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Quantum cooling evaporation process in regular black holes

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Abstract

We investigate a universal behavior of thermodynamics and evaporation process for the regular black holes. We observe an important point where the temperature is maximum, the heat capacity is changed from negative infinity to positive infinity, and the free energy is minimum. Furthermore, this point separates the evaporation process into the early stage with negative heat capacity and the late stage with positive heat capacity. The latter represents the quantum cooling evaporation process. As a result, the whole evaporation process could be regarded as the inverse Hawking–Page phase transition.

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Keywords: Regular black holes; Thermodynamics; Evaporation; Hawking–Page phase transition

1. Introduction

Hawking's semiclassical analysis of the black hole radiation suggests that most information about initial states is shielded behind the event horizon and will not back to the asymptotic region far from the evaporating black hole [1]. This means that the unitarity is violated by an evaporating black hole. However, this conclusion has been debated by many authors for three decades [2–4]. It is closely related to a long standing puzzle of the information loss paradox, which states the question of whether the formation and subsequent evaporation of a black hole is unitary. One of the most urgent problems in black hole physics is to resolve the unitarity issue. In this direction a complete description of black hole evaporation is an important issue. In order to determine the final state of evaporation process, a more precise treatment including quantum gravity effects and backreaction is generally required. At present, two leading candidates for quantum gravity are the string theory and the loop quantum gravity. Interestingly, the semiclassical analysis of the loop quantum black hole provides a regular black hole (RBH) without singularity in contrast to the classical one [5]. Its minimum size r_c is at Planck scale $\ell_{\rm Pl}$. On the other hand, in the continuing search for quantum gravity, the black hole thermodynamics may be related to a future experimental result at the LHC [6].

RBHs have been considered, dating back to Bardeen [7], for avoiding the curvature singularity beyond the event horizon in black hole physics [8]. Their causal structures are similar to the Reissner–Nordström (RN) black hole with the singularity replaced by de Sitter space–time with curvature radius $r_0 = \sqrt{3/\Lambda}$ [11]. Recently, Hayward has discussed the formation and evaporation process of a RBH with minimum size l [12], which can be identified with the minimal length induced from the string theory [13]. A more rigorous treatment of the evaporation process was carried out for the renormalization group (RG) improved black hole with minimum size $r_{\rm cr} = \sqrt{\tilde{\omega} G}$ [14,15]. The noncommutativity also provides another RBH with minimum scale $\sqrt{\theta}$: noncommutative black hole [16]. Very recently, we have investigated thermodynamics and evaporation process of

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the noncommutative black hole [17]. The RN black hole with charge Q also belongs to the RBH [18], even though it has a timelike singularity [19]. It turned out that the final state of the evaporation process for all RBHs is a cold, Planck size remnant of the extremal black hole. The connection between their minimum sizes is given by $r_c \sim r_0 \sim l \sim r_{\rm cr} \sim \sqrt{\theta} \sim Q \sim \ell_{\rm Pl}$.

It is very important to study the terminal phase of black hole evaporation. In the semiclassical study of the Schwarzschild black hole, the temperature ($T_H \propto 1/m$) and the luminosity ($L_{\rm Sch} \propto 1/m^2$) diverge as m approaches zero. This means that the semiclassical approach breaks down for very light holes. Furthermore, one has to take into account the backreaction. It was shown that the effect of quantum gravity could cure this pathological short distance behavior [20].

In this Letter, we first study universal thermodynamic properties of RBHs by analyzing the minimal model proposed by Hayward [12] and then investigate its evaporation process. We wish to remind the reader that the RBH is closely related to effects of quantum gravity.

Hawking temperature drops to zero at $m = m_{\rm crit}$, and the temperature-mass diagram has a maximum between $m_{\rm crit}$ and $m \to \infty$, where a specific heat is broken and changes sign testifying to a second-order phase transition in the course of Hawking evaporation [9] (and suggesting symmetry restoration in the origin [10]).

Especially, we observe an important point at $r_+ = r_m$ where the temperature is maximum, the heat capacity is changed from negative infinity to positive infinity, and the free energy is minimum. This point separates the whole evaporation process into the early stage with negative heat capacity and the late stage with positive heat capacity. The latter is described by the quantum cooling evaporation process (QCEP) which is a necessary step to reach extremal black hole. For the QCEP, the temperature decreases near Planck scale as the mass of black hole decreases, while for the early evaporation process, the temperature increases as the mass of black hole decreases. It is important to note that we do not need to take into account the backreaction for RBHs due to the QCEP.

We could understand the thermodynamic process for RBHs from the analogy of the Hawking–Page (HP) phase transition in the AdS black hole [21,22]. The relevant thermodynamic quantities are temperature $T_{\rm SAdS}$, heat capacity $C_{\rm SAdS}$, free energy $F_{\rm SAdS}$ and, off-shell free energy $F^{\rm off}_{\rm SAdS}$.[1] In the HP transition, one generally starts with thermal radiation in AdS space. A small black hole appears with negative heat capacity. The heat capacity changes from negative infinity to positive infinity at the minimum temperature T_0. Finally, the large black hole with positive heat capacity comes out as a stable object. There is a change of the dominance at the critical temperature T_1: from thermal radiation to black hole.

In contrast to the HP case, we start with the large unstable black hole with negative heat capacity for RBHs. The heat capacity changes from negative infinity to positive infinity at the maximum temperature. Then, the small black hole with positive heat capacity comes out. There is a change of the dominance at the critical temperature near $T = 0$: from a large black hole to a different, extremal black hole. Consequently, we regard the evaporation process of RBHs as the inverse HP transition because this is the process from initial (unstable) large black hole to final (stable) extremal black hole. We note that the QCEP plays a crucial role in the inverse HP transition. However, it takes an infinite time to reach the final remnant of extremal black hole using the quantum-corrected Vaidya metric.

2. Thermodynamics of regular black holes

It was shown that in order to obtain a RBH, we need to introduce an anisotropic fluid whose energy–momentum tensor is given by $T^{\mu}{}_{\nu} = {\rm diag}[-\rho, p_r, p_\perp, p_\perp]$ with energy density ρ, radial pressure p_r, and tangential pressure $p_\perp$. For simplicity, we study the minimal model [12] provided by the energy–momentum tensor

$$\rho = \frac{3l^2m^2}{2\pi(r^3+2l^2m)^2} = -p_r, \qquad p_\perp = \frac{3l^2m^2(r^3-l^2m)}{\pi(r^3+2l^2m)^3} \tag{1}$$

with the Planck units of $c = \hbar = G = \ell_{\rm Pl} = 1$. Solving the Einstein equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$ leads to the solution

$$ds^2_{\rm RBH} \equiv g_{\mu\nu}\,dx^\mu\,dx^\nu = -F(r)\,dt^2 + F(r)^{-1}\,dr^2 + r^2\,d\Omega_2^2. \tag{2}$$

The metric function $F(r)$ is given by

$$F(r) = 1 - \frac{2mr^2}{r^3+2l^2m}, \tag{3}$$

where l denotes the curvature radius of de Sitter space–time near the center and $m = 4\pi\int_0^\infty \rho(r)r^2\,dr$ represents the Arnowitt–Deser–Misner mass. We have de Sitter space–time $F(r) \simeq 1 - r^2/l^2$ as $r \to 0$, while an asymptotically Schwarzschild space–time $F(r) \simeq 1 - 2m/r$ appears as $r \to \infty$. Hence, ρ connects the de Sitter vacuum in the origin with the Minkowski vacuum at infinity.

From the condition of horizon $F = 0$, we obtain the horizon masses $m_\pm = r_\pm^3/2(r_\pm^2 - l^2)$. Here we find the minimum mass $m_* = 3\sqrt{3}l/4$ at $r_* = \sqrt{3}l$. For definiteness, we consider three different types: (i) For $m > m_*$, two distinct horizons appear with the inner cosmological horizon $r = r_-$ ($l < r_- \leqslant r_*$) and the outer event horizon $r = r_+$ ($r_* \leqslant r_+ < \infty$). They are analytically derived by $r_+ = \frac{m}{3}(2 + 4\cos\frac{\alpha}{3})$, $r_- = \frac{m}{3}(2 + 4\cos(\frac{\alpha}{3} - \frac{2\pi}{3}))$ where $\cos\alpha = 1 - \frac{2m_*^2}{m^2}$ with $\frac{2m_*}{m} < \alpha \leqslant \pi$. In particular, for $m \gg m_*(\alpha \to \frac{2m_*}{m} \simeq 0)$, the outer horizon is located at $r_+ \simeq 2m$, while the inner horizon is at $r_- \simeq l$. (ii) In case of

[1] Their explicit forms are $T_{\rm SAdS} = \frac{1}{4\pi}(\frac{3r_+}{\ell^2} + \frac{1}{r_+})$, $C_{\rm SAdS} = 2\pi r_+^2 \times \frac{(3r_+^2+\ell^2)^2}{(3r_+^2-\ell^2)^2}$, $F_{\rm SAdS} = \frac{r_+}{4}(1 - \frac{r_+^2}{\ell^2})$, $F^{\rm off}_{\rm SAdS} = \frac{r_+}{2}(1 + \frac{r_+^2}{\ell^2}) - \pi r_+^2 T$ with the curvature radius ℓ of AdS_4 spacetime. Here we have $r_0 = \ell/\sqrt{3}$ where the heat capacity blows up and the temperature has the minimum value $T_0 = \frac{\sqrt{3}}{2\pi\ell}$ and $r_1 = \ell$ where the free energy is zero and the temperature has the critical value $T_1 = \frac{1}{\pi\ell}$. For numerical computations, we choose $\ell = 10$.

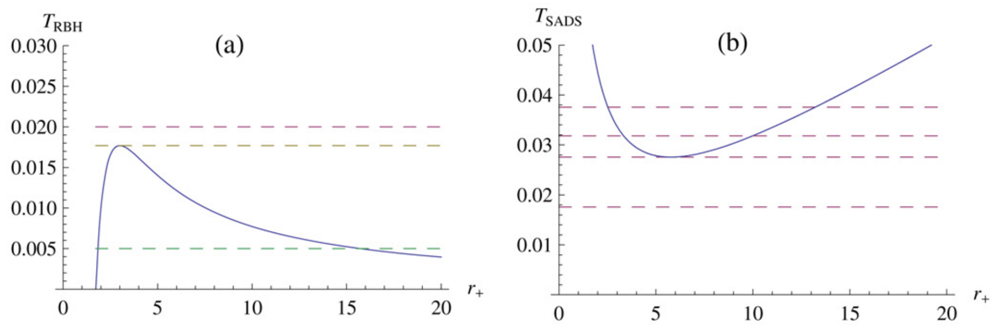

Fig. 1. (a) The solid line represents temperature $T_{\rm RBH}$ with the maximum point at $r_+ = r_m$ and minimum point at $r_+ = r_*$. The near-horizon region where the QCEP takes place is $r_* < r_+ < r_m$. Three horizontal dashed lines denote the temperature $T = 0.02$, T_m and 0.005 from the top to the bottom. (b) Temperature for Schwarzschild–AdS black hole with the minimum point at $r_+ = r_0$.

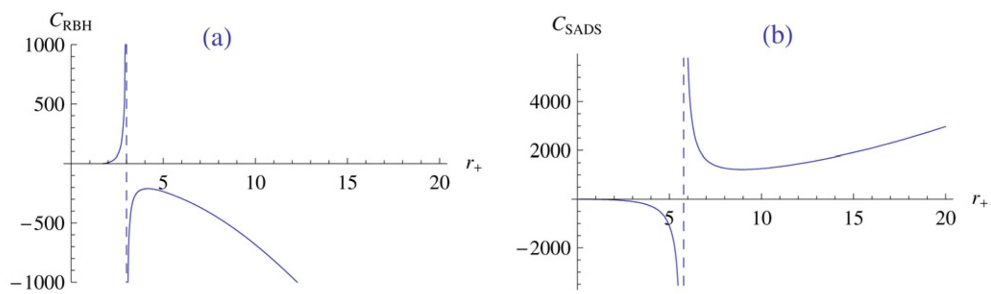

Fig. 2. (a) The solid line represents heat capacity $C_{\rm RBH}$ as a function of the black hole radius r_+. The QCEP takes place in the near-horizon region of $r_* < r_+ < r_m (C \geqslant 0)$. (b) The heat capacity of Schwarzschild–AdS black hole is negative for $r_+ < r_0$ and positive value for $r_+ > r_0$.

$m = m_*(\alpha = \pi)$, one has a degenerate horizon at $r = r_*$, which corresponds to the extremal black hole. (iii) For $m < m_*$, there is no horizon.

The black hole temperature can be calculated to be

$$T_{\rm RBH}(r_+) = \frac{1}{4\pi}\left[\frac{dF}{dr}\right]_{r=r_+} = \frac{1}{4\pi r_+}\frac{r_+^2 - r_*^2}{r_+^2} \tag{4}$$

with a fixed core radius $l = 1$. For $r_+^2 \gg 1$, one recovers the Hawking temperature $T_H = 1/4\pi r_+$ of the Schwarzschild black hole. Therefore, at the early stage of the Hawking radiation, the black hole temperature increases as the horizon radius decreases. It is important to investigate what happens as $r_+ \to 0$. In the Schwarzschild case, T_H diverges and this puts the limit on the validity of the evaporation process via the Hawking radiation. Against this scenario, the temperature $T_{\rm RBH}$ includes quantum effects, which are relevant at short distance comparable to the Planck scale of $r_+ \simeq 1$ [14,16]. As is shown in Fig. 1, the temperature of the RBH grows until it reaches to the maximum value $T_m = 0.017$ at $r_+ = r_m = 3(m = m_m = 1.68)$ and then falls down to zero at $r_+ = r_* = \sqrt{3}(m = m_*)$ which the extremal black hole appears with $T_* = 0$. As a result, the evaporation process is split into the right branch of $r_m < r_+ < \infty$ called the early stage of evaporation and the left branch of $r_* < r_+ < r_m$ called the QCEP. In the region of $r < r_*$, there is no black hole for $m < m_*$ and thus the temperature cannot be defined. For $m > m_*$, we have the inner horizon at $r = r_-$ inside the outer horizon but an observer at infinity does not recognize the presence of this cosmological horizon. Hence, we regard this region as the forbidden region.

The entropy $S_{\rm RBH} = \int_{r_*}^{r_+}(m'/T_{\rm RBH})\,dr$ of the RBH can be obtained from the first-law of thermodynamics $dm = T_{\rm RBH}\,dS_{\rm RBH}$ as

$$S_{\rm RBH}(r_+) = \frac{A}{4} + \frac{\pi}{50}\Bigg[-\frac{10}{2r_+^2 - 1} + 108\ln\left(r_+^2 - r_*^2\right) + 17\ln\left(2r_+^2 - 1\right)\Bigg] \tag{5}$$

with the area of the event horizon $A = 4\pi r_+^2$. We have negative infinity-entropy for the extremal black hole at $r_+ = r_*$ due to the third term. Hence we cannot find logarithmic correction to the extremal black hole. On the other hand, we have the area-law behavior of $S_{\rm BH} \simeq \pi r_+^2$ for $r_+ \gg 1$.

In order to check the thermal stability of the RBH, we have to know the heat capacity [14]. Its heat capacity $C_{\rm RBH} = dm/dT_{\rm RBH}$ is given by

$$C_{\rm RBH}(r_+) = -2\pi r_+^2 \frac{r_+^4(r_+^2 - r_*^2)}{(r_+^2 - 1)^2(r_+^2 - r_m^2)} \tag{6}$$

and its variation is plotted in Fig. 2. Here, we find the near-horizon region of $C_{\rm RBH} > 0$, where the QCEP takes place. This means that the RBH could be thermodynamically stable in the range of $r_* < r_+ < r_m$. The heat capacity becomes singular at $r_+ = r_m$, which corresponds to the maximum temperature $T = T_m$. We also observe that a thermodynamically unstable region ($C_{\rm RBH} < 0$) appears for $r_+ > r_m$. We note that in the Hawking regime of $r_+ \gg 1$, $C_{\rm RBH}$ is consistent with the specific heat $C_{\rm RBH} \simeq -2\pi r_+^2$ of the Schwarzschild black hole. Also we have $C_{\rm RBH}|_{r_+ = r_*} = 0$ for the extremal black hole.

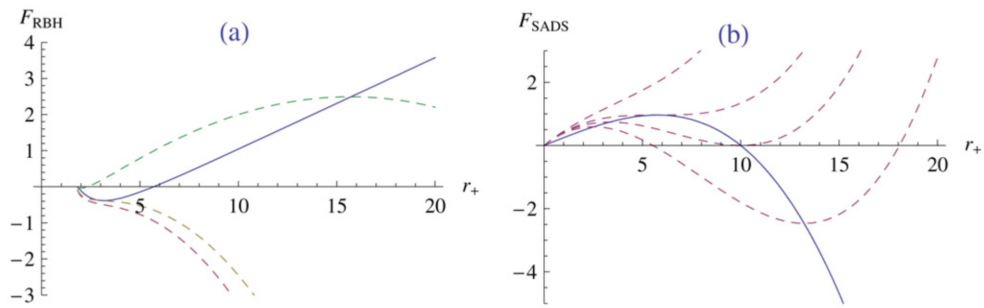

Fig. 3. (a) The solid line represents plot of the free energy F_{RBH} as a function of r_+. The dashed curves denote $F^{\text{off}}_{\text{RBH}}(r_+, T = 0.005)$, $F^{\text{off}}_{\text{RBH}}(r_+, T = T_m)$, $F^{\text{off}}_{\text{RBH}}(r_+, T = 0.02)$ from top to bottom. $r_i = 15.72$ and $r_* = 1.84$ represent the starting point and the ending point for an evaporation process at $T = 0.005$, respectively. (b) The free energy and off-shell free energy for are shown for the Schwarzschild–AdS black hole. We find the HP transition along the bottom dashed curve: starting with thermal radiation at $r_+ = 0$ and ending with large stable black hole at $r_+ = r_s$.

Now, we are in a position to discuss a possible phase transition. For this purpose, we introduce the on-shell free energy as

$$F_{\text{RBH}}(r_+) = m(r_+) - m_* - T_{\text{RBH}}(r_+)S_{\text{RBH}}(r_+), \tag{7}$$

where for fixed $l = 1$, we have to use the extremal black hole as background [23]. Its graph is shown in Fig. 3. Interestingly, the free energy has the minimum value at $r_+ = r_m$. The QCEP takes place for $r_* < r_+ < r_m$. For $r_+ \gg 1$, one recovers $F_{\text{RBH}} \simeq r_+/4$ for the Schwarzschild black hole. Further, one needs to know the off-shell free-energy

$$F^{\text{off}}_{\text{RBH}}(r_+, T) = m(r_+) - m_* - T S_{\text{RBH}}(r_+) \tag{8}$$

with the temperature T of the heat reservoir.

Finally, let us describe the inverse HP phase transition, which is closely related to the evaporation process of the RBH. For $T = 0.02 > T_m$, there is no meeting point between $F^{\text{on}}_{\text{RBH}}$ and $F^{\text{off}}_{\text{RBH}}$ except $r_+ = r_*$. For $T = T_m$, we find one meeting point (the minimum point) at $r_+ = r_m$. For $T = 0.005 < T_m$, we find two meeting points: unstable large black hole at $r_+ = r_i$ and extremal black hole at $r_+ = r_*$. Actually, there is a change of dominance at the critical temperature $T = 0.005$: from unstable large black hole to stable extremal back hole. Explicitly, the off-shell (non-equilibrium) process starts with $r_+ = r_i = 15.72$ and ends at $r_+ = r_e = 1.84$. We observe that $r_i \to \infty$ and $r_e \to r_*$, as $T \to 0$. Hence this could be regarded as the inverse HP transition for the RBH.

3. Evaporation of the regular black holes

We remind that the RBH looks like the RN black hole with the singularity replaced by a regular center. The evaporating process will terminate at the extremal point ($r_+ = r_*$) before arriving at $r_+ = 0$. Hence, as far as the evaporation process, there is no difference between the regular and singular black holes. Following Hayward [12] and Bonanno and Reuter [15], we find that the early stage of evaporation is given by that of Schwarzschild black hole. The late stage of the evaporation process for the RBH is totally different from the Schwarzschild case. Instead, this is described by the QCEP. We obtain the approximate forms for temperature and luminosity:

$$T_{\text{RBH}}(m) \simeq \alpha\sqrt{m - m_*}, \tag{9}$$

$$L_{\text{RBH}}(m) \simeq \beta(m - m_*)^2 \tag{10}$$

with $\alpha = 3/8\pi m_*^2 = 0.07$ and $\beta = \sigma A\alpha^4 = 9\sigma/64\pi^3 m_*^6 = 0.00016$. One finds

$$m(v) - m_* \propto \frac{1}{v}, \tag{11}$$

where v is the advanced time coordinate. It was shown that $m(v) - m_*$ vanishes as v^{-1} for the RG-improved Vaidya metric [24,15]. Hence, we obtain the late stage of the evaporation process: $T_{\text{RBH}}(v) \propto v^{-1}$ and $L_{\text{RBH}} \propto v^{-4}$. We confirm that the RBHs lead to concrete predictions on the final state of the evaporation process. We note again that $m = m_*$ is the mass of a cold remnant, which is an extremal black hole with the Planck size. It takes an infinite time to reach the extremal black hole, in compared with the Schwarzschild black hole.

4. Discussions

First of all, we mention that the local thermal stability is given by positive heat capacity with $C > 0$, while the global stability is guaranteed for positive heat capacity $C > 0$ and the negative free energy $F < 0$.

We distinguish the difference between the Hawking–Page transition in the Schwarzschild–AdS black hole and the inverse Hawking–Page transition in the minimal model of the RBH. The Hawking–Page transition is a thermodynamic process by absorbing radiations in heat reservoir: thermal radiation ($C = 0$, $F = 0$) → unstable small black hole ($C < 0$, $F > 0$) → stable large black hole ($C > 0$, $F < 0$). Hence, the ending point is a globally stable black hole. At the critical temperature $T = T_1$, there is a change of the dominance from thermal radiation to a black hole.

On the other hand, the inverse Hawking–Page transition is an evaporation process by emitting radiations through the Hawking radiation: unstable large black hole ($C < 0$, $F > 0$) → locally stable black hole ($C > 0$, $F > 0$) → stable extremal black hole ($C = 0$, $F = 0$). The ending state is supposed to be a vacuum state because it has $T = C = F = 0$ and $S \neq 0$. At the critical temperature near $T = 0$, there is a change of the dominance from locally stable black hole to extremal black hole. These are summarized in Table 1.

Table 1
Summary for the Hawking–Page transition (HPT) and Inverse Hawking–Page phase transition (IHPT). In the bottom, TR (GSBH) means thermal radiation (globally stable black hole), and UBH (EBH) means unstable black hole (extremal black hole)

	HPT		IHPT	
	Starting point	Ending point	Starting point	Ending point
r_+	$r_+ = 0$	$r_+ = r_s$	$r_+ = r_i$	$r_+ = r_*$
C_{RBH}	0	$+$	$-$	0
F_{RBH}	0	$-$	$+$	0
Stability	TR	GSBH	UBH	EBH

Concerning the temperature T, which defines the inverse Hawking–Page transition, we have still some arguments for regarding T as the temperature of heat reservoir. This is because we did not introduce any reservoir such as the cavity for the Schwarzschild black hole [25] and the negative cosmological constant for the AdS black hole [26]. These are necessary devices to derive the Hawking–Page transition from thermal radiation to large stable black hole. Here, we have used the external temperature T by assuming the reservoir.

Moreover, in this work the backreaction effect is trivial because the temperature approaches zero (not divergent) as $m \to m_*$. For the Schwarzschild case, one expects relevant backreaction effects during the terminal stage of the evaporation because of huge increase of temperature as approaches $m = 0$. However, there is a suppression of quantum backreaction for the RBH, since it emits less and less energy as the QCEP does.

In summary, we have shown that the whole evaporation process in the minimal model of the RBH could be regarded as the inverse Hawking–Page phase transition comparing with the Hawking–Page phase transition in the AdS black hole. Its early stage is described by the evaporation of Schwarzschild black hole and the late stage is described by the QCEP with $C > 0$ and $F > 0$. In fact, our result is universal for any RBHs although we have newly investigated the thermodynamics and the evaporation process by choosing a minimal model suggested by Hayward. This is because the temperature in Fig. 1, the heat capacity in Fig. 2, and the free energy in Fig. 3 show the universal behaviors for all known RBHs including the loop quantum and RN black holes.

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