



Effective field theory, large number of particle species, and holography

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ABSTRACT

An effective quantum field theory (QFT) with a manifest UV/IR connection, so as to be valid for arbitrarily large volumes, can successfully be applied to the cosmological dark energy problem as well as the cosmological constant (CC) problem. Motivated by recent approaches to the hierarchy problem, we develop such a framework with a large number of particle species. When applying to systems on the brink of experiencing a sudden collapse to a black hole, we find that the entropy, unlike the total energy, now becomes an increasing function of the number of field species. An internal consistency of the theory is then used to infer the upper bound on the number of particle species, showing consistency with the holographic Bekenstein–Hawking bound. This may thus serve to fill in a large gap in entropy of any non-black hole configuration of matter and the black holes. In addition, when the bound is saturated the entanglement entropy matches the black hole entropy, thus solving the multiplicity of species problem. In a cosmological setting, the maximum allowable number of species becomes a function of cosmological time, reaching its minimal value in a low-entropy post-reheating epoch.

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For an effective quantum field theory (QFT) in a box of size L (providing an IR cutoff) and with the UV cutoff Λ , the entropy scales extensively, $S_{\text{QFT}} \sim L^3 \Lambda^3$, and therefore there is always a sufficiently large volume for which S_{QFT} would exceed the absolute Bekenstein–Hawking bound $S_{\text{BH}} \sim L^2 M_{\text{Pl}}^2$. Thus, considerations for the maximum possible entropy suggest that ordinary QFT may not be valid for arbitrarily large volumes, unless the UV and the IR cutoffs obey a constraint, $L \Lambda^3 \lesssim M_{\text{Pl}}^2$ [1]. However, at saturation, this bound means that an effective QFT should also be capable to describe systems containing black holes, since it necessarily includes many states with Schwarzschild radius much larger than the box size. There are however arguments for why an effective QFT appears unlikely to provide an adequate description of any system containing black holes [2,3]. So, ordinary QFT may not be valid for much smaller volumes, but would apply provided a more stringent constraint, $L^{3/2} \Lambda^3 \lesssim M_{\text{Pl}}^{3/2}$, is obeyed [1].

The above field-theoretical setup with the encoded holographic information has recently triggered a novel variable CC approach, generically dubbed that of ‘holographic dark energy’ (HDE) [4–6]. For the saturated case, $\Lambda \sim L^{-1/2}$, Λ gets depleted in an expanding universe so that at present times the effective cosmological constant (CC) generated by vacuum fluctuation (always dominated by UV modes) becomes so low that the need for fine-tuning in the ‘old’ CC problem gets eliminated. Moreover, with L of order of the present Hubble radius, the CC energy density $\sim L^{-2} M_{\text{Pl}}^2$ becomes of the same order as the observed dark energy of the universe [7].

An approach to the hierarchy problem put forward a decade ago [8] demonstrates that the true UV cutoff can be made many orders of magnitude smaller than the Planck mass M_{Pl} , provided proliferation of a large number of quantum fields does occur in the theory. Besides the higher-dimensional scenario of [8], hosting additional particles of the Kaluza–Klein type, a similar scenario has appeared recently [9–11] in four dimensions, where the stability of the weak scale was explained by postulating the existence of $N \sim 10^{32}$ gravitationally interacting species beyond the standard model. In a different context, looking at the renormalization-group running effects of M_{Pl} , the same conclusion was reached in [12]. Another, more natural explanation for the weakness of gravity in particle physics requires a switch of statistic from Bose/Fermi to infinite one at high energies and no introduction of artificially large numbers [13]. On nonperturbative grounds, a cutoff M_{Pl}/\sqrt{N} makes quantum entanglement universal [11], offering thus a resolution of the species problem in the physics of black holes.

In the present Letter, we examine a large- N formulation of the effective QFT with UV/IR mixing underlying the saturated HDE models, i.e. obeying $L^{3/2} \Lambda^3 \simeq M_{\text{Pl}}^{3/2}$. Note that the present size of the universe is large enough to reduce the UV cutoff down even to the dark-energy scale of 10^{-3} eV. Hence there is no need to introduce a large number N of particle species to reduce Λ any further, the motivation here being different than in models motivated by a stabilization of the weak scale. Next we list a few obvious benefits of the large- N formulation of an effective QFT with the proposed relationship between the UV and the IR cutoffs: (i) For systems on the verge of gravitational collapse we find their entropy to scale as $N^{1/4} L^{3/2} M_{\text{Pl}}^{3/2}$, realizing thus a possibility to complete a large

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gap in entropy between those systems, $L^{3/2}M_{\text{pl}}^{3/2}$ [2] and that of black holes, $L^2M_{\text{pl}}^2$; (ii) An internal consistency of the theory yields a bound, $N_{\text{max}} \simeq L^2M_{\text{pl}}^2$, and for the particular model we can trace L to the earliest moments in the history of the universe to obtain a minimum value for N_{max} ; (iii) For systems with $N = N_{\text{max}}$, when the access to the inside region of the system becomes impossible for the outside observer, we find that the entanglement entropy (scaling up with N) matches the black hole entropy (a universal quantity independent of the black-hole past history that should not depend on N). We note that a large- N scenario with the UV/IR connection in higher-dimensional settings was also recently discussed [14].

We begin our considerations by recapitulating the scenario of Cohen, Kaplan and Nelson in a different manner, such as to allow us to lay down an extra feature not exposed in [1], which turns out to be crucial for our arguments: a lower bound on the QFT energy density ρ_Λ (or equivalently on Λ).¹ For that purpose also the Bekenstein bound S_B [15] needs to be invoked. For a macroscopic system in which self-gravitation effects can be disregarded, the Bekenstein bound is given by a product of the energy and the linear size of the system, EL . In the context of the effective QFT it therefore becomes proportional to $\rho_\Lambda^4 L^4$. It is convenient to use this entropic bound to derive a lower bound on the energy density ρ_Λ since S_B is more extensive than S_{QFT} . The obvious hierarchy between $S_{\text{QFT}} \sim L^3 \Lambda^3$ and the entropic bounds, $S_B \sim \rho_\Lambda L^4$ and $S_{\text{BH}} \sim L^2 M_{\text{pl}}^2$,

$$S_{\text{QFT}} \leq S_B \leq S_{\text{BH}}, \quad (1)$$

yields ρ_Λ which is constrained from both above and below²:

$$\Lambda^3 L^{-1} \leq \rho_\Lambda \leq L^{-2} M_{\text{pl}}. \quad (2)$$

From (1) and (2) one sees that the concept of HDE emerges whenever $S_B \leq S_{\text{BH}}$; i.e., for a weakly gravitating system. This requirement automatically prevents formations of black holes, as the Bekenstein bound, in spite of its original connection with black hole physics, does not involve the Newton constant. The most commonly used saturated models do obey $S_B \simeq S_{\text{BH}}$; this requirement brings a system on the verge of gravitational collapse since then $L \simeq L_S$, with $L_S \sim M_{\text{pl}}^{-2} \rho_\Lambda L^3$ being the Schwarzschild radius. The only remaining bound is then the lower bound on ρ_Λ , which for $\rho_\Lambda \sim \Lambda^4$ simply becomes $\Lambda \gtrsim L^{-1}$. Below we are going to consider this bound as a consistency condition which will allow us to set the upper bound on N .

Now we are going to implement the new N degrees of freedom into the setup described by (1) and (2). The main observation is that while both S_{QFT} and S_B scale up with the number N of the species, S_{BH} stays a universal quantity that does not depend on N . The setup underlying the saturated HDE models with $N \gg 1$ thus becomes

$$NS_{\text{QFT}} \leq NS_B \simeq S_{\text{BH}}. \quad (3)$$

From (3) one readily sees that the lower bound $\Lambda \gtrsim L^{-1}$ remains unchanged. What we would like to find out is the upper bound on the entropy in QFT, $N\Lambda^3 L^3$. From $NS_B \simeq S_{\text{BH}}$, i.e.,

$$N\Lambda^4 L^4 \simeq L^2 M_{\text{pl}}^2, \quad (4)$$

¹ Obviously, ρ_Λ is the energy density corresponding to a zero-point energy and the cutoff Λ , being proportional to Λ^4 ($\Lambda \gtrsim m$) or $m\Lambda^3$ ($\Lambda \lesssim m$), where m is the mass of the QFT field. Throughout the Letter we shall explore the consequences of the former choice which turn out to be more interesting, and mention the latter case only at the end of the Letter.

² We note that the entropic bound of type (1) prevents to assign the CC a zero value.

one finds that the upper bound on NS_{QFT} becomes N -dependent,³

$$N\Lambda^3 L^3 \simeq N^{1/4} L^{3/2} M_{\text{pl}}^{3/2}. \quad (5)$$

Extracting Λ/L^{-1} from (5) and setting $\Lambda/L^{-1} \gtrsim 1$,

$$\frac{\Lambda}{L^{-1}} \simeq N^{-1/4} L^{1/2} M_{\text{pl}}^{1/2} \gtrsim 1, \quad (6)$$

one obtains the upper bound for N in the form

$$N \lesssim L^2 M_{\text{pl}}^2, \quad (7)$$

being of order of the Bekenstein–Hawking entropy itself. From (5) we see that the gap between systems on the verge of gravitational collapse having $S \sim L^{3/2} M_{\text{pl}}^{3/2}$ [2] and black holes begins to populate when N is increasing.⁴ When the bound on N (7) begins to saturate, a (normal) system begins to sustain a black hole entropy. Note that (7) corresponds to a loose bound in the late-time universe, $N \lesssim 10^{122}$. Also, at saturation, $L \simeq \Lambda^{-1}$, Eq. (7) does reproduce the gravity cutoff M_{pl}/\sqrt{N} obtained in the scenario [9–11].

We note the ratio Λ/L^{-1} in (6) as the increasing function of the IR cutoff L , which means that in an expanding universe the upper bound on N can be strengthened considerably, provided we have some knowledge on the behavior of L in the past. For that purpose we have to resort to a particular model. This makes the upper bound on N model-dependent. For the sake of illustration, we consider the popular Li's model [5]. This model belongs to a class of noninteracting and saturated HDE models, with a choice for L in the form of the future event horizon,

$$d_E = a \int_a^\infty \frac{da}{a^2 H}, \quad (8)$$

with a being a scale factor. Furthermore, we assume that our vacuum energy ρ_Λ is not responsible for the early-time inflation, and that all particle species came into being when early vacuum energy density decays into matter, in the process of reheating (see e.g., [17]). Ignoring subtle details of reheating, we assume an instantaneous process, occurring at T_{reh} . This amounts to knowing the behavior of $\rho_\Lambda \simeq L^{-2} M_{\text{pl}}^2$ during the radiation-dominated era, in which ρ_Λ occupies only a tiny fraction of the total energy density. In a two-component universe ρ_Λ evolution is governed by [5,18]

$$\Omega'_\Lambda = \Omega_\Lambda^2 (1 - \Omega_\Lambda) \left[\frac{1}{\Omega_\Lambda} + \frac{2}{c\sqrt{\Omega_\Lambda}} \right], \quad (9)$$

where the prime denotes the derivative with respect to $\ln a$. In (9) $\Omega_\Lambda = \rho_\Lambda/\rho_{\text{crit}}$, where ρ_{crit} is the critical density and ρ_Λ was parametrized as $\rho_\Lambda = (3/8\pi)c^2 M_{\text{pl}}^2 L^{-2}$. With $\Omega_\Lambda \ll 1$, $c \simeq 1$ and $\rho_{\text{crit}} \simeq \rho_{\text{rad}}$ we obtain

$$\rho_\Lambda \simeq \rho_{\text{rad}0} a^{-3}, \quad (10)$$

where $\rho_{\text{rad}0}$ denotes the radiation energy density at the present time. This in turn determines $L(a)$ as

$$L(a) \simeq M_{\text{pl}} \rho_{\text{rad}0}^{-1/2} a^{3/2}. \quad (11)$$

Equipped with these relationships and $T \sim a^{-1}$, we obtain a final expression for $N_{\text{max}} = L^2 M_{\text{pl}}^2$ as

³ In contrast, the upper bound for the total energy $N\Lambda^3 L^3$ stays N -independent. Also, (4) and (5) still continue to describe systems on the verge of gravitational collapse since now the Schwarzschild radius scales up with N .

⁴ Another resolution of this problem involves curved space configurations called *monsters* [16].

$$N_{\max} \simeq M_{\text{Pl}}^4 \rho_{\text{rad}0}^{-1} (T_{\text{reh}}/T_0)^{-3}, \quad (12)$$

where T_0 is the present temperature of the universe. In supergravity theories if the reheating temperature after inflation is too high one inevitably overproduce gravitinos. Plugging in the relevant numbers with $T_{\text{reh}}^{\max} \simeq 10^7$ GeV [19] as to avoid troubles with the overproduction of gravitinos, one gets $N_{\max} \simeq 10^{68}$, a considerably stringent requirement than 10^{122} . The bound $N_{\max} \simeq 10^{32}$ [9–12], obtained in QFTs without the holographic constraint, by noting that we have not seen any strong gravity in the particle collisions, is still more stringent. We stress once again that the bound obtained here is quite model-dependent. In addition, a higher T_{reh} could reduce N_{\max} considerably. Also, if ρ_Λ is to play any role in early-time inflation, one expects much severe constraints on N_{\max} . Still, our bound is much less than the entropy of the CMB photons or relic neutrinos in the present universe ($\sim 10^{88}$).

Now we show that when the bound (7) is saturated, the entanglement entropy S_{ent} [20] computed in the proposed QFT setup can be the origin of black hole entropy. When the bound (7) is not saturated, an observer outside of the box of size L has (at least theoretically) an unlimited access to the interior of the box. Consequently, the entanglement entropy, measuring quantum-mechanical correlations between the box and the space outside of the box, is zero. However, at saturation of (7) the physical horizon forms, and consequently an outside observer lacks any information about the interior of the box. Thus, both the entanglement and the black hole entropy then become nonzero.⁵ A pressing problem in identification of S_{BH} with S_{ent} is the multiplicity of species problem [22]. Incidentally, S_{ent} should depend on N , while S_{BH} lacks any information about number of species. We show below that the proposed UV/IR mixing, together with the bound (7), easily resolves this dilemma. In dealing with an overall pure state, S_{ent} should behave nonextensively, i.e., should depend only on the surface $A(L) \sim L^2$ separating the box from the rest. On the other hand, quantum correlations between the subsystems in local QFT are taken care by the UV cutoff. Hence we have

$$S_{\text{ent}}(L) \simeq \Lambda^2 A(L). \quad (13)$$

By invoking (4) we have

$$\Lambda^2 \simeq N^{-1/2} L^{-1} M_{\text{Pl}}. \quad (14)$$

Noting that S_{ent} should scale up with N one has

$$S_{\text{ent}}(L, N) \simeq N \times (N^{-1/2} L^{-1} M_{\text{Pl}}) \times A(L) \simeq N^{1/2} M_{\text{Pl}} L. \quad (15)$$

With $N_{\max} = L^2 M_{\text{Pl}}^2$, one immediately arrives at

$$S_{\text{ent}}(L, N_{\max}) \simeq M_{\text{Pl}}^2 L^2 \simeq S_{\text{BH}}. \quad (16)$$

Thus, we have seen how the proposed UV/IR mixing, together with the bound on the number of species, settles the problem of species.

In considering the second option, $\rho_\Lambda \sim m\Lambda^3$ ($\Lambda \lesssim m$), one should replace the consistency relation $\Lambda \gtrsim L^{-1}$ with $m \gtrsim L^{-1}$,

which is nothing but a trivial statement about encompassment of the modes within the box. So, N cannot be restricted before $\Lambda \gtrsim L^{-1}$ is imposed by hand. This is, of course, a quite plausible constraint for any QFT. The upper bound for $N L^3 \Lambda^3$ then becomes $m^{-1} L M_{\text{Pl}}^2$, which means that the gap between normal systems and black holes can never be fully populated. The same is also true for the entanglement entropy. This makes this case less interesting.

In conclusion, we have developed a promising QFT setup of Cohen, Kaplan and Nelson with a large- N new degrees of freedom. The holographic ingredient implemented via the specific UV/IR mixing makes the setup valid in an arbitrarily large volume, such that successful application both to particle physics and cosmology becomes possible. By using thermodynamics with large N , we have shown that it is possible to bridge a gap in entropy between the systems on the verge of gravitational collapse and the black holes themselves. Drawing on the internal consistency of the theory and cosmological evolution of the IR cutoff, we have obtained the upper bound for the number of particle species N . Finally, a resolution of the species problem comes out naturally due to the proposed UV/IR relationship.

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⁵ A small nonvanishing entanglement entropy emerges even for systems with $L \simeq L_S$ and $N \lesssim N_{\max}$, as for the systems with artificially created horizons [21]. However, the only possible way to physically prevent the access to a part of the system is to put a closed surface separating the two subsystems on the event horizon.