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www.elsevier.com/locate/physletbFinal state interactions in $B^\pm \rightarrow K^+K^-K^\pm$ decaysA. Furman^a, R. Kamiński^b, L. Leśniak^{b,*}, P. Żenczykowski^b^a *Ul. Bronowicka 85/26, 30-091 Kraków, Poland*^b *Henryk Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences, PL 31-342 Kraków, Poland*

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ABSTRACT

Charged B decays to three charged kaons are analysed in the framework of the QCD factorization approach. The strong final state K^+K^- interactions are described using the kaon scalar and vector form factors. The scalar non-strange and strange form factors at low K^+K^- effective masses are constrained by chiral perturbation theory and satisfy the two-body unitarity conditions. The latter stem from the properties of the meson–meson amplitudes which describe all possible S -wave transitions between three coupled channels consisting of two kaons, two pions and four pions. The vector form factors are fitted to the data on the electromagnetic kaon interactions. The model results are compared with the Belle and BaBar data. Away from $\phi(1020)$ resonance, in the S -wave dominated K^+K^- mass spectra, a possibility for a large CP asymmetry is identified.

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1. Introduction

Recently, charmless three-body decays of B mesons have been intensively studied both experimentally and theoretically. On the experimental side, Dalitz plot analyses of the charged B decays were performed by Belle [1] and BaBar [2] Collaborations. Likewise, several theoretical studies involving the $B^\pm \rightarrow K^+K^-K^\pm$ decays have been published [3,4] and [5].

Since charged kaons interact strongly, their long distance interactions in the final states have to be well understood if one aims at extracting weak decay amplitudes from the B to KKK decays. In this Letter we go beyond an isobar model parameterization of the B decay amplitudes and introduce additional theoretical constraints on the S -wave two-body K^+K^- interaction amplitudes, which follow, in particular, from unitarity. In order to satisfy unitarity in two-body interactions we construct scalar strange and non-strange form factors which enter into the S -wave parts of the decay amplitudes. These amplitudes are calculated in the framework of the QCD factorization approach. In the construction of form factors we use experimental information on the K^+K^- interactions coming from experiments other than B decays, for example from K^+K^- production processes in hadronic collisions or from e^+e^- reactions. We apply also some low-energy constraints coming from the chiral perturbation theory. Preliminary results of our analysis concerning the $B^\pm \rightarrow K^+K^-K^\pm$ reactions can be found in Ref. [6].

In Section 2 we formulate the theoretical model of the B^+ and B^- decay amplitudes. Presentation of results and their comparison with the experimental data are given in Section 3. Our conclusions are presented in Section 4.

2. $B^\pm \rightarrow K^+K^-K^\pm$ decay amplitudes

Inspection of the Dalitz plots of the Belle [1] and BaBar [2] experiments reveals an accumulation of events for the K^+K^- effective masses below 1.8 GeV. Indeed, several mesonic resonances which can decay into the K^+K^- pairs exist in this range [7]. Among them there are scalar and vector resonances which are formed via the S - and P -wave final state interactions. In the first approximation one can neglect their interaction with the third kaon. This justifies using the QCD quasi-two-body factorization approach for the limited range of the effective K^+K^- masses (see, for example Ref. [8]). The $B^- \rightarrow K^+K^-K^-$ amplitude is then expressed in terms of the following matrix element of the weak effective Hamiltonian H :

$$\langle K^-(p_1)K^+(p_2)K^-(p_3) | H | B^- \rangle = A_S^- + A_P^-, \quad (1)$$

where the S -wave part is

$$A_S^- = \frac{G_F}{\sqrt{2}} \left\{ -\sqrt{\frac{1}{2}} \chi f_K (M_B^2 - s_{23}) F_0^{B \rightarrow (K^+K^-)_S}(m_K^2) y \Gamma_2^{n*}(s_{23}) + \frac{2B_0}{m_b - m_s} (M_B^2 - m_K^2) F_0^{BK}(s_{23}) v \Gamma_2^{S*}(s_{23}) \right\}, \quad (2)$$

the P -wave part is

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$$A_p^- = \frac{G_F}{\sqrt{2}} \left\{ \frac{f_K}{f_\rho} A_0^{B\rho}(m_K^2) y F_u^{K^+K^-}(s_{23}) - F_1^{BK}(s_{23}) [w_u F_u^{K^+K^-}(s_{23}) + w_d F_d^{K^+K^-}(s_{23}) + w_s F_s^{K^+K^-}(s_{23})] \right\} 4\vec{p}_1 \cdot \vec{p}_2 \quad (3)$$

and the interacting kaons are taken to be kaons 2 and 3. Furthermore, s_{23} is the square of the $K^+(p_2)K^-(p_3)$ effective mass $m_{23} \equiv m_{K^+K^-}$, while \vec{p}_1 and \vec{p}_2 are the kaon 1 and kaon 2 momenta in the center of mass system of the kaons 2 and 3. The scalar product of the kaon momenta can be written in terms of the helicity angle Θ_H :

$$\vec{p}_1 \cdot \vec{p}_2 = -|\vec{p}_1||\vec{p}_2| \cos \Theta_H. \quad (4)$$

In these equations G_F is the Fermi coupling constant, $f_K = 0.1555$ GeV and $f_\rho = 0.220$ GeV are the kaon and the ρ meson decay constants, M_B , m_K , $m_b = 4.9$ GeV, $m_s = 0.1$ GeV, $m_u = 0.004$ GeV and $m_d = 0.004$ GeV are the masses of the B meson, kaon, b -quark, strange quark, down- and up-quarks, respectively.

The functions Γ_2^n and Γ_2^s , present in the S -wave amplitude in Eq. (2), are the kaon non-strange and strange scalar form factors. The vector form factors $F_q^{K^+K^-}$ (for $q = u, d$ and s), introduced in Eq. (3), are defined through matrix elements

$$\langle K^+(p_2)K^-(p_3) | \bar{q} \gamma_\mu q | 0 \rangle = (p_2 - p_3)_\mu F_q^{K^+K^-}(s_{23}), \quad (5)$$

where $|0\rangle$ is the vacuum state. The K^+K^- pair in the S -wave is then denoted by $R_S \equiv (K^+K^-)_S$. Similarly $R_P \equiv (K^+K^-)_P$ stands for the P -state. Furthermore $F_0^{B \rightarrow (K^+K^-)_S}$ in Eq. (2) is the form factor of the transition from the B meson to the K^+K^- pair in the S -state, χ is the constant related to the decay of the $(K^+K^-)_S$ state into two kaons, and $B_0 = m_\pi^2/(m_u + m_d)$, where m_π is the pion mass. We take $F_0^{B \rightarrow (K^+K^-)_S}(m_K^2) = 0.13$ [9] and we fit χ to the data. Functions $F_0^{BK}(s_{23})$ and $F_1^{BK}(s_{23})$ are the $B \rightarrow K$ scalar and vector transition form factors and $A_0^{B\rho}(m_K^2) = 0.37$ [8] is the $B \rightarrow \rho$ transition form factor. In our approximation, the ratio $A_0^{B\rho}/f_\rho$ represents a general factor related to the transition from B^- to any $(K^+K^-)_P$ state and then its decay into the final K^+K^- pair. For the case of the ρ meson this coupling to the pair of kaons is effectively realized only above the K^+K^- threshold.

The weak decay amplitudes depend on QCD factorization coefficients a_j^p and on the products $\Lambda_u = V_{ub}V_{us}^*$, $\Lambda_c = V_{cb}V_{cs}^*$, where V_{ij} are the CKM quark-mixing matrix elements. In order to describe B decay into mesons M_1 and M_2 we follow Ref. [8] and calculate the coefficients $a_j^p(M_1M_2)$ at the next-to-leading order in the strong coupling constant at the renormalization scale equal to $m_b/2$. Here the M_1 meson has a common spectator quark with the decaying B meson. In the case of the $B^- \rightarrow K^+K^-K^-$ decays, M_1 or M_2 can be either kaon K^- , or systems R_S, R_P . We take into account one-loop vertex and penguin corrections to $a_j^p(M_1M_2)$ but neglect those due to hard scattering or the annihilation since they are expected to be generally suppressed. In the QCD factorization approach they receive logarithmically divergent contributions due to soft gluon interaction which are “unavoidably model dependent” (see Ref. [8]). We treat such soft interactions by introducing the form factors constrained by data on meson-meson interactions, taken from analyses of reactions other than the B decays. Under these conditions we have $a_j^p(R_S M_2) = a_j^p(R_P M_2)$, with their common value denoted below by $a_j^p(R_{S,P} M_2) \equiv a_{jy}^p$. We also use the abbreviations: $a_{jw}^p \equiv a_j^p(K^- R_P)$ and $a_{jv}^p \equiv a_j^p(K^- R_S)$. The values of coefficients $a_j^p(M_1M_2)$ are given in Table 1.

In terms of the quantities introduced above one defines:

$$y = \Lambda_u [a_{1y} + a_{4y}^u + a_{10y}^u - (a_{6y}^u + a_{8y}^u) r_\chi^K] + \Lambda_c [a_{4y}^c + a_{10y}^c - (a_{6y}^c + a_{8y}^c) r_\chi^K], \quad (6)$$

where

$$r_\chi^K = \frac{2m_K^2}{(m_b + m_u)(m_u + m_s)}, \quad (7)$$

$$w_u = \Lambda_u (a_{2w} + a_{3w} + a_{5w} + a_{7w} + a_{9w}) + \Lambda_c (a_{3w} + a_{5w} + a_{7w} + a_{9w}), \quad (8)$$

$$w_d = \Lambda_u \left[a_{3w} + a_{5w} - \frac{1}{2}(a_{7w} + a_{9w}) \right] + \Lambda_c \left[a_{3w} + a_{5w} - \frac{1}{2}(a_{7w} + a_{9w}) \right], \quad (9)$$

$$w_s = \Lambda_u \left[a_{3w} + a_{4w}^u + a_{5w} - \frac{1}{2}(a_{7w} + a_{9w} + a_{10w}^u) \right] + \Lambda_c \left[a_{3w} + a_{4w}^c + a_{5w} - \frac{1}{2}(a_{7w} + a_{9w} + a_{10w}^c) \right], \quad (10)$$

and

$$v = \Lambda_u \left(-a_{6v}^u + \frac{1}{2}a_{8v}^u \right) + \Lambda_c \left(-a_{6v}^c + \frac{1}{2}a_{8v}^c \right). \quad (11)$$

One can notice that in the expressions for the decay amplitudes there are no transitions to the K^+K^- states of spin 2 or higher. This results from the application of the factorization approach in which matrix elements to spin states higher than one vanish. The contribution of $f_2(1270)$ with its rather small branching fraction to $K\bar{K}$ (4.6%) is thus not included in this study.

Since two identical charged kaons appear in the final state of the $B^- \rightarrow K^+K^-K^-$ decay, the amplitude of Eq. (1) has to be symmetrized

$$A_{sym}^- = \frac{1}{\sqrt{2}} \left[\langle K^-(p_1)K^+(p_2)K^-(p_3) | H | B^- \rangle + \langle K^-(p_3)K^+(p_2)K^-(p_1) | H | B^- \rangle \right]. \quad (12)$$

The symmetrized amplitude for the $B^+ \rightarrow K^+K^-K^+$ reaction reads

$$A_{sym}^+ = A_{sym}^- (\Lambda_u \rightarrow \Lambda_u^*, \Lambda_c \rightarrow \Lambda_c^*, B^- \rightarrow B^+). \quad (13)$$

The final state kaon-kaon S -wave interactions are dynamically coupled with systems consisting of two and four pions. Thus a system of three coupled channels: $\pi\pi, \bar{K}K$ and 4π (effective $(2\pi)(2\pi)$ or $\sigma\sigma, \rho\rho$, etc.), labelled by $j = 1, 2, 3$, is considered in the construction of scalar form factors Γ_2^n and Γ_2^s . Here we use an approach initiated in [4] and recently developed in [10] for the $B^\pm \rightarrow \pi^+\pi^-\pi^\pm$ decays. A set of the 3×3 transition amplitudes T , describing all possible transitions between the three channels, is taken from a unitary model of Ref. [11] (solution A). We introduce two kinds of production functions $R_j^{n,s}$, labeled by n (non-strange) or by s (strange):

$$R_j^{n,s}(E) = \frac{\alpha_j^{n,s} + \tau_j^{n,s} E + \omega_j^{n,s} E^2}{1 + cE^4}, \quad j = 1, 2, 3, \quad (14)$$

where $\alpha_j^{n,s}$, $\tau_j^{n,s}$, $\omega_j^{n,s}$ and c are constant parameters, while E represents the total energy and is related to the center of mass momenta $k_j = \sqrt{E^2 - m_j^2}$, with $m_1 = m_\pi$, $m_2 = m_K$, $m_3 = 700$ MeV, and $s \equiv E^2 \equiv m_{K^+K^-}^2$. The three scalar form factors, written in the compact row matrix form $\Gamma^{n,s*}$, are given by

$$\Gamma^{n,s*} = R^{n,s} + TGR^{n,s}, \quad (15)$$

Table 1
Leading order (LO) and next-to-leading order (NLO) coefficients a_{iy}^p , a_{iv}^p and a_{iw}^p entering into Eqs. (6)–(11). The NLO coefficients are the sum of the LO coefficients plus next-to-leading order vertex and penguin corrections. The superscript p is omitted for $i = 1, 2, 3, 5, 7$ and 9 , the penguin corrections being zero for these cases.

	a_{iy}^p		a_{iv}^p		a_{iw}^p	
	LO	NLO	LO	NLO	LO	NLO
a_1	1.039	1.066 + $i0.039$				
a_2					0.084	−0.041 − $i0.114$
a_3					0.004	0.010 − $i0.005$
a_4^u	−0.044	−0.029 − $i0.02$			−0.044	−0.032 − $i0.019$
a_4^c	−0.044	−0.035 − $i0.004$			−0.044	−0.038 − $i0.006$
a_5					−0.012	−0.010 − $i0.007$
a_6^u	−0.062	−0.057 − $i0.017$	−0.062	−0.075 − $i0.017$		
a_6^c	−0.062	−0.062 − $i0.004$	−0.062	−0.079 − $i0.004$		
a_7					0.0001	0.0 + $i0.0001$
a_8^u	0.0007	0.0008 + $i0.0$	0.0007	0.0007 + $i0.0$		
a_8^c	0.0007	0.0008 + $i0.0$	0.0007	0.0006 + $i0.0$		
a_9					−0.0094	−0.0097 − $i0.0003$
a_{10}^u	−0.0009	0.0005 + $i0.0013$			−0.0009	0.0006 + $i0.001$
a_{10}^c	−0.0009	0.0005 + $i0.0013$			−0.0009	0.0006 + $i0.001$

where $R^{n,s}$ are rows of the production functions and G is the matrix of the Green's functions multiplied by the convergence factors $F_j(p) = (k_j^2 + \kappa^2)/(p^2 + \kappa^2)$. These factors, which reduce to unity on shell ($p = k_j$), make finite the relevant integrals over the intermediate momenta p . The parameter κ will be fitted to the data of the BaBar [2] and Belle [1] Collaborations.

For both the non-strange and strange form factors we also constrain their low energy behaviour using the chiral perturbation model of Refs. [12,13]. At low s values one writes the following expansion:

$$\Gamma_j^{n,s}(s) \cong d_j^{n,s} + f_j^{n,s}s, \quad j = 1, 2, 3, \quad (16)$$

with real coefficients $d_j^{n,s}$ and $f_j^{n,s}$. Explicit formulae for the set of non-strange form factors, in particular for the Γ_2^n presented in Eq. (2), are given in Eqs. (24)–(35) of Ref. [10]. For the strange form factors we have

$$d_1^s = \frac{\sqrt{3}}{2} \left[\frac{16m_\pi^2}{f^2} (2L_6^r - L_4^r) - \frac{m_\pi^2}{72\pi^2 f^2} \left(1 + \log \frac{m_\eta^2}{\mu^2} \right) \right], \quad (17)$$

$$f_1^s = \frac{\sqrt{3}}{2} \left[\frac{8L_4^r}{f^2} - \frac{1}{32\pi^2 f^2} \left(1 + \log \frac{m_K^2}{\mu^2} \right) + \frac{m_\pi^2}{432\pi^2 m_\eta^2 f^2} \right], \quad (18)$$

and

$$d_2^s = 1 + \frac{8(2L_6^r - L_4^r)}{f^2} (m_\pi^2 + 4m_K^2) - \frac{16L_5^r}{f^2} m_K^2 + \frac{32L_8^r}{f^2} m_K^2 + \frac{m_\eta^2}{48\pi^2 f^2} \log \frac{m_\eta^2}{\mu^2} + \frac{m_K^2}{36\pi^2 f^2} \left(1 + \log \frac{m_\eta^2}{\mu^2} \right), \quad (19)$$

$$f_2^s = \frac{8L_4^r}{f^2} + \frac{4L_5^r}{f^2} - \frac{m_K^2}{216\pi^2 f^2 m_\eta^2} - \frac{1}{32\pi^2 f^2} \left(1 + \log \frac{m_\eta^2}{\mu^2} \right) - \frac{3}{64\pi^2 f^2} \left(1 + \log \frac{m_K^2}{\mu^2} \right). \quad (20)$$

In these equations m_η is the η meson mass, μ is the scale of the dimensional regularization and $f = f_\pi/\sqrt{2}$. Using $f = 92.4$ MeV and the chiral perturbation theory constants L_k^r , $k = 4, 5, 6, 8$, given in Table X of Ref. [14], we obtain the non-strange-sector parameters: $d_1^n = 1.1957$, $f_1^n = 3.1329$ GeV $^{-2}$, $d_2^n = 0.7193$ and $f_2^n = 1.6719$ GeV $^{-2}$ and their strange-sector counterparts: $d_1^s = -0.0016$, $f_1^s = 0.2393$ GeV $^{-2}$, $d_2^s = 1.0410$ and $f_2^s = 0.6235$ GeV $^{-2}$. For the form factors related to the third channel at low energies

we make the simplest assumptions $d_3^n = d_3^s = f_3^n = f_3^s = 0$, as in Ref. [20].

The coefficients $\alpha_j^{n,s}$, $\tau_j^{n,s}$ and $\omega_j^{n,s}$ are constrained by the values of the form factors at low energies. They are calculated using the low energy expansion of Eq. (15) and are listed in Table 2. The parameter c , which controls the high energy behaviour of R , is fixed while fitting the data.

Our scalar form factors satisfy the following unitarity conditions:

$$\text{Im } \Gamma^* = T^\dagger D \Gamma^*, \quad (21)$$

where D is the diagonal matrix of the kinematical coefficients which are proportional to the channel momenta k_j in the center of mass frame:

$$D_{ij} = -\frac{k_j \sqrt{s}}{8\pi} \delta_{ij} \theta(\sqrt{s} - 2m_j), \quad i, j = 1, 2, 3. \quad (22)$$

Presence of the resonances in the K^+K^- effective mass distributions (see Refs. [2,1]) is a direct manifestation of the K^+K^- final state interactions. The most prominent resonance in the P -wave is $\phi(1020)$. In 2005 Bruch, Khodjamirian and Kühn [15] described the electromagnetic form factors for charged and neutral kaons in terms of additive contributions from eight vector mesons: $\rho \equiv \rho(770)$, $\rho' \equiv \rho(1450)$, $\rho'' \equiv \rho(1700)$, $\omega \equiv \omega(782)$, $\omega' \equiv \omega(1420)$, $\omega'' \equiv \omega(1650)$, $\phi \equiv \phi(1020)$ and $\phi' \equiv \phi(1680)$. Using quark model assumptions and isospin symmetry as in Ref. [15] one can deduce the following expressions for the three P -wave form factors $F_q^{K^+K^-}$ defined in Eq. (5):

$$F_u^{K^+K^-} = \frac{1}{2} (c_\rho BW_\rho + c_{\rho'} BW_{\rho'} + c_{\rho''} BW_{\rho''} + c_\omega BW_\omega + c_{\omega'} BW_{\omega'} + c_{\omega''} BW_{\omega''}), \quad (23)$$

$$F_d^{K^+K^-} = \frac{1}{2} (-c_\rho BW_\rho - c_{\rho'} BW_{\rho'} - c_{\rho''} BW_{\rho''} + c_\omega BW_\omega + c_{\omega'} BW_{\omega'} + c_{\omega''} BW_{\omega''}), \quad (24)$$

$$F_s^{K^+K^-} = -c_\phi BW_\phi - c_{\phi'} BW_{\phi'}. \quad (25)$$

In the above equations BW_i , $i = 1, \dots, 8$, are the energy-dependent Breit–Wigner functions, defined for each resonance of mass m_i and width Γ_i as

$$BW_i(s) = \frac{m_i^2}{m_i^2 - s - i\sqrt{s} \Gamma_i(s)}, \quad (26)$$

and c_i are the constants given in Table 2 of Ref. [15] for the constrained fit.

Table 2Parameters of production functions $R_i^n(E)$ and $R_i^s(E)$ defined in Eq. (14) for $\kappa = 3.506$ GeV.

i	α_i^n	τ_i^n (GeV $^{-1}$)	ω_i^n (GeV $^{-2}$)	α_i^s	τ_i^s (GeV $^{-1}$)	ω_i^s (GeV $^{-2}$)
1	0.6731	-0.2511	1.5301	0.3743	-0.1090	0.1008
2	0.6116	0.0428	1.5232	0.7075	-0.1029	0.3256
3	1.2055	0.3589	3.1556	1.0028	+0.0979	0.4653

The B to K transition form factors have been parametrized according to Ref. [16]:

$$F_0^{BK}(s) = \frac{r_0}{1 - \frac{s}{s_0}}, \quad (27)$$

where $r_0 = 0.33$, $s_0 = 37.46$ GeV 2 , and

$$F_1^{BK}(s) = \frac{r_1}{1 - \frac{s}{m_1^2}} + \frac{r_2}{(1 - \frac{s}{m_1^2})^2}, \quad (28)$$

where $r_1 = 0.162$, $r_2 = 0.173$ and $m_1 = 5.41$ GeV.

3. Results

Partial wave analysis of the decay amplitudes helps in the investigation of the density distributions in the Dalitz diagrams. In Eqs. (2), (3) we have defined the S - and P -wave amplitudes to which the double differential $B^- \rightarrow K_1^- K_2^+ K_3^-$ branching fraction Br is related through the symmetrized amplitude A_{sym}^- of Eq. (12):

$$\frac{d^2 Br^-}{dm_{23} d \cos \Theta_H} = \frac{1}{\Gamma_B} \frac{m_{23} |\vec{p}_1| |\vec{p}_2|}{8(2\pi)^3 M_B^3} |A_{sym}^-(m_{23}, \Theta_H)|^2. \quad (29)$$

Here Γ_B is the total width of the B^- meson and the kaon momenta are:

$$|\vec{p}_1| = \frac{1}{2} \sqrt{m_{23}^2 - 4m_K^2}, \quad (30)$$

$$|\vec{p}_2| = \frac{1}{2m_{23}} \sqrt{[M_B^2 - (m_{23} + m_K)^2][M_B^2 - (m_{23} - m_K)^2]}. \quad (31)$$

The helicity angle Θ_H is kinematically related to the effective mass m_{12} of the $K_1^- K_2^+$ system:

$$\cos \Theta_H = \frac{1}{2|\vec{p}_1| |\vec{p}_2|} \left[m_{12}^2 - \frac{1}{2}(M_B^2 - m_{23}^2 + 3m_K^2) \right]. \quad (32)$$

Due to the symmetry of the Dalitz plot density under the exchange of the kaons K_1^- and K_3^- , one can define the effective mass m_{23} distribution integrated over the m_{12} masses larger than m_{23} :

$$\frac{dBr^-}{dm_{23}} = \int_{\cos \Theta_g}^1 \frac{d^2 Br^-}{dm_{23} d \cos \Theta_H} d \cos \Theta_H, \quad (33)$$

where $\cos \Theta_g$ corresponds to the value of $\cos \Theta_H$ in Eq. (32) with $m_{12} = m_{23}$. The helicity angle distribution $dBr^-/d \cos \Theta_H$ can be obtained from Eq. (29) by integration over the specific range of the effective mass m_{23} .

Our aim is to describe the data of the Belle [1] and BaBar [2] Collaborations in one common fit. The data chosen by us include the total branching fraction for the decay $B^\pm \rightarrow [\phi(1020)K^\pm, \phi(1020) \rightarrow K^+K^-]$, the averaged effective mass distributions dBr^\pm/dm_{23} for m_{23} smaller than 1.8 GeV, and the averaged helicity angle distribution $dBr^\pm/d \cos \Theta_H$ for $m_{23} < 1.05$ GeV. The distributions of the $B^\pm \rightarrow K^+K^-K^\pm$ events are obtained from the published data by subtraction of the background components. The total number of data points for ten plots from both collaborations is equal to 175. The theoretical distributions are

normalized to the total number of experimental events corresponding to each data set. In our fit we used the averaged $B^\pm \rightarrow [\phi(1020)K^\pm, \phi(1020) \rightarrow K^+K^-]$ branching fraction equal to $(4.06 \pm 0.34) \cdot 10^{-6}$ [7]. There are four fitted parameters: χ , κ , c and N_P . The first three parameters are related to the S -wave decay amplitudes and the fourth one, N_P , is the common P -wave normalization constant by which the amplitudes A_P^- and A_P^+ are multiplied. We have performed the fit to the 176 data points obtaining the total value of χ^2 equal to 343 and the following parameters: $\chi = (6.44 \pm 0.44)$ GeV $^{-1}$, $\kappa = (3.51 \pm 0.20)$ GeV, $c = (0.084 \pm 0.010)$ GeV $^{-4}$ and $N_P = 1.037 \pm 0.014$. For $N_P = 1$ we obtain the averaged $B^\pm \rightarrow [\phi(1020)K^\pm, \phi(1020) \rightarrow K^+K^-]$ branching fraction equal to $3.73 \cdot 10^{-6}$ which is within one standard deviation from the experimental value of $(4.06 \pm 0.34) \cdot 10^{-6}$. One sees that the absolute normalization of the P -wave is very close to 1 which means that the decay amplitudes calculated in our model are adequate.

Our value of χ parametrizes a large range of K^+K^- effective mass up to 1.8 GeV and not just the region of $f_0(980)$. Therefore it cannot be directly compared with the value given in Ref. [4]. In addition, the estimate of χ given in Eq. (18) of [4] involves the coupling constant of $f_0(980)$ to $\pi\pi$ while here we have coupling to KK . Using $g_{f_0\bar{K}K}/g_{f_0\pi\pi} = 4.2$ from Ref. [19], a very rough estimate similar to that given in Ref. [4] leads to $\chi \approx 5.6$ GeV $^{-1}$. The κ parameter was not used in Ref. [4] where only the on shell contributions to the form factors were taken into account. The value of $\kappa = 3.51$ GeV $^{-1}$ is reasonably larger than the typical KK mass considered. We have also done an analogous fit to the data using the three P -wave form factors based on the parameterization of Ref. [5] obtaining similar values of parameters as those written above, however with a higher χ^2 value of 354.

Fig. 1 shows the moduli of scalar form factors $\Gamma_2^{n*}(s_{23})$ and $\Gamma_2^{s*}(s_{23})$ which determine the functional dependence of the S -wave amplitudes on the K^+K^- effective mass. There are two prominent maxima of both form factors, one related to the $f_0(980)$ resonance and the second one forming cusps due to the opening of the third channel at 1400 MeV (in the present model responsible effectively for the production of four pions). Presence of $f_0(980)$ leads to the threshold enhancement of the S -wave amplitude. This effect can be directly studied in high statistics experiment with a very good effective K^+K^- mass resolution of about 1 MeV and should be seen only a few MeV above the threshold.

In Fig. 2 the K^+K^- effective mass distributions are shown for two mass ranges and for the data from the BaBar Collaboration. At low $m_{K^+K^-}$ the spectrum is influenced by the P -wave amplitude and dominated by the $\phi(1020)$ resonance. Above 1.05 GeV the S -wave amplitude is much more important than the P -wave one. According to our analysis which uses the approach of Refs. [11,21], the experimental maximum near 1.5 GeV can be attributed to the $f_0(1400-1460)$ found therein in solution A. We recall that in Ref. [21] the coupling constant of the $f_0(1400)$ decay to $\bar{K}K$ is much smaller than the corresponding coupling to $\pi\pi$. Let us notice that the model distribution depends on the sharp 4π threshold located at $2 \cdot m_3 = 1.4$ GeV which in reality should be smoothed out by the four-body pion interactions not taken into account in this quasi-two-body approximation. We have

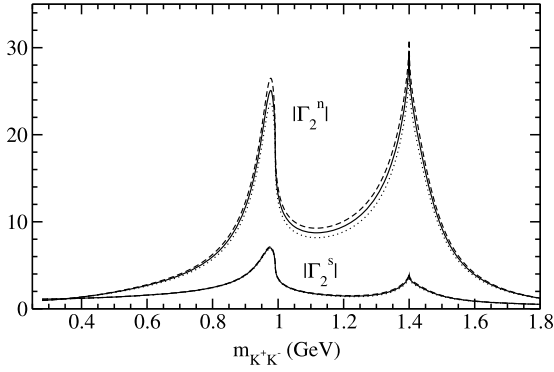


Fig. 1. Moduli of kaon scalar non-strange and strange form factors (solid lines) obtained in our fit. The dashed and dotted lines represent the variation of their moduli when parameter κ varies within its error band.

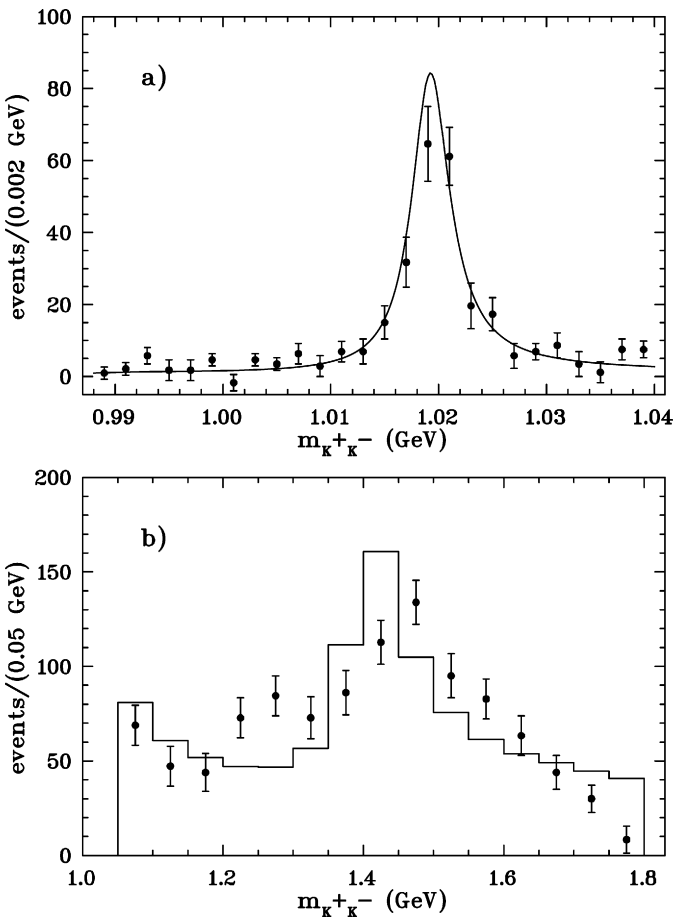


Fig. 2. The K^+K^- effective mass distributions from the fit to BaBar experimental data [2] in the $\phi(1020)$ range (a) and between 1.05 GeV and 1.8 GeV (b). Theoretical results are shown as solid line in (a) and as histogram in (b).

also studied the Belle [1] K^+K^- effective mass spectra and found that the quality of their description is similar to that shown in Fig. 2 for the BaBar data. Fig. 3 shows a more detailed comparison of the $m_{K^+K^-}$ theoretical distributions with the Belle data [1], with events grouped in five ranges of m_{12} which is the other combination of the K^+K^- effective masses. One observes an overall general agreement of theoretical histograms with experiment, with some surplus of experimental events in Fig. 2e for the case of the highest slice of m_{12} (larger than 20 GeV^2) where our model is not fully applicable due to the proximity of the Dalitz plot edge.

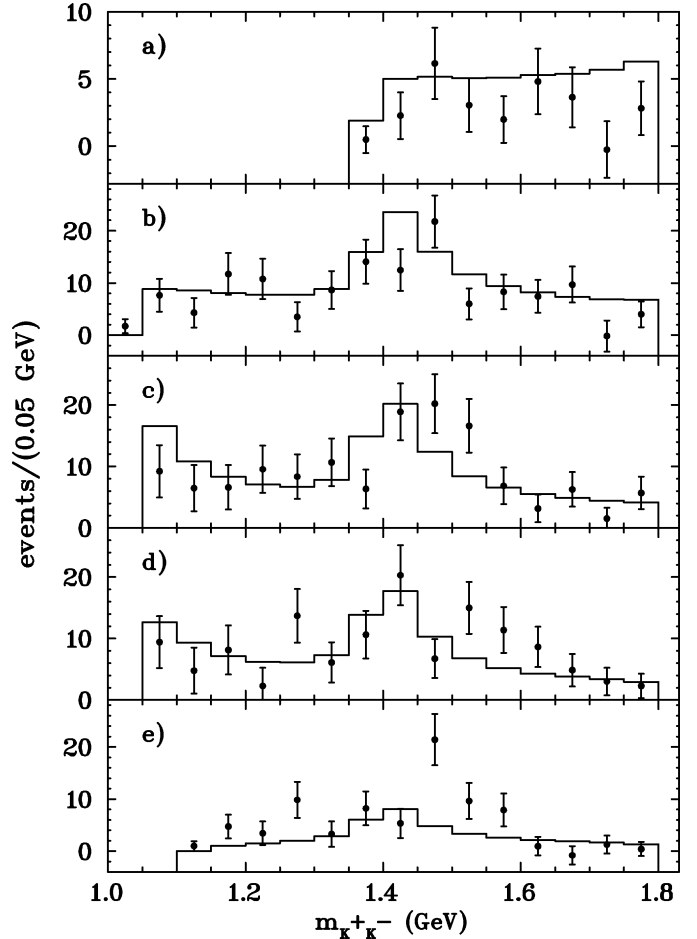


Fig. 3. The K^+K^- effective mass distributions from the fit to Belle experimental data [1] (a) for $m_{12}^2 < 5 \text{ GeV}^2$, (b) for $5 \text{ GeV}^2 < m_{12}^2 < 10 \text{ GeV}^2$, (c) for $10 \text{ GeV}^2 < m_{12}^2 < 15 \text{ GeV}^2$, (d) for $15 \text{ GeV}^2 < m_{12}^2 < 20 \text{ GeV}^2$ and (e) for $20 \text{ GeV}^2 < m_{12}^2$. Theoretical results are shown as histograms.

Finally, in Fig. 4 we present the helicity angle distribution in the K^+K^- mass range dominated by the $\phi(1020)$ resonance. Without the S -wave component of the decay amplitude the distribution should be symmetric with respect to $\cos \Theta_H = 0$. However, we observe an interference effect which distorts the distribution. This is a direct evidence of a non-zero part of the S -wave present even under the huge peak of the $\phi(1020)$ resonance. A theoretical integration of the S -wave contribution to the spectrum in the $m_{K^+K^-}$ range from threshold till 1.05 GeV leads to about 12% relative branching fraction. It corresponds to the average branching fraction of $4.83 \cdot 10^{-7}$ which is in agreement with the experimental upper bound of $2.9 \cdot 10^{-6}$ found in Ref. [1]. This agrees also with the BaBar estimate $(9 \pm 6)\%$ of the S -wave fraction in the region of masses between 1.013 and 1.027 GeV [2]. In the range of the K^+K^- effective mass from 1200 to 1800 MeV, which might be relevant for the $X_0(1550)$ discussed in Ref. [2], the CP averaged branching fraction corresponding to the S -wave is equal to $4.42 \cdot 10^{-6}$ which is larger than the total contribution of the $\phi(1020)$ resonance.

We have also studied the CP violation effects comparing the magnitudes of the decay amplitudes of the B^- and B^+ decays. While the moduli of the P -wave amplitudes for these charge conjugated decays are rather similar, the S -wave amplitudes behave differently indicating an important CP violation effect which depends on the $m_{K^+K^-}$ range. For the S -wave parameters written above, starting from the K^+K^- threshold up to about 1.4 GeV,

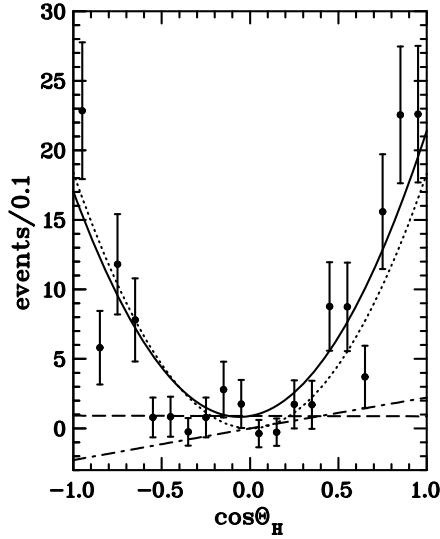


Fig. 4. Helicity angle distribution for the Belle data [1] in the K^+K^- effective mass up to 1.05 GeV. The dashed line represents the S -wave contribution of our model, the dotted line – that of the P -wave, the dot-dashed – that of the interference term and the solid line corresponds to the sum of these contributions.

the modulus of the B^+ S -wave amplitude is larger than the corresponding modulus of the B^- amplitude. Then, above 1.4 GeV, the B^- moduli become larger than the B^+ ones. Defining the CP asymmetry as

$$A_{CP}(m_{23}) = \left(\frac{dBr^-}{dm_{23}} - \frac{dBr^+}{dm_{23}} \right) / \left(\frac{dBr^-}{dm_{23}} + \frac{dBr^+}{dm_{23}} \right), \quad (34)$$

one gets very large asymmetries if one takes into account solely the contribution of the S -wave. For example, $A_{CP}^S(1 \text{ GeV}) = -0.51$, $A_{CP}^S(1.020 \text{ GeV}) = -0.54$, $A_{CP}^S(1.25 \text{ GeV}) = -0.95$, and $A_{CP}^S(1.50 \text{ GeV}) = +0.59$ (here the superscript S stands for the S -wave asymmetry). When the P -wave is included then the CP asymmetry is reduced to: $A_{CP}(1 \text{ GeV}) = -0.25$, $A_{CP}(1.020 \text{ GeV}) = +0.029$, $A_{CP}(1.25 \text{ GeV}) = -0.85$ and $A_{CP}(1.50 \text{ GeV}) = +0.495$. Let us note a particularly small asymmetry in the range of the ϕ resonance, where the P -wave amplitude dominates, and an inversion of the A_{CP} sign above 1.4 GeV. Due to cancellations between the ranges of the negative and positive asymmetries the resulting CP asymmetry averaged over the $m_{K^+K^-}$ range from threshold up to 1.8 GeV is rather small, equal to -0.05 . The averaged branching fraction for the same mass range equals to $9.6 \cdot 10^{-6}$. It is worthwhile to add that the S -wave gives to it the dominant contribution of $5.8 \cdot 10^{-6}$.

4. Conclusions

We have studied final state interactions between kaons in the $B^\pm \rightarrow K^+K^-K^\pm$ decays. An overall general agreement with the Belle and BaBar data has been obtained. Our formalism is based on the QCD factorization supplemented with the inclusion of the long distance K^+K^- interactions. The latter are taken

into account through the functional dependence of the scalar and vector form factors on the effective K^+K^- masses. A unitary model is constructed for the scalar non-strange and strange form factors in which three scalar resonances $f_0(600)$, $f_0(980)$ and $f_0(1400-1460)$ are naturally incorporated. The scalar resonance $f_0(980)$ leads to the threshold enhancement of the S -wave K^+K^- amplitude. The K^+K^- structure seen near 1.5 GeV can be attributed to the third scalar resonance. A potentially large CP asymmetry is obtained in the mass spectrum dominated by the S -wave. It originates from violent phase variations of the two kaon scalar form factors which affect the K^+K^- effective mass dependence of the S -wave decay amplitudes. In general one can best study this effect away from the $\phi(1020)$ peak. We have shown, however, that even under the ϕ maximum one observes nonnegligible helicity angle asymmetry. This effect originates from the interference between the S - and P -waves.

Our approach presented here for the $B^\pm \rightarrow K^+K^-K^\pm$ decays can be extended to study the $B^0 \rightarrow K^+K^-K_S^0$ reactions for which results of the time-dependent Dalitz analyses have been recently published by the Babar [17] and Belle [18] Collaborations. For further studies of the charged B decays new experimental data with better statistics are needed. Such data already exist! For example, the Belle Collaboration has now five times larger data sample than that used in their publication [1] analysed by us here. Future results from LHCb and from super-B factories would also be very useful.

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