$\sum_{x \neq y} \mu_B^N(xy) = \frac{1}{2} \sum_{x \neq y} \max(\mu_A^N(x), \mu_A^N(y)).$$

Proof Let $G = (A, B)$ be a self complement bipolar fuzzy graph of a graph $G^* = (V, E)$. Then, there exist a weak isomorphism g from G to $\overline{G}$ such that for every $x, y \in V$ we have

$$\mu_A^P(x) = \overline{\mu_A^P}(g(x)), \mu_A^N(x) = \overline{\mu_A^N}(g(x)) \text{ for all} x \in V$$

and

$$\mu_B^P(xy) = \overline{\mu_B^P}(g(x)g(y)), \mu_B^N(xy) = \overline{\mu_B^N}(g(x)g(y)) \text{for all } x, y \in V.$$

Now by definition of $\overline{G}$, for every $x, y \in V$, we have

$$\begin{cases} \overline{\mu_B^P}(g(x)g(y)) = \min(\mu_A^P(g(x)), \mu_A^P(g(y))) - \mu_B^P(g(x)g(y)), \\ \overline{\mu_B^N}(g(x)g(y)) = \max(\mu_A^N(g(x)), \mu_A^N(g(y))) - \mu_B^N(g(x)g(y)), \end{cases}$$

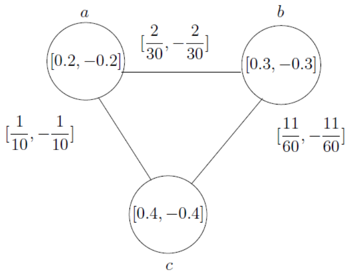
i.e., $$\begin{cases} \mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y)) - \mu_B^P(g(x)g(y)), \\ \mu_B^N(xy) = \max(\mu_A^N(x), \mu_A^N(y)) - \mu_B^N(g(x)g(y)), \end{cases}$$

i.e., $$\begin{cases} \sum_{x \neq y} \mu_B^P(xy) + \sum_{x \neq y} \mu_B^P(g(x)g(y)) = \sum_{x \neq y} \min(\mu_A^P(x), \mu_A^P(y)), \\ \sum_{x \neq y} \mu_B^N(xy) + \sum_{x \neq y} \mu_B^N(g(x)g(y)) = \sum_{x \neq y} \max(\mu_A^N(x), \mu_A^N(y)), \end{cases}$$

i.e., $$\begin{cases} 2 \sum_{x \neq y} \mu_B^P(xy) = \sum_{x \neq y} \min(\mu_A^P(x), \mu_A^P(y)), \\ 2 \sum_{x \neq y} \mu_B^N(xy) = \sum_{x \neq y} \max(\mu_A^N(x), \mu_A^N(y)), \end{cases}$$

$$\Rightarrow \begin{cases} \sum_{x \neq y} \mu_B^P(xy) = \frac{1}{2} \sum_{x \neq y} \min(\mu_A^P(x), \mu_A^P(y)), \\ \sum_{x \neq y} \mu_B^N(xy) = \frac{1}{2} \sum_{x \neq y} \max(\mu_A^N(x), \mu_A^N(y)). \end{cases}$$

Example 4 In this example, we show that the reverse of the above theorem in the general is not true. We suppose that $G = (A, B)$ is the bipolar fuzzy graph defined as follows.



G

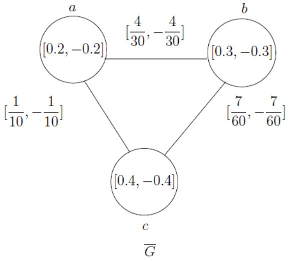


Fig. 4 Bipolar fuzzy graphs G and $\overline{G}$

By a routine computations, it is easy to see that G satisfies the conditions

$$\sum_{x\neq y}\mu_B^P(xy)=\frac{1}{2}\sum_{x\neq y}\min(\mu_A^P(x),\mu_A^P(y))$$

and

$$\sum_{x\neq y}\mu_B^N(xy)=\frac{1}{2}\sum_{x\neq y}\max(\mu_A^N(x),\mu_A^N(y)),$$

but G is not self complement because there is not an isomorphism between G and $\overline{G}$.

Theorem 2 *Let $G=(A,B)$ be a self weak complement bipolar fuzzy graph of a graph $G^*=(V,E)$. Then*

$$\sum_{x\neq y}\mu_B^P(xy)\leq\frac{1}{2}\sum_{x\neq y}\min(\mu_A^P(x),\mu_A^P(y))$$

and

$$\sum_{x\neq y}\mu_B^N(xy)\geq\frac{1}{2}\sum_{x\neq y}\max(\mu_A^N(x),\mu_A^N(y)).$$

Proof Let $G=(A,B)$ be a self weak complement bipolar fuzzy graph of a graph $G^*=(V,E)$. Then there exist a weak isomorphism h from G to $\overline{G}$ such that for all $x,y\in V$, we have

$$\mu_A^P(x)=\overline{\mu_A^P}(h(x))=\mu_A^P(h(x)),\mu_A^N(x)=\overline{\mu_A^N}(h(x))=\mu_A^N(h(x))$$

and

$$\mu_B^P(xy)\leq\overline{\mu_B^P}(h(x)h(y)),\mu_B^N(xy)\geq\overline{\mu_B^N}(h(x)h(y)).$$

Using the definition of complement in the above inequality, for all $x, y \in V$, we have

$$\mu_B^P(xy) \leq \overline{\mu_B^P}(h(x)h(y)) = \min(\mu_A^P(h(x)), \mu_A^P(h(y))) - \mu_B^P(h(x)h(y)),$$
$$\mu_B^N(xy) \geq \overline{\mu_B^N}(h(x)h(y)) = \max(\mu_A^N(h(x)), \mu_A^N(h(y))) - \mu_B^N(h(x)h(y)).$$

So,

$$\mu_B^P(xy) + \mu_B^P(h(x)h(y)) \leq \min(\mu_A^P(h(x)), \mu_A^P(h(y)))$$

and

$$\mu_B^N(xy) + \mu_B^N(h(x)h(y)) \geq \max(\mu_A^N(h(x)), \mu_A^N(h(y))).$$

Hence,

$$\sum_{x\neq y} \mu_B^P(xy) + \sum_{x\neq y} \mu_B^P(h(x)h(y)) \leq \sum_{x\neq y} \min(\mu_A^P(h(x)), \mu_A^P(h(y)))$$

and

$$\sum_{x\neq y} \mu_B^N(xy) + \sum_{x\neq y} \mu_B^N(h(x)h(y)) \geq \sum_{x\neq y} \max(\mu_A^N(h(x)), \mu_A^N(h(y))).$$

Thus

$$2\sum_{x\neq y} \mu_B^P(xy) \leq \sum_{x\neq y} \min(\mu_A^P(x), \mu_A^P(y))$$

and

$$2\sum_{x\neq y} \mu_B^N(xy) \geq \sum_{x\neq y} \max(\mu_A^N(x), \mu_A^N(x)(y)).$$

Therefore,

$$\sum_{x\neq y} \mu_B^P(xy) \leq \frac{1}{2}\sum_{x\neq y} \min(\mu_A^P(x), \mu_A^P(y))$$

and

$$\sum_{x\neq y} \mu_B^N(xy) \geq \frac{1}{2}\sum_{x\neq y} \max(\mu_A^N(x), \mu_A^N(y)).$$

Remark 1 Example 4 shows the converse of above theorem is not true.

Theorem 3 *Let $G = (A, B)$ be a bipolar fuzzy graph of a graph $G^* = (V, E)$. If for all $x, y \in V$, $\mu_B^P(xy) \leq \frac{1}{2}\min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) \geq \frac{1}{2}\max(\mu_A^N(x), \mu_A^N(y))$, then $G = (A, B)$ is a self weak complement bipolar fuzzy graph.*

Proof Consider the identity map $I : V \to V$, that for all $x \in V$, $\mu_A^P(x) = \mu_A^P(I(x))$ and $\mu_A^N(x) = \mu_A^N(I(x))$. By definition of $\overline{\mu_B}$, we have $\overline{\mu_B^P}(xy) = \min(\mu_A^P(x), \mu_A^P(y)) - \mu_B^P(xy)$ for all $x, y \in V$, and $\overline{\mu_B^N}(xy) = \max(\mu_A^N(x), \mu_A^N(y)) - \mu_B^N(xy)$. Hence, for every $x, y \in V$, we have

$$\overline{\mu_B^P}(xy) \geq \min(\mu_A^P(x), \mu_A^P(y)) - \frac{1}{2}\min(\mu_A^P(x), \mu_A^P(y))$$
$$= \frac{1}{2}\min(\mu_A^P(x), \mu_A^P(y)) \geq \mu_B^P(xy),$$

and

$$\overline{\mu_B^N}(xy) \leq \max(\mu_A^N(x), \mu_A^N(y)) - \frac{1}{2}\max(\mu_A^N(x), \mu_A^N(y))$$
$$= \frac{1}{2}\max(\mu_A^N(x), \mu_A^N(y)) \leq \mu_B^N(xy).$$

So,

$$\mu_B^P(xy) \leq \overline{\mu_B^P}(I(x)I(y)) \text{ for all } x, y \in V \text{ and } \mu_B^N(xy) \geq \overline{\mu_B^N}(I(x)I(y)).$$

4. Complement and Isomorphism on Bipolar Fuzzy Graphs

In this section, we discuss some of the properties of isomorphism and complement on bipolar fuzzy graphs.

Theorem 4 *Let* $G_1 = (A_1, B_1)$ *and* $G_2 = (A_2, B_2)$ *be two bipolar fuzzy graphs such that* $V_1 \cap V_2 = \emptyset$. *Then* $\overline{G_1 + G_2} \cong \overline{G_1} \cup \overline{G_2}$.

Proof We shall prove that the identity map is the required isomorphism.

Let $I : V_1 \cup V_2 \to V_1 \cup V_2$ be the identity map. We prove that for all $x, y \in V$

$$\left(\overline{\mu_{A_1}^P + \mu_{A_2}^P}\right)(x) = \left(\overline{\mu_{A_1^P}} \cup \overline{\mu_{A_2^P}}\right)(x), \left(\overline{\mu_{A_1}^N + \mu_{A_2}^N}\right)(x) = \left(\overline{\mu_{A_1^N}} \cup \overline{\mu_{A_2^N}}\right)(x)$$

and

$$\left(\overline{\mu_{B_1}^P + \mu_{B_2}^P}\right)(xy) = \left(\overline{\mu_{B_1^P}} \cup \overline{\mu_{B_2^P}}\right)(xy), \left(\overline{\mu_{B_1}^N + \mu_{B_2}^N}\right)(xy) = \left(\overline{\mu_{B_1^N}} \cup \overline{\mu_{B_2^N}}\right)(xy).$$

Let $x, y \in V$. Then

$$\left(\overline{\mu_{A_1^P} + \mu_{A_2^P}}\right)(x) = \left(\mu_{A_1}^P \cup \mu_{A_2}^P\right)(x) = \begin{cases} \mu_{A_1}^P(x), & \text{if } x \in V_1 \\ \mu_{A_2}^P(x), & \text{if } x \in V_2 \end{cases}$$
$$= \begin{cases} \overline{\mu_{A_1}^P}(x), & \text{if } x \in V_1 \\ \overline{\mu_{A_2}^P}(x), & \text{if } x \in V_2 \end{cases}$$
$$= \left(\overline{\mu_{A_1^P}} \cup \overline{\mu_{A_2^P}}\right)(x),$$

$$\left(\overline{\mu_{A_1^N} + \mu_{A_2^N}}\right)(x) = \left(\mu^N_{A_1} + \mu^N_{A_2}\right)(x) = \begin{cases} \mu^N_{A_1}(x), & \text{if } x \in V_1 \\ \mu^N_{A_2}(x), & \text{if } x \in V_2 \end{cases}$$
$$= \begin{cases} \overline{\mu^N_{A_1}}(x), & \text{if } x \in V_1 \\ \overline{\mu^N_{A_2}}(x), & \text{if } x \in V_2 \end{cases}$$
$$= \left(\overline{\mu_{A_1^N}} \cup \overline{\mu_{A_2^N}}\right)(x).$$

Also

$$\left(\overline{\mu_{B_1^P} + \mu_{B_2^P}}\right)(xy)$$
$$= \min\left((\mu^P_{A_1} + \mu^P_{A_2})(x), (\mu^P_{A_1} + \mu^P_{A_2})(y)\right) - \left(\mu^P_{B_1} + \mu^P_{B_2}\right)(xy)$$
$$= \begin{cases} \min\left((\mu^P_{A_1} \cup \mu^P_{A_2})(x), (\mu^P_{A_1} \cup \mu^P_{A_2})(y)\right) - \left(\mu^P_{B_1} \cup \mu^P_{B_2}\right)(xy), & \text{if } xy \in E_1 \cup E_2 \\ \min\left((\mu^P_{A_1} \cup \mu^P_{A_2})(x), (\mu^P_{A_1} \cup \mu^P_{A_2})(y)\right) - \min\left(\mu^P_{A_1}(x), \mu^P_{A_2}(y)\right), & \text{if } xy \in E' \end{cases}$$
$$= \begin{cases} \min\left(\mu^P_{A_1}(x), \mu^P_{A_2}(y)\right) - \mu^P_{B_1}(xy), & \text{if } xy \in E_1 \\ \min\left(\mu^P_{A_1}(x), \mu^P_{A_2}(y)\right) - \mu^P_{B_2}(xy), & \text{if } x \in E_2 \\ \min\left(\mu^P_{A_1}(x), \mu^P_{A_2}(y)\right) - min\left(\mu^P_{A_1}(x), \mu^P_{A_2}(y)\right), & \text{if } xy \in E' \end{cases}$$
$$= \begin{cases} \overline{\mu^P_{B_1}}(xy), & \text{if } xy \in E_1 \\ \overline{\mu^P_{B_2}}(xy), & \text{if } xy \in E_2 \\ 0, & \text{if} xy \in E' \end{cases} = \left(\overline{\mu^P_{B_1}} \cup \overline{\mu^P_{B_2}}\right)(xy).$$

$$\left(\overline{\mu_{B_1^N} + \mu_{B_2^N}}\right)(xy)$$
$$= \max\left((\mu_{A_1^N} + \mu_{A_2^N})(x), (\mu_{A_1^N} + \mu_{A_2^N})(y)\right) - (\mu_{B_1^N} + \mu_{B_2^N})(xy)$$
$$= \begin{cases} \max\left((\mu^N_{A_1} \cup \mu^N_{A_2})(x), (\mu^N_{A_1} \cup \mu^N_{A_2})(y)\right) - \left(\mu^N_{B_1} \cup \mu^N_{B_2}\right)(xy), & \text{if } xy \in E_1 \cup E_2 \\ \max\left((\mu^N_{A_1} \cup \mu^N_{A_2})(x), (\mu^N_{A_1} \cup \mu^N_{A_2})(y)\right) - \max\left(\mu^N_{A_1}(x), \mu^N_{A_2}(y)\right), & \text{if } xy \in E' \end{cases}$$
$$= \begin{cases} \max\left(\mu^N_{A_1}(x), \mu^N_{A_2}(y)\right) - \mu^N_{B_1}(xy), & \text{if } xy \in E_1 \\ \max\left(\mu^N_{A_2}(x), \mu^N_{A_2}(y)\right) - \mu^N_{B_2}(xy), & \text{if } xy \in E_2 \\ \max\left(\mu^N_{A_1}(x), \mu^N_{A_2}(y)\right) - \max\left(\mu^N_{A_1}(x), \mu^N_{A_2}(y)\right), & \text{if } xy \in E' \end{cases}$$
$$= \begin{cases} \overline{\mu^N_{B_1}}(xy), & \text{if } xy \in E_1 \\ \overline{\mu^N_{B_2}}(xy), & \text{if } xy \in E_2 \\ 0, & \text{if } xy \in E' \end{cases} = \left(\overline{\mu^N_{B_1}} \cup \overline{\mu^N_{B_2}}\right)(xy).$$

Theorem 5 *Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two bipolar fuzzy graphs such that $V_1 \cap V_2 = \emptyset$. Then $\overline{G_1 \cup G_2} \cong \overline{G_1} + \overline{G_2}$.*

Proof We shall prove that the identity map is the required isomorphism. For all $x, y \in V_1 \cup V_2$, then we have

$$\left(\overline{\mu_{A_1^P} \cup \mu_{A_2^P}}\right)(x) = \left(\mu^P_{A_1} \cup \mu^P_{A_2}\right)(x) = \begin{cases} \mu^P_{A_1}(x), & \text{if } x \in V_1 \\ \mu^P_{A_2}(x), & \text{if } x \in V_2 \end{cases}$$
$$= \left(\overline{\mu_{A_1^P}} \cup \overline{\mu_{A_2^P}}\right)(x) = \left(\overline{\mu_{A_1^P}} + \overline{\mu_{A_2^P}}\right)(x),$$

$$\left(\overline{\mu_{A_1^N} \cup \mu_{A_2^N}}\right)(x) = \left(\mu^N_{A_1} \cup \mu^N_{A_2}\right)(x) = \begin{cases} \mu^N_{A_1}(x), & \text{if } x \in V_1 \\ \mu^N_{A_2}(x), & \text{if } x \in V_2 \end{cases}$$
$$= \begin{cases} \overline{\mu^N_{A_1}}(x), & \text{if } x \in V_1 \\ \overline{\mu^N_{A_2}}(x), & \text{if } x \in V_2 \end{cases}$$
$$= \left(\overline{\mu_{A_1^N}} \cup \overline{\mu_{A_2^N}}\right)(x) = \left(\overline{\mu_{A_1^N}} + \overline{\mu_{A_2^P}}\right)(x),$$

$$\left(\overline{\mu_{B_1^P} \cup \mu_{B_2^P}}\right)(xy) = \min\left((\mu^P_{A_1} \cup \mu^P_{A_2})(x), (\mu^P_{A_1} \cup \mu^P_{A_2})(y)\right) - \left(\mu^P_{B_1} \cup \mu^P_{B_2}\right)(xy)$$
$$= \begin{cases} \min\left(\mu^P_{A_1}(x), \mu^P_{A_1}(y)\right) - \mu^P_{B_1}(xy), & \text{if } xy \in E_1 \\ \min\left(\mu^P_{A_2}(x), \mu^P_{A_2}(y)\right) - \mu^P_{B_2}(xy), & \text{if } xy \in E_2 \\ \min\left(\mu^P_{A_1}(x), \mu^P_{A_2}(y)\right) - 0, & \text{if } x \in V_1, y \in V_2 \end{cases}$$
$$= \begin{cases} \overline{\mu^P_{B_1}}(xy), & \text{if } xy \in E_1 \\ \overline{\mu^P_{B_2}}(xy), & \text{if } xy \in E_2 \\ \min\left(\mu^P_{A_1}(x), \mu^P_{A_2}(y)\right), & \text{if } x \in V_1, y \in V_2 \end{cases}$$
$$= \begin{cases} \left(\overline{\mu^P_{B_1}} \cup \overline{\mu^P_{B_2}}\right)(xy), & \text{if } xy \in E_1 \\ \left(\overline{\mu^P_{B_1}} \cup \overline{\mu^P_{B_2}}\right)(xy), & \text{if } xy \in E_2 \\ \min\left(\mu^P_{A_1}(x), \mu^P_{A_2}(y)\right), & \text{if } xy \in E' \end{cases}$$
$$= \left(\overline{\mu^P_{B_1}} + \overline{\mu^P_{B_2}}\right)(xy),$$

$$\left(\overline{\mu^N_{B_1} \cup \mu^N_{B_2}}\right)(xy) = \max\left((\mu^N_{A_1} \cup \mu^N_{A_2})(x), (\mu^N_{A_1} \cup \mu^N_{A_2})(y)\right) - \left(\mu^N_{B_1} \cup \mu^N_{B_2})\right)(xy)$$
$$= \begin{cases} \max\left(\mu^N_{A_1}(x), \mu^N_{A_1}(y)\right) - \mu^N_{B_1}(xy), & \text{if } xy \in E_1 \\ \max\left(\mu^N_{A_2}(x), \mu^N_{A_2}(y)\right) - \mu^N_{B_2}(xy), & \text{if } xy \in E_2 \\ \max\left(\mu^N_{A_1}(x), \mu^N_{A_2}(y)\right) - 0, & \text{if } x \in V_1, y \in V_2 \end{cases}$$
$$= \begin{cases} \overline{\mu^N_{B_1}}(xy), & \text{if } xy \in E_1 \\ \overline{\mu^N_{B_2}}(xy), & \text{if } xy \in E_2 \\ \max\left(\mu^N_{A_1}(x), \mu^N_{A_2}(y)\right), & \text{if } x \in V_1, y \in V_2 \end{cases}$$
$$= \begin{cases} \left(\overline{\mu^N_{B_1}} \cup \overline{\mu^N_{B_2}}\right)(xy), & \text{if } xy \in E_1 \\ \left(\overline{\mu^N_{B_1}} \cup \overline{\mu^N_{B_2}}\right)(xy), & \text{if } xy \in E_2 \\ \max\left(\mu^N_{A_1}(x), \mu^N_{A_1}(y)\right), & \text{if } xy \in E' \end{cases}$$
$$= \left(\overline{\mu^N_{B_1}} + \overline{\mu^N_{B_2}}\right)(xy).$$

Remark 2 If there is a weak isomorphism between two bipolar fuzzy graphs G_1 and G_2, there need not to be a weak isomorphism between $\overline{G_1}$ and $\overline{G_2}$.

Consider the following example.

Example 5 Let $V_1 = \{a, b, c, d\}$ and $V_2 = \{u, v, x, w\}$. Consider two bipolar fuzzy graphs $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ defined by

Table 3: Weight of vertices and edges of G_1.

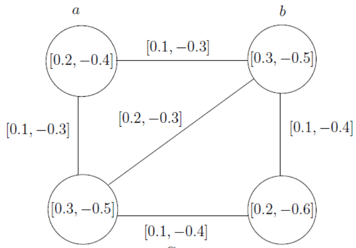
	a	b	c	d
$\mu^P_{A_1}$	0.2	0.3	0.3	0.2
$\mu^N_{A_1}$	−0.4	−0.5	−0.5	−0.6

	ab	ac	bc	bd	cd
$\mu^P_{B_1}$	0.1	0.1	0.2	0.1	0.1
$\mu^N_{B_1}$	−0.3	−0.3	−0.3	−0.4	−0.4

Table 4: Weight of vertices and edges of G_2.

	u	v	x	w
$\mu^P_{A_2}$	0.2	0.3	0.3	0.2
$\mu^N_{A_2}$	−0.4	−0.5	−0.5	−0.6

	uv	vx	ux	vw	xw
$\mu^P_{B_2}$	0.15	0.25	0.15	0.1	0.1
$\mu^N_{B_2}$	−0.4	−0.4	−0.4	−0.5	−0.5



G_1

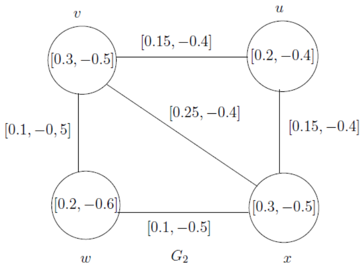


Fig. 5 Bipolar fuzzy graphs G_1 and G_2

It is easy to check that the mapping $g : V_1 \to V_2$, $g(a) = u$, $g(b) = v$, $g(c) = x$ and $g(d) = w$, is a weak isomorphism from G_1 to G_2. Now by definition of complement, we have

$$\overline{\mu^P_{B_1}}(ab) = 0.1, \overline{\mu^N_{B_1}}(ab) = -0.1, \overline{\mu^P_{B_1}}(ac) = 0.1, \overline{\mu^N_{B_1}}(ac) = -0.1, \overline{\mu^P_{B_1}}(bc) = 0.1,$$
$$\overline{\mu^N_{B_1}}(bc) = -0.2, \overline{\mu^P_{B_1}}(bd) = 0.1, \overline{\mu^N_{B_1}}(bd) = -0.1, \overline{\mu^P_{B_1}}(cd) = 0.1, \overline{\mu^N_{B_1}}(cd) = -0.1.$$
$$\overline{\mu^P_{B_2}}(uv) = 0.05, \overline{\mu^N_{B_2}}(uv) = 0, \overline{\mu^P_{B_2}}(vx) = 0.05, \overline{\mu^N_{B_2}}(vx) = -0.1, \overline{\mu^P_{B_2}}(ux) = 0.05$$
$$\overline{\mu^N_{B_2}}(ux) = 0, \overline{\mu^P_{B_2}}(vw) = 0.1, \overline{\mu^N_{B_2}}(vw) = 0, \overline{\mu^P_{B_2}}(xw) = 0.1,$$
$$\overline{\mu^N_{B_2}}(xw) = 0.$$

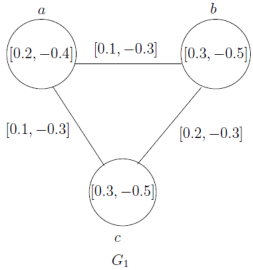
Hence, there is not a weak isomorphism between $\overline{G_1}$ and $\overline{G_2}$, because

$$\overline{\mu^P_{B_2}}(uv) = 0.05 < 0.1 = \overline{\mu^P_{B_1}}(xy) \quad \text{for all } x, y \in V_1.$$

Remark 3 If there is a co-weak isomorphism between bipolar fuzzy graphs G_1 and G_2, then there need not to be a co-weak isomorphism between $\overline{G_1}$ and $\overline{G_2}$.

The following example shows the above statement.

Example 6 Let $V_1 = \{a, b, c\}$ and $V_2 = \{u, v, w\}$. Consider two bipolar fuzzy graphs $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ as follows.



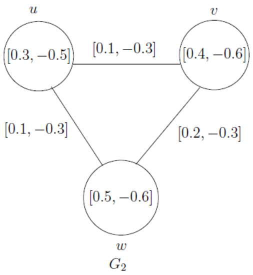


Fig. 6 Bipolar fuzzy graphs G_1 and G_2

There is a co-weak isomorphism $h : V_1 \to V_2$ with $h(a) = u$, $h(b) = v$, $h(c) = w$. Also we have

$$\overline{\mu^P_{B_1}}(ab) = 0.1, \overline{\mu^N_{B_1}}(ab) = -0.1, \overline{\mu^P_{B_1}}(ac) = 0.1, \overline{\mu^N_{B_1}}(ac) = -0.1,$$
$$\overline{\mu^P_{B_1}}(bc) = 0.1, \overline{\mu^N_{B_1}}(bc) = -0.2, \overline{\mu^P_{B_2}}(uv) = 0.2, \overline{\mu^N_{B_2}}(uv) = -0.2,$$
$$\overline{\mu^P_{B_2}}(uw) = 0.2, \overline{\mu^N_{B_2}}(uw) = -0.2, \overline{\mu^P_{B_2}}(vw) = 0.2, \overline{\mu^N_{B_2}}(vw) = -0.3.$$

So, $\overline{G_1}$ is not a co-weak isomorphism with $\overline{G_2}$ because

$$\overline{\mu^P_{B_2}}(uw) = 0.2 \neq 0.1 = \overline{\mu^P_{B_1}}(xy) \ \text{ for all } x, y \in V_1.$$

Theorem 6 *Let $G = (A, B)$ be a bipolar fuzzy graph of a graph $G^* = (V, E)$. Then the automorphism group G and $\overline{G}$ are identical.*

Proof We show that for any injective map $h : V \to V$, $h \in Aut(G)$ if and only if $h \in Aut(\overline{G})$, we have

$$\overline{\mu^P_A}(h(x)) = \mu^P_A(h(x)) = \mu^P_A(x) = \overline{\mu^P_A}(x) \ \text{ for all } x \in V,$$
$$\overline{\mu^N_A}(h(x)) = \mu^N_A(h(x)) = \mu^N_A(x) = \overline{\mu^N_A}(x) \ \text{ for all } x \in V.$$

Also, for all $x, y \in V$,

$$\begin{aligned}\overline{\mu^P_B}(h(x)h(y)) &= \overline{\mu^P_B}(xy)\\ &\Leftrightarrow \min\left(\mu_{A^P}(h(x)), \mu_{A^P}(h(y))\right) - \mu^P_B(h(x)h(y)) = \min\left(\mu_{A^P}(x), \mu_{A^P}(y)\right)\\ &\quad -\mu^P_B(xy)\\ &\Leftrightarrow \mu_{B^P}(h(x)h(y)) = \mu_{B^P}(xy),\end{aligned}$$

and

$$\begin{aligned}\overline{\mu_B^N}(h(x)h(y)) &= \overline{\mu_B^N}(xy)\\ &\Leftrightarrow \min\left(\mu_{A^N}(h(x)), \mu_{A^N}(h(y))\right) - \mu_B^N(h(x)h(y)) = \min\left(\mu_{A^N}(x), \mu_{A^N}(y)\right)\\ &\quad -\mu_B^N(xy)\\ &\Leftrightarrow \mu_{B^N}(h(x)h(y)) = \mu_{B^N}(xy).\end{aligned}$$

This complete the proof.

5. Conclusion

Graph theory is an extremely useful tool in solving the combinatorial problems in different areas including geometry, algebra, topology, optimization and computer science. In this paper, we discussed some properties of the self complement and self weak complement bipolar fuzzy graphs, and get a sufficient condition for a bipolar fuzzy graph to be the self-weak complement bipolar fuzzy graph. Also we investigated relations between operations union, join, and complement on bipolar fuzzy graphs.

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