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Statistical exponential distribution function as distance indicator to stellar groups

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Abstract In this paper, statistical distribution functions are developed for distance determination of stellar groups. This method depends on the assumption that, absolute magnitudes and apparent magnitudes follow an exponential distribution function. The developed approaches have been implemented to determine distances of some clusters and stellar associations. The comparison with the distances derived by different authors revealed good agreement.

1. Introduction

One of the most important quantities in astronomy is the distance of the celestial objects, it is considered as premier data aimed at understanding the physics of the universe.

There are many methods for the determination of distance, e.g. trigonometric, spectroscopic and dynamical parallax. Moving star clusters, zero age main sequence fitting are also two other methods for distance determination. However, standard candle is considered as the most important procedure to calculate distances (Sandage and Tammann, 1971; Iben and Tuggle, 1975).

Due to the great increase of data in astronomy, statistical methods can be considered as a bridge between data and models. Many astrophysical challenges are tightly connected to statistics, e.g. time series analysis of variable objects including dynamical models of extrasolar planetary systems, treatments of faint source and other Poisson processes.

Statistical distance determination are carried out by many authors. Among them, Sistero (1988) constructed a distance indicator for Sc galaxies from statistics of angular distances of H II regions. Sharaf et al. (2003) developed Gaussian distribution function for distance determination of stellar groups. Recently, Abdel-Rahman et al. (2009) developed three different Gaussian distribution function as a distance indicator to stellar groups.

In the present paper, exponential distribution function is developed to carry out distance determination of some stellar groups and associations. The organization of the paper is as follows: the method of calculation is discussed in Section 2, Section 3 is devoted to the data sources, while the numerical results and discussions are represented in Section 4.

2. The method

In what follows we shall regard the distribution of magnitudes is exponential and all the members in a given cosmic group are

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assumed at the same distance, r (parsecs) from the sun. The frequency function of the absolute magnitudes of the members is assumed exponential having the general form,

$$\Phi(M) = \frac{1}{\theta} e^{-\frac{M}{\theta}}, M > 0, \theta > 0. \tag{1}$$

The relation between the apparent magnitude m and the absolute magnitude M of a cosmic object at a distance r (in parsecs) is:

$$M = m + 5 - 5\log r. \tag{2}$$

The frequency function of the apparent magnitudes could be given by (Sharaf et al., 2003)

$$\Psi(m) = \frac{\partial(r, M)}{\partial(r, m)} \Phi[M(m, r)], \tag{3}$$

where $\frac{\partial(r,M)}{\partial(r,m)}$ is the Jacobean transformation from absolute to apparent magnitude and $\Phi[M(m, r)]$ is the frequency function of absolute magnitude, which is a function of apparent magnitude and distance.

Since r is nearly the same for all members of a group, we can prove from Eqs. (1)–(3), that

$$\Psi(m) = \frac{1}{\theta} e^{-\frac{(m+5-5\log r)}{\theta}}. \tag{4}$$

We shall fix the limiting apparent magnitude at the faintest limit reached by Hubble Space Telescope at $m_L = 26$ magnitude. Then, the mean apparent magnitude $\bar{m}$ is given as

$$\bar{m}_e = \frac{\int_0^{m_L} m\Psi(m)dm}{\int_0^{m_L} \Psi(m)dm}, \tag{5}$$

Substituting from (4), we get

$$\bar{m}_e = \frac{\int_0^{m_L} m e^{-\frac{(m+5-5\log r)}{\theta}} dm}{\int_0^{m_L} e^{-\frac{(m+5-5\log r)}{\theta}} dm}. \tag{6}$$

Let

$$\theta x = m + 5 - 5\log r, \tag{7.1}$$

and

$$\theta y_e = m_L + 5 - 5\log r. \tag{7.2}$$

Subtract Eq. (7.1) from Eq. (7.2) leads to;

$$\left.\begin{array}{l} m_L - m = \theta(y_e - x) \Rightarrow m = m_L - \theta(y_e - x) \\ \text{at } \ m = 0 \Rightarrow x = y_e - \frac{m_L}{\theta} \\ \text{at } m = m_L \Rightarrow x = y_e \\ dm = \theta dx \end{array}\right\} \tag{8}$$

Using Taylor series in Eq. (1) leads to

$$\Phi(M) = \frac{1}{\theta} e^{-\frac{M}{\theta}} = \frac{1}{\theta}\left[1 - \frac{M}{\theta} + \frac{M^2}{\theta^2 2!} - \frac{M^3}{\theta^3 3!} + \frac{M^4}{\theta^4 4!} - \cdots\right] \tag{9}$$

As a first approximation we shall confine ourselves with the first two terms, where

$$\Phi(M) = \frac{1}{\theta}\left[1 - \frac{M}{\theta}\right]. \tag{10}$$

Using Eqs. (2) and (3), we get

$$\Psi(m) = \frac{1}{\theta}\left[1 - \frac{(m + 5 - 5\log r)}{\theta}\right] \tag{11}$$

substituting in Eq. (6) yields,

$$\bar{m}_e = \frac{\int_0^{m_L} m\left[1 - \frac{(m+5-5\log r)}{\theta}\right] dm}{\int_0^{m_L}\left[1 - \frac{(m+5-5\log r)}{\theta}\right] dm}. \tag{12}$$

Substituting Eqs. (7.1), (7.2) and (8) in Eq. (12) we get

$$\bar{m}_e = \frac{\int_{y_e - \frac{m_L}{\theta}}^{y_e} [m_L - \theta(y_e - x)][1 - x]dx}{\int_{y_e - \frac{m_L}{\theta}}^{y_e} [1 - x]dx}. \tag{13}$$

Hence

$$m_L - \bar{m}_e = m_L + \frac{\int_{y_e - \frac{m_L}{\theta}}^{y_e} [-m_L + \theta(y_e - x)][1 - x]dx}{\int_{y_e - \frac{m_L}{\theta}}^{y_e} [1 - x]dx},$$

which leads to

$$\frac{m_L - \bar{m}_e}{\theta} = y_e - \frac{\left\{\frac{y_e^2}{2} - \frac{(y_e - \frac{m_L}{\theta})^2}{2} - \frac{y_e^3}{3} + \frac{\left(y_e - \frac{m_L}{\theta}\right)^3}{3}\right\}}{\{y_e - (y_e - \frac{m_L}{\theta}) - \frac{y_e^2}{2} + \frac{(y_e - \frac{m_L}{\theta})^2}{2}\}}, \tag{14}$$

proceeding more, we have

$$\alpha_e = F(y_e), \tag{15}$$

where

$$F(y_e) = \frac{-\frac{1}{2} y_e + \frac{1}{2}\frac{m_L}{\theta} + \frac{1}{3}\left(\frac{m_L}{\theta}\right)^2}{1 - y_e + \frac{1}{2}\frac{m_L}{\theta}}, \tag{16}$$

and

$$\alpha_e = \frac{m_L - \bar{m}_e}{\theta}$$

Eq. (15), can be written as,

$$G(y_e) = F(y_e) - \alpha_e = 0. \tag{17}$$

Solving the transcendental Eq. (17) by Newton's method we could get y. Getting y, we can solve for the distance in the distance equation

$$r_e = 10^{1+(m_L - \theta y_e)/5}. \tag{18}$$

The interstellar absorption A and metallicity [Fe/H] as well can be taken into account, then Eq. (18) must be written as:

$$r_e = 10^{1+(m_L - \theta y_e - A - [\mathrm{Fe/H}])/5} \tag{19}$$

The distance modulus is then given as

$$(m - M)_e = m_L - \theta y_e - A - [\mathrm{Fe/H}] \tag{20}$$

Finally, we estimate the parameter θ by Maximum Likelihood method as follows

$$\Phi(M) = \frac{1}{\theta} e^{-\frac{M}{\theta}}$$

$$L(\Phi) = \prod_{i=1}^{n} \Phi(M) = \frac{1}{\theta^n} e^{-\sum_{i=1}^{n} \frac{M_i}{\theta}} \tag{21}$$

taking the logarithm of both sides

$$\ln L(\Phi) = -n\ln\theta - \sum_{i=1}^{n} \frac{M_i}{\theta} \tag{22}$$

Partial differentiation for θ, leads to

$$\frac{d\ln L(\Phi)}{d\theta} = -\frac{n}{\hat{\theta}} + \sum_{i=1}^{n} \frac{M_i}{\hat{\theta}^2} = 0, \tag{23}$$

where

$$\hat{\theta} = \sum_{i=1}^{n} \frac{M_i}{n} = M_0.$$

Then θ is substituted by M_0.

Estimates of distances were tried for the first two, four and six terms of the Taylor series. These calculations, when compared with that determined by other investigators, indicate that using the first two terms of the series are accurate enough.

3. Data sources

Two types of cosmic objects are used for the present application, open star clusters and stellar associations.

3.1. Open star clusters

UBVRI photometric observations for NGC 2658 and NGC 2439 open star clusters (Ramsay and Pollaco, 1992) were used. The observations were obtained in 1990 Jan. 9–22 with the 1-m telescope in Sutherland station of the South African Astronomical observatory.

Vazquez and Will (1995), and Vazque and Baume (1996) used the 0.61-m telescope of the university of Toronto Southern observatory at las Campanas observatory, Chile, to get photometric observations for the open clusters Pis 20 and Tr 14, respectively.

3.2. Stellar associations

Brown et al. (1994) used the Dutch 0.91-m telescope at ESO in the VBLUW Walraven system, to get photoelectric observation for Orion OB1 associations.

The final HIPPARCOS Input Catalog contains 699 stars of the sample under study (1318 stars in Orion OB1) of which 236 are of priority 1, and 463 are of priority 2. These stars are the subject of this work.

Kaltcheva, 1998 observed Carina OB1 and Carina OB2 using the half meter telescope at Sutherland site of the South African Astronomical observatory. The observations were calibrated to the standard system using standard stars from the lists of Cousins (1987), Perry et al. (1987), Crawford et al. (1971) and Olsen (1983). The reduction to the standard system was carried out by the procedure adopted at SAAO.

To correct the apparent magnitude for absorption we use the relation

$$m_0 = m - A_v$$

where

$$A_v = 3.1E(B\text{–}V)$$

and A_v, B, V and $E\,(B\text{–}V)$ are the visual absorption, the magnitudes in the blue and visual filters and the color excess, respectively.

4. Results and discussions

We presented an approach in which an assumed exponential distribution function for both absolute and apparent magnitudes are studied. Based on these distributions, distance of some stellar groups and associations are determined.

In Table 1: column 1 includes the object name, the next columns are devoted for the mean apparent magnitude $\bar{m}$, the unbiased dispersion σ, the unbiased mean absolute magnitude M_0, the parameter α, the parameter y where is the solution of Eq. (17), r_s the statistical distance computed by the present method, r_c the distance computed by others and for the references of the comparison distances, respectively.

Practical applications showed that the formula $m_L = \frac{\bar{m}}{2} - 1$ is the best to control the dispersion values. The dispersion springs up as the difference between m_L and $\bar{m}$ decreases.

As it is shown in Table 1, our distances r_s differ slightly from the distances r_c obtained by other methods. In some cases NGC 2658, NGC 2439, Pis 20, Tr 14, Car OB1 and Orion OB1-1d the difference is positive while in others the difference is negative. One must mention that r_c obtained mostly photoelectrically which is affected by many factors among which the evolutionary factors. Fig. 1 plots the comparison between the

Table 1 The exponential parameters and distances r_s of the present work in comparison to others r_c.

Objects	$\bar{m}$	σ	M_0	α	y	r_s (pc)	r_c (pc)	Ref. of r_c
NGC 2658	12	6.325	56.1823	0.249	0.228701	4267.87 ± 6.33	3631 ± 11 2021	Tadross et al. (2002) Loktin et al. (2001)
NGC 2439	12	6.357	55.978	0.250	0.225887	4688.8 ± 6.36	4450 ± 11 3855	Ramsay and Pollaco (1992), Loktin et al. (2001)
Pis 20	11.489	5.298	37.4801	0.387	0.352149	3633.17 ± 7	3600 ± 30 2018	Vazquez and Will (1995), Loktin et al. (2001)
Tr 14	12.934	6.388	56.8302	0.246	0.237495	3166.77 ± 6	3100 ± 300 2733	Vazque and Baume (1996), Loktin et al. (2001)
Orion OB1- 1a	8.3223	3.46	17.3418	1.019	1.05519	346.929 ± 7.34	380.18 ± 13 398 ± 12	Brown et al. (1994), Warren and Hesser (1978)
Orion OB1-1b	7.9914	3.413	16.5933	1.08529	1.05603	339.469 ± 7	363.07 ± 12 443 ± 12	Brown et al. (1994), Warren and Hesser (1978)
Orion OB1-1c	8.2574	3.447	17.2717	1.0273	1.06495	332.123 ± 5.7	398.10 ± 13 437 ± 13	Brown et al. (1994), Warren and Hesser (1978)
Orion OB1-1d	6.714	3.287	13.5616	1.422	1.29777	478.647 ± 5	380.18 ± 11 479 ± 13	Brown et al. (1994), Warren and Hesser (1978)
Car OB1	6.9864	3.284	10.0519	1.892	1.36136	2905.04 ± 4.24	2850 ± 961	Kaltcheva (1998)
Car OB2	7.4581	3.328	11.1031	1.66998	1.25534	2584.55 ± 4.8	2590 ± 260	Kaltcheva (1998)

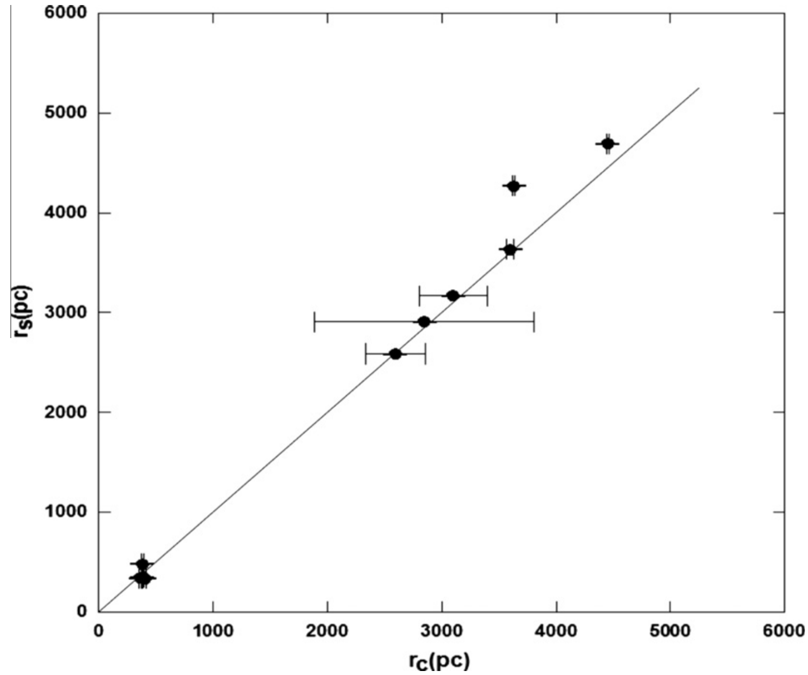

Fig. 1 Comparison between the distance computed in the present work (r_s) with the distance (r_c) in the first row of the eights column in Table 1.

statistical distance and the distance in the first row of the eights column in Table 1. As it is shown, the comparison is fairly good except for NGC 2658 and NGC 2439.

It should be mentioned however, that different author's distance does not agree fully among themselves. For NGC 2658 quite a big difference between distances given by Tadross et al. (2002) and Loktin et al. (2001). The same also for NGC 2439. Differences also among other author's distances are found in Orion OB1 (1a, 1b, 1c). In our opinion such differences can be due to the location of the stars used to determine the distances.

References

Abdel-Rahman, H.I., Issa, I.A., Sharaf, M.A., Nouh, M.I., Bakry, A., Osman, A.I., Saad, A.S., Kamal, F.Y., Essam, A., 2009. J. Korean Astron. Soc. 42, 71.

Brown, A.G.A., Degeus, E.J., Gezeeuw, P.T., 1994. Astron. Astrophys. 289, 101.

Cousins, A.W.J., 1987. SAAO Circ. 11, 93.

Crawford, D.L., Barnes, J.V., Golson, J.C., 1971. Astron. J. 76, 1058C.

Iben Jr., I., Tuggle, R.S., 1975. Astrophys. J. 197, 39.

Kaltcheva, N.T., 1998. Astron. Astrophys. Suppl. Ser. 128, 309.

Loktin, A.V., Gerasimenko, T.P., Malisheva, L.K., 2001. Astron. Astrophys. Trans. 20, 607.

Olsen, E.H., 1983. Astron. Astrophys. Suppl. Ser. 54, 55.

Perry, C.L., Olsen, E.H., Crawford, D.L., 1987. Publ. Astron. Soc. Pac. 99, 1184.

Ramsay, G., Pollaco, D.L., 1992. Astron. Astrophys. Suppl. Ser. 94, 73.

Sandage, A., Tammann, G.A., 1971. Astrophys. J. 167, 293.

Sistero, Roberto F., 1988. Astrophys. Lett. Commun., 27–41.

Sharaf, M.A., Issa, I.A., Saad, A.S., 2003. N. Astron. 8, 15.

Tadross, A.L., Werner, P., Osman, A., Marie, M., 2002. N. Astron. 7, 553.

Vazquez, R.A., Will, J.M., 1995. Astron. Astrophys. Suppl. Ser. 111, 85.

Vazque, R.A., Baume, G., 1996. Astron. Astrophys. Suppl. Ser. 116, 75.

Warren Jr., W.H., Hesser, J.E., 1978. ApJS. 36, 497W.