Trajectory Analysis of Fuel Injection into Supersonic Cross Flow Based on Schlieren Method

YANG Hui, LI Feng*, SUN Baigang

School of Jet Propulsion, Beihang University, Beijing 100191, China

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Abstract

Trajectory analysis of fuel injection into supersonic cross flow is studied in this paper. A directly-connected wind tunnel is constructed to provide stable supersonic freestream. Based on the test rig, the schlieren system is established to reveal the fuel injection process visually. Subsequently, the method of quantitative schlieren is adopted to obtain data of both fuel/air interface and bow shock with the aid of Photoshop and Origin. Finally, the mechanism based on two influential factors of fuel injection angle and fuel injection driven pressure, is researched by vector analysis. A dimensionless model is deduced and analyzed. The curve fitting result is achieved. The relationship between the data and the two influential factors is established. The results provide not only the quantitative characteristics of the fuel injection in supersonic cross flow but also the valuable reference for the future computational simulation.

Keywords: fuel injection; schlieren; supersonic flow; injection driven pressure; injection angle; characteristic velocity

1. Introduction

Due to increasing interest in hypersonic propulsion, airbreathing scramjet engine research has become a trend for both military and commercial applications. To realize stable and efficient combustion in the engine, two issues should be addressed: what kind of fuels to select and how to conduct combustion.

Hydrocarbons and liquid hydrogen are two main candidates of fuels for the scramjet engine [1-5]. Compared with liquid hydrogen fuels, hydrocarbon fuels have several potential advantages, including higher densities, lower cost, and ambient storability for reduced operational costs. Moreover, the engine using hydrocarbon fuels is helpful during the vehicle design. The high density quality makes fuel tank more compact, which results in reducing dry mass, aerodynamic drag and increasing payload ratio. Meanwhile, hydrocarbon fuels also have some disadvantages, such as long ignition delays and limited cooling capability. However, there is a general consensus that hydrocarbon fuels can be used when flight speed is up to Mach 6-8.

Combustion is usually sustained in two ways: a) to capture indispensable ignition energy from the external heat source, such as an uninterrupted igniter; b) to absorb the energy from its own high temperature gas with the aid of optimized structure of combustor, which seems to be an internal “igniter”. In view of hostile work condition of combustor, the latter way is more popular in application. Several studies have been carried out to design and refine the structure, which focus on three fields: strut structure [6-9], ramp structure [10-14] and cavity structure [15-19].

According to the above discussion, it is necessary to discuss the fundamental process of hydrocarbon fuel injection into supersonic cross flow. The existing works may generally focus on the research where the injector’s axis is usually parallel to or perpendicular to airflow direction, namely, the injection angle is equal
to 0° or 90°. The research on other injection angles is an open field. To explore this field directly, an experimental investigation is carried out in this paper. A directly-connected wind tunnel, which reserves a current interface for different injection modules, is built to meet stable experimental condition. Several modules of different injection angles are manufactured and mounted on the interface. Schlieren, based on the equipment, is adopted to achieve the visual result of fuel injection in supersonic cross flow. Then the precise data of both fuel/air interface and bow shock, originated from the schlieren image, are obtained and analyzed. A novel analytic method based on vector analysis is developed to evaluate injection effect, which is influenced by fuel injection angle and driven pressure.

The rest of the paper is organized as follows: Section 2 introduces the equipment and diagnostics used in this work, and illustrates the process of obtaining data; Section 3 analyzes and evaluates the injection effect by using vector analysis; the last section is the conclusion.

2. Experimental Facility and Diagnostics

To study the fuel injection and atomization in supersonic cross flow, test rigs are constructed, which are composed of three units: a directly-connected wind tunnel, a system of fuel injection and a system of schlieren.

As illustrated in Fig. 1, the directly-connected wind tunnel consists of a setting chamber, a nozzle, an experimental channel and a diffuser. The compressed air whose experimental conditions are that the total temperature is 300 K and the total pressure is 0.5 MPa (absolute pressure) is admitted into the wind tunnel from the inlet. The setting chamber adjusts direction of airflow to the axis of nozzle, reduces the turbulence of airflow and improves the uniformity of flow distribution. According to the method of characteristics, the two-dimensional (2D) nozzle is designed to produce the \( M_\infty = 2 \) (where \( M_\infty \) is the freestream Mach number). To minimize the influence resulting from assembly, the supersonic nozzle and experimental channel are designed and manufactured as a mechanical unity, as illustrated in Fig. 2. The unity consists of two parts, a 150 mm length nozzle and a 450 mm length experimental channel, and their width is 80 mm. Then the heights of nozzle entry, nozzle throat and nozzle exit are 60 mm, 16.72 mm, and 30 mm respectively. But the upper and lower walls of the channel expand 0.5° at each side to dispel the influence of the boundary layer. The experimental channel has current interface to install various modules of fuel injection. In addition, a diffuser is designed to recover the static pressure of the airflow and enhance the efficiency of the wind tunnel.

The aviation kerosene RP-3, saved in tank, is driven into the injection module by high pressure nitrogen. The injector diameter is 1 mm in the experiment, while the injection angle will vary. A 31 mm × 24 mm × 5 mm slot sits in the front of the fuel injector to reduce fluctuation of driven pressure. And the fuel injection is finally regulated by solenoid valve. Figure 3 shows the fuel injection system.

Schlieren is applied widely to the experimental investigations of the aerodynamics [20-23]. The fundamental principle is that light transmitted through a transparent medium is refracted when it encounters a density gradient. This refracted light is partially blocked to create a schlieren image of the density gradients in the fluid. The system of schlieren is designed as a z-type configuration, as shown in Fig. 4. A 12 V metal halide light source is used in the setups. The slit sits at the focal point of the first lens, where the light is
collimated, transmitted through the test region and reflected to a focal point where the knife-edge is located. Then the flat reflecting mirrors are placed to save the space. Finally the image is photographed by the camera.

The fuel injection into supersonic flow field is a typical mode in the scramjet combustors. The mechanism is that: the fuel is injected into the supersonic flow field, which does not parallel to each other usually. Due to the obstruction from injection, a bundle of bow shocks is formed in the front of the injector. Simultaneously the separation and backwash occur in the boundary layer, which is propitious to combust. Figure 5 shows the basic structure of the flow field of the fuel injection. This paper explores these by the schlieren method. The schlieren images are processed and analyzed by Photoshop and Origin. Then the data of fuel/air interface and shock surface are obtained.

Figure 6 shows the schlieren image and data graphs, of which the injector diameter \( d \) is 1 mm, the injection angle \( \alpha \) is 45° and the injection driven pressure \( p_2 \) is 1.0 MPa. As shown in Fig. 6(a), the black opaque part in the red region (I) could be seen as the area filled with fuel, while the light and grey regions are regarded as the flow field which is only filled with air. The boundary between them is defined as the interface of fuel/air. Then the deepest line in the blue region (II) is regarded as the shock surface. As mentioned, a Cartesian coordinate system is established in the paper. Set the lower wall as the horizontal axis \((x)\) axis, the point of intersection of the horizontal axis and fuel/air interface as the origin of coordinate. Then create a line perpendicular to the horizontal axis through the point and set it as the vertical axis \((y)\) axis. Then the coordinate system setting is accomplished.

### 3. Results and Discussion

Several cases are studied in the experiment, of which parameters and values are shown in Table 1. What is more, all the pressure parameters are expressed as the absolute pressure. The conditions of the freestream keep constant \((M_{\infty}=2, \ d=1 \ mm, \ and \ total \ pressure \ of \ freestream \ p_0=0.5 \ MPa)\), while the differences focus on the fuel injection system. Then the fuel injection angle \( \alpha \) is defined as the angle between the axis of the injector and the wall of the module (see Fig. 2). Figure 7 shows the schlieren images visually.

<table>
<thead>
<tr>
<th>No.</th>
<th>( \alpha / (°) )</th>
<th>( p_2/\text{MPa} )</th>
<th>No.</th>
<th>( \alpha / (°) )</th>
<th>( p_2/\text{MPa} )</th>
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<tr>
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<td>13</td>
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<td>1.0</td>
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<td>9</td>
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<td>1.5</td>
<td>18</td>
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3.1. Vector analysis

It is expected for fuel injection to break through the boundary layer and permeate deeply in the mainstream. The penetration of the fuel is strongly coupled to the initial motivity of the fuel, which comes from both freestream and injection. To measure the two aspects, the characteristic velocities would be discussed in detail.

First, the characteristic velocity \( V_1 \), named inertial velocity, is applied to scaling the driven power from the freestream. The fuel injected into the flow field is influenced by the inertia of the freestream, which could be measured ideally by the dynamic pressure of the freestream. So \( V_1 \) is calculated by

\[
V_1 = \sqrt{\frac{2(p_1 - p_0)}{\rho_t}} \tag{1}
\]

where \( p_1 \) is the static pressure of the freestream, and \( \rho_t \) the fuel density. And \( p_1 \) is calculated by

\[
p_1 = p_0 \left(1 + \frac{k-1}{2} \frac{M_{max}^2}{k}\right)^{\frac{k}{k-1}} \tag{2}
\]

where \( k \) is specific heat ratio of gas.

Second, the characteristic velocity \( V_2 \), named injection velocity, is applied to representing the driven component of velocity, \( V_y \) vertical component of velocity, and \( \beta \) is the vector angle between \( V_x \) and \( V_y \).

Select \( V_1 \) as characteristic parameter and transform Eq. (4) to dimensionless format, expressed as

\[
I_x = \frac{V_x}{V_1} = 1 + q \cos \alpha \tag{4}
\]

or

\[
I_y = \frac{V_y}{V_1} = \sqrt{q} \sin \alpha \tag{5}
\]

where \( q = \frac{p_2 - p_1}{p_0 - p_t} \) is the driven pressure ratio. Respectively, \( I_x, I_y \) and \( I \), are dimensionless formats of \( V_x, V_y \) and \( V \). It could be deduced that:

\[
\begin{align*}
q & = 1, \text{ when } p_2 = 0.5 \text{ MPa} \\
q & = 2.146, \text{ when } p_2 = 1.0 \text{ MPa} \\
q & = 3.293, \text{ when } p_2 = 1.5 \text{ MPa}
\end{align*}
\tag{6}
\]

Hence, to research Eq. (5), more attention should be paid to the domain, of which \((\alpha, q) \in D = [(\alpha, q) | \alpha \in [0\degree, 180\degree], q \in (0, 4)]\), though the parameters are just some scattered points picked up from domain \( D \).
Figure 8 shows the 3D surface charts of \( I_x, I_y, I \) and \( \tan \beta \) as the function of \( \alpha \) and \( q \). Judging from both mathematic analysis and straightforward figures, it could be concluded as follows:

1) Treat \( q \) as fixed constant. In domain \( D \), \( I_x \) and \( I \) are the decreasing functions of \( \alpha \), while \( I_y \) is the bell-shaped function of \( \alpha \). In addition, \( \tan \beta \) is the bell-shaped function of \( \alpha \) when \( q \in (0,1) \), while there is a sudden change when \( q \in [1,4) \), namely, 

\[
\lim_{q \to 1} \tan \beta = 90^\circ.
\]

2) Treat \( \alpha \) as fixed constant. In domain \( D \), the function \( I_x \) is the increasing function of \( q \) when \( \alpha \in [0^\circ, 90^\circ] \), while \( I_x \) is the decreasing function of \( q \) when \( \alpha \in (90^\circ,180^\circ] \). And \( I_y \) is the increasing function of \( q \). Then \( I \) is the increasing function of \( q \) except some domain of \( \{ (\alpha, q) : \alpha \in (90^\circ,180^\circ], q \in (0,1) \} \). In addition, \( \tan \beta \) is the increasing function of \( q \), while there is also a sudden change mentioned as before.

3.2. Analysis of influential factors

The previous section focuses on the mathematic analysis and the results are shown in Fig. 8. In this section, the influential factors of the injection would be discussed.

1) Injection driven pressure

There are three scenarios of \( p_2 \) in the experiment: 0.5 MPa, 1.0 MPa and 1.5 MPa, and driven pressure ratio \( q \) is calculated shown in Eq. (6). Classified by different \( \alpha \), the data graphs of the fuel/air interface are shown in Fig. 9, while the data graphs of the shock surface are shown in Fig. 10. Although the distinctions of the shock in the cases are subtle, the shock surface is tilted upstream slightly, which means the intensity of the shock is enhanced respectively.
When the influences resulted from increasing $p_2$, namely increasing $q$, are discussed, it should be paid more attention to the change of Eq. (5), in which $\alpha$ is treated as fixed value. As mentioned previously, $I_x$ increases in step with $q$ if $\alpha$ is an acute angle, or else $I_x$ decreases. Then $I_y$ and $I$ increase in step with $q$.

Compared with the results shown in Figs. 9-10, it would be found that both the penetrability of the fuel and the intensity of the shock would be enhanced when $q$ is increased, which is similar to the changes of the magnitude of $I_y$. In addition, the magnitude of $I_y$ may have the same change between the two groups, for example, the results shown in Fig. 9(c) and Fig. 10(c) (named Group 1) and the ones shown in Fig. 9(d) and Fig. 10(d) (named Group 2). However, the distinctions among the graphs in Group 1 are more obvious than those in Group 2. These could be explained by the change of $I_x$. $I_x$ in Group 1 increases with the increase of $q$, whereas in Group 2 it shows a contrary trend. And its magnitude in both groups is positive, namely, the injection is downstream in the beginning. If both the penetrability of the fuel and the intensity of the shock only depend on the change of $I_y$, the flow fields in two groups should have been the same. Then, $I_x$ in Group 1 is larger than the one in Group 2 when $q$ is a fixed value (as shown in Eq. (6)). In Group 1, there will be a larger airflow resistance in the horizontal direction. Finally, the fuel in Group 1 is more difficult to flow downstream. The penetrability of the fuel is forced to be enhanced. The increased penetrability also leads to the enhanced shock. That is to say, larger $I_x$ is more helpful for the injection. The similar results could be obtained in other groups in Figs. 9-10.

To sum up, it could be concluded that the magnitude of $I_x$ is positively related to both the penetrability of the fuel and intensity of the shock, while larger $I_x$ is more helpful for the injection.

2) Injection angle

There are six scenarios of fuel injection angle in the
experiment: 30°, 45°, 60°, 120°, 135° and 150°, which could be divided into two groups: the acute group (30°, 45° and 60°) and the obtuse group (120°, 135° and 150°). Classified by $p_2$, the data graphs of the fuel/air interface are shown in Fig. 11, while the data graphs of the shock surface are shown in Fig. 12. The foreside of the graphs in the acute group is fuller than in the obtuse group, as shown in Fig. 11. When $p_2$ is fixed, both the penetration of the fuel and the intensity of the bow shock satisfy the following conditions: in the acute angle group, the result is the weakest when the angle is 30°, the result will be stronger when the angle is 45°, and the result is the strongest when the angle is 60°. Then in the obtuse group, the result is the weakest when the angle is 150°, the result will be stronger when the angle is 135°, and the result is the strongest when the angle is 120°.

When the influences resulted from changing $\alpha$ are discussed, the change of Eq. (5) should be paid more attention, in which $p_2$ is treated as fixed value. As mentioned previously, $I_x$ and $I$ would decrease with the increase of $\alpha$, while $I_y$ changes in step with $\sin \alpha$. Compared with the results shown in Figs. 11-12, it would be found that the changes of both the penetrability of the fuel and the intensity of the shock are similar to the change of $\sin \alpha$, that is to say, both of them would change in step with $I_y$. In addition, when $q$ and $\alpha$, drawn from Eq. (6) and Table 1, are set as known value, the other parameters in Eq. (5) could be
calculated. Figure 13 shows the change of $I_\alpha$, $I_\beta$ and $I$, and Table 2 shows the calculated results of $\beta$. It is easy to find that $\beta$ is acute angle in the six scenarios when $q=1$, but $\beta$ has obtuse value when $q=2.146$ and $q=3.293$. The similarity of the latter two could decide the similar relations in Figs. 11-12.

![Graph](image)

**Fig. 13**  Calculation graphs of $I_\alpha$, $I_\beta$ and $I$.

Table 2  Calculated results of $\beta$

<table>
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<tr>
<th>$\alpha$ (°)</th>
<th>$\beta$ (°)</th>
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<tr>
<td></td>
<td>$q=1$</td>
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<tr>
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</tr>
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<td>135</td>
<td>67.5</td>
</tr>
<tr>
<td>150</td>
<td>75</td>
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</table>

3.3. Results of curve fitting

In this section, some quantitative results could be expected. Several parameters would be utilized.

First, the variation of parameter $\gamma$ with parameter $x$ seems to be exponential, as shown in Fig. 9 and Fig. 11. To broaden the applicability, the parameters should be dimensionless. Set the injector diameter $d$ as the characteristic length and transform the data $(x, \gamma)$ to the non-dimensional data $(, )$. Then it could be expressed as

$$\gamma = a \alpha^b \quad a > 0, 0 < b < 1$$  (7)

where $a$ and $b$ are undetermined coefficients.

Second, the two influential factors could make some difference between each cases. To represent these effects, there are two schemes, ($I$, $\beta$) and ($\alpha$, $q$). The later one is selected in this paper, though the former one depends on the vector analysis. That is because the two parameters in the later one are independent from each other, and directly related to the two influential factors, which seems to be convenient for practical use. In addition, it should be assumed that the two parameters have an effect on the coefficient $a$ (as shown in Eq. (7)), expressed as

$$a \propto (\cos \alpha)^c \cdot (\sin \alpha)^\gamma \cdot q^f$$  (8)

Finally, the result of curve fitting would be obtained, expressed as

$$\overline{\gamma} = 1.702(e^{\cos \alpha})^{-0.117} \cdot (\sin \alpha)^{1.807} \cdot q^{0.553} \cdot \frac{x}{d}^{0.296}$$

or

$$\frac{\gamma}{d} = 1.702(e^{\cos \alpha})^{-0.117} \left(\frac{p_2 - p_1}{p_0 - p_1}\right)^{0.553} \left(\frac{x}{d}\right)^{0.296}$$  (9)

4. Conclusions

The paper focuses on the trajectory analysis of fuel injection in supersonic cross flow. With the aid of the test rigs and related software, several schlieren images are obtained and analyzed by using vector analysis. All the facilities and experience are helpful for the next exploration. And two influential factors, fuel injection angle and driven pressure, are discussed in detail. The research establishes the quantitative relationship between the injection performance and the two factors, which is also valuable for the future computational simulation. Future work will address the quantitative relationship between the fuel penetration and the intensity of the bow shock.

References


Biographies:

YANG Hui is a Ph.D. candidate in Beihang University. His main research interest is scramjet design and optimization. E-mail: yanghui_tj1983@yahoo.com.cn

LI Feng is a professor and Ph.D. supervisor in Beihang University. His main research interest is hypersonic vehicle design and flight control. E-mail: lifeng1966@263.net