Large Magnetoresistance in Diluted Magnetic Semiconductors in Quasi-two Dimensional Geometry

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Abstract

The carrier-induced magnetic solitons in diluted magnetic semiconductors have been introduced by using the gauge-invariant effective Lagrangian. We have discussed the large magnetoresistance effects in diluted magnetic semiconductors in quasi-two dimensional geometry, by using the path-integral method and the supersymmetric sigma formula. We have considered the interaction effects of the localization in DMS. The lowest order contribution in the long-range interactions, which are mediated with massless gauge fields between the magnetic solitons, is given by Hartree and Fock processes for the self-energy.

Keywords: diluted magnetic semiconductor, magnetoresistance, metal-insulator transition

1 Introduction

Diluted magnetic semiconductors (DMSs) have attracted much attention because of the possibility of utilizing both charge and spin degrees of freedom in spintronics devices [1-5]. Especially, Mn doping in InAs and GaAs leads to ferromagnetism and interest magneto-optical transport properties. Recently the successful synthesis and properties of new series of DMSs, BaK(Zn,Mn)As [6,7], Li(Zn,Mn)As [8], and Li(Zn,Mn)P [9,10], have been reported. The spintronic applications of DMSs may soon become a reality once the Curie temperature $T_c$ exceeds room temperature [11]. Nonetheless, understanding the physical properties of DMSs remains a major challenge. Large magnetoresistance effects in diluted magnetic semiconductors (DMS) have been reported [12]. In addition, interesting phenomena such as the photo-induced magnetic polaron in diluted magnetic semiconductors have been discovered [13,14]. These works stimulated us to the study of the carrier-induced magnetic soliton, which is an interesting and challenging subject. The microscopic-formation mechanism of the hole-induced magnetic soliton is very difficult to understand, because of cooperation phenomena such as the remarkable change of spin exchange interaction among Mn ions by the hole [15]. III-V-based DMSs [16,17], especially ferromagnetic p-type (In, Mn)As [18], offer an opportunity to explore various aspects, localization and negative magnetoresistance of carrier transport in the presence of cooperation phenomena [19]. Kanazawa [20-22] has discussed the localization mechanism in the...
metal-insulator transition, using the gauge-invariant Lagrangian density for the hole-induced magnetic solitons. In this study, we have discussed the large magnetoresistance effects in diluted magnetic semiconductors in quasi-two dimensional geometry, extending the previous formula [20-22] and the supersymmetric sigma formula.

2 A model system

It has been suggested that the ferromagnetic interaction induced by the hole in DMS seems to be cooperative and non-linear. In order to argue in the gauge-invariant formula, we shall introduce the non-linear gauge fields (Yang-Mills fields) $A_\mu^a$, which mediate the effective ferromagnetic interaction induced by the hole. It has been proposed that the hedgehog-like soliton in three-dimensional system is specified by rigid-body rotation, which is related to gauge fields of SO(4) symmetry for $S^3$ [23-25]. Thus it is thought that the non-linear gauge fields $A_\mu^a$ introduced by the hole have a local SO(4) symmetry. Then we have assumed that the SO(4) quadruplet fields, $A_\mu^a$, are spontaneously broken around the doped hole through the Anderson-Higgs mechanism, in the III-V-based DMS with magnetic manganese ion-doping. We set the symmetry breaking $\langle 0|\phi_a|0 \rangle = \langle 0, 0, 0, \mu \rangle$ of the Bose field $\phi_a$ in the Lagrangian density as follows,

$$
L = \frac{1}{2} \left( \partial_i S_j - g_1 \varepsilon_{ijk} A_{\mu}^a S^k \right)^2 \\
+ \psi^+ \left( i \partial_0 - g_2 T_a A_{\mu}^a \right) \psi \\
- \frac{1}{2m} \psi^+ \left( i \nabla - g_2 T_a A_{\mu}^a \right) \psi \\
- \frac{1}{4} \left( \partial_\nu A_{\mu}^a - \partial_\mu A_{\nu}^a + g_3 \varepsilon_{abc} A_{\mu}^b A_{\nu}^c \right)^2 \\
+ \frac{1}{2} \left( \partial_\mu \phi_a - g_4 \varepsilon_{abc} A_{\mu}^b \phi_c \right)^2 \\
- \lambda^2 \left( \phi_a \phi_a - \mu^2 \right)^2.
$$

(1)
After the symmetry breaking \( \langle 0 | \phi_\alpha | 0 \rangle = \langle 0, 0, 0, \mu \rangle \), we can obtain the effective Lagrangian density,

\[
L_{\text{eff}} = \frac{1}{2} \left( \partial_i S^i - g_1 \varepsilon_{ijk} A^b_i S^k \right)^2 \\
+ \psi^+ (i \partial_0 - g_2 T_a A^a_0) \psi \\
- \frac{1}{2m} \psi^+ \left( i \nabla - g_2 T_a A^a_{(\mu \neq 0)} \right)^2 \psi \\
- \frac{1}{4} \left( \partial_\mu A^a_\mu - \partial_\mu A^c_\mu + g_3 \varepsilon_{abc} A^b_\mu A^c_\mu \right)^2 \\
+ \frac{1}{2} \left( \partial_\mu \phi_a - g_4 \varepsilon_{abc} A^b_\mu \phi_c \right)^2 \\
+ \frac{1}{2} m_1^2 \left[ (A^1_\mu)^2 + (A^2_\mu)^2 + (A^3_\mu)^2 \right] \\
+m_1 \left[ A^1_\mu \partial_\mu \phi_2 - A^2_\mu \partial_\mu \phi_1 \right] \\
+m_1 \left[ A^2_\mu \partial_\mu \phi_3 - A^3_\mu \partial_\mu \phi_2 \right] \\
+m_1 \left[ A^3_\mu \partial_\mu \phi_1 - A^1_\mu \partial_\mu \phi_3 \right] \\
+ g_4 m_1 \left\{ \phi_4 \left[ (A^1_\mu)^2 + (A^2_\mu)^2 + (A^3_\mu)^2 \right] \right\} \\
- g_4 m_1 \left\{ A^1_\mu \left[ \phi_1 A^1_\mu + \phi_2 A^2_\mu + \phi_3 A^3_\mu \right] \right\} \\
- \frac{m_2^2}{2} (\phi_4)^2 - \frac{m_2^2 g_4}{2m_1} (\phi_4)^2 - \frac{m_2^2 g_4}{8m_1^2} (\phi_4 \phi_4)^2,
\]

(2)

where \( S^i \) is the spin of Mn, \( \psi \) is the Fermi field of the hole, \( m_1 = \mu \cdot g_4, m_2 = 2(2)^{1/2} \nu \cdot \mu \). Recent study [26] shows that carriers of the hole seem to be coupled to Mn spins by an antiferromagnetic Heisenberg exchange interaction. Thus \( \hat{j} \) corresponds to the reverse direction of the spin one of the hole. The effective Lagrangian describes three massive gauge fields \( A^1_\mu, A^2_\mu, \) and \( A^3_\mu \) and one massless gauge field \( A^4_\mu \). The generation function \( Z[J] \) for Green functions is shown as follows,

\[
Z[J] = \int \mathcal{D}A \mathcal{D}B \mathcal{D}C \mathcal{D} \psi^+ \mathcal{D} \psi \mathcal{D} \phi \\
\cdot \exp i \int d^4x \left( L_{\text{eff}} + L_{\text{GF+FP}} + J \cdot \Phi \right),
\]

(3)

\[
L_{\text{GF+FP}} = B^a \partial^\mu A^a_\mu + \frac{1}{2} \alpha B^a B^a + i \bar{\phi}^a \partial^\mu \mathcal{D}_\mu C^a,
\]

(4)

where \( B^a \) and \( C^a \) are the Nakanishi-Lautrup fields and Faddeev-Popov fictitious fields, respectively.

\[
J \cdot \Phi = J^a \mu A^a_\mu + J^a B^a + J_S \cdot S + J^a_C \cdot C^a + J^a_C C^a \\
+ \eta \bar{\psi} + \eta \bar{\psi} + J^a_\phi \phi_a
\]

(5)

BRS-quartet [27,28] in the present theoretical formula are \( (\phi_1, B^1, C^1, \bar{C}^1), (\phi_2, B^2, C^2, \bar{C}^2), (\phi_3, B^3, C^3, \bar{C}^3), \) and \( (A^4_{\mu \nu}, B^4, C^4, \bar{C}^4) \). Where \( A^4_{\mu \nu} \) is the longitudinal component of \( A^4_\mu \). Thus we need these fields for the unitality condition, although these fields are unobservable.
and fictitious ones. Because masses of \( A_1^μ, A_2^μ \) and \( A_3^μ \) are created through the Anderson-Higgs mechanism by introducing the hole, the fields \( A_1^μ, A_2^μ \) and \( A_3^μ \) exist around the hole within the length of \( \sim 1/m_1 \equiv R_C \). From the first term in Eq. (1), the spins \( S \) of Mn atoms are induced in the ferromagnet state, where the average spin is parallel to \( \hat{j} \) direction, within the length of \( \sim R_C \) around the hole. That is, the effective Lagrangian represents that the ferromagnetically aligned and fictitious ones. Because masses of \( A_1^μ, A_2^μ \) and \( A_3^μ \) exist around the hole within the length of \( \sim 1/m_1 \equiv R_C \). From the first term in Eq. (1), the spins \( S \) of Mn atoms are induced in the ferromagnet state, where the average spin is parallel to \( \hat{j} \) direction, within the length of \( \sim R_C \) around the hole. That is, the effective Lagrangian represents that the ferromagnetically aligned Mn spins form clusters, in which the hole is trapped, with the radius, \( R_C \sim 1/m_1 \). Especially Katsumoto et al. [12] indicated that the finite localization length \( l_c \) of the wave functions of holes plays a crucial role in the metal-insulator transition in (Ga, Mn)As. It looks like that the \( l_c \) might correspond to \( R_C \sim 1/m_1 \). Now we shall consider the localization mechanism of the hole-induced magnetic solitons by the random potential due to the alloy potential fluctuation in DMS. From Eq. (3), we can obtain the Green functions of the massive gauge fields \( A_μ^a, A_μ^3, \) and \( A_μ^3 \) around the hole in ’t Hooft-Feynmann gauge as follows, that is, the Fourier transform of \( \langle A_μ^a A_μ^3 \rangle_{a=1,2,3} \) is

\[
D_R(p_1, \varepsilon_1) \sim \frac{g_{\mu\nu}}{\varepsilon_1^2 - (p_1^2 + m^2)} + \Pi_1. \tag{6}
\]

The thermal Green function \( g(k, \varepsilon_n) \), \( (\varepsilon = (2n + 1)\pi T, T \) being the temperature), of the hole is given as follows,

\[
g(k, \varepsilon_n) = \frac{1}{i\varepsilon_n - \xi - \sum(k, \varepsilon_n)}, \tag{7}
\]

where \( \xi = k^2/2m - E_p \)

\[
\sum(k, \varepsilon_n) = -\frac{g_2^2}{(2\pi)^2}\int dp_1 \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\varepsilon_1 \frac{\text{Im}g(k-p_1, \omega)\text{Im}D_R(p_1, \varepsilon_1)}{\omega + \varepsilon_1 - i\varepsilon_n - i\delta} \left( \tanh \frac{\varepsilon_1}{2T} + \coth \frac{\omega}{2T} \right). \tag{8}
\]

According to Hagston et al. [29], the random potential by the alloy potential fluctuations (APF) is approximated by a following analytic form,

\[
V_{APF}(r) \propto -|E|\exp[-2r^2m*|E|^2/\delta], \tag{9}
\]

where \( E < 0 \) is a localization energy and \( m^* \) is the conduction band mass. \( \delta \) is the interaction range. For simplicity, we adopt the constant value, \( V_{APF}^0 \equiv V_{APF}(r = \delta) \). The propagator \( \Gamma(q, \omega_l) \) of the hole by the scattering is represented as,

\[
\Gamma(q, \omega_l) \sim \frac{n_i(V_{APF}^0)^2}{1 - n_i(V_{APF}^0)^2X(q, \omega_l)}. \tag{10}
\]

Here \( \omega_l \) is \( 2\pi lT, l \) is integer, and \( n_i \) is average concentration of the random potential due to APF.

\[
X(q, \omega_l) = \sum_k g(k + q, \varepsilon_n + \omega_l) \cdot g(-k, \varepsilon_n) \times \int_{-\infty}^{\infty} d\xi \int d\Omega \left[ i\varepsilon_n + i\omega_l - \xi - vq \right. \\
+ i\text{Im} \sum(k, \varepsilon_n + \omega_l) \cdot \text{sgn}(\varepsilon_n + \omega_l)]^{-1} \cdot \left[ i\varepsilon_n - \xi + i\text{Im} \sum(k, \varepsilon_n) \cdot \text{sgn}(\varepsilon_n) \right]^{-1}. \tag{11}
\]

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where $v = k/m$.

The integration over $d\Omega$ is over the angle of $v$. If $\varepsilon_n(\varepsilon_n + \omega_l) < 0$, $X(q, \omega_l)$ is evaluated for small $q$ and $\omega_l$ as follows,

$$X(q, \omega_l) \propto \frac{1}{\text{Im} \sum(k, \varepsilon_n)} \left[ 1 - Dq^2 \frac{1}{\text{Im} \sum(k, \varepsilon_n)} - |\omega_l| \frac{1}{\text{Im} \sum(k, \varepsilon_n)} \right]$$

(12)

Here $D = \frac{v_f^2}{d \cdot \text{Im} \sum(k, \varepsilon_n)}$ is the diffusion constant. $v_f$ and $d$ are Fermi velocity and dimension of system, respectively.

Thus $\Gamma(q, \omega_l)$ is represented as

$$\Gamma(q, \omega_l) \propto \frac{1}{\left( \text{Im} \sum(q, \varepsilon_n) \right)^2} \left[ Dq^2 + |\omega_l| \right]$$

(13)

Taking into account $\text{Im} \sum(k, \varepsilon_n)$ in the diffusion constant $D$, Eq. (12) shows that the massive gauge fields $A_{\mu}^1$, $A_{\mu}^2$, and $A_{\mu}^3$ around the hole influence strongly the diffusive and localization property of the hole transport. Now we shall consider the interaction effects of the localization in DMS. The lowest order contributions in the long-range interactions, which are mediated with massless gauge fields $A_{\mu}^4$ in Eq. (2), between the magnetic solitons are given by the Hartree and Fock processes for the self-energy. From Eq. (2), we can obtain the Green function of the massless gauge field $A_{\mu}^4$, that is, the Fourier transform of $<A_{\mu}^4A_{\nu}^4>$ is $G_4(k, \omega) \sim \{g_{\mu\nu}/(\omega^2 - k^2 + \Pi)\}$. In the random potential, Hartree-Fock processes coupled with random potential by the alloy potential fluctuation give important contributions. First, we shall consider the Fock processes. The lowest order contribution of the Fock processes is evaluated as follows,

$$\Sigma_F(k, \epsilon) \simeq -\frac{T}{\left( \text{Im} \sum(k, \varepsilon_n) \right)^2} \sum_{\omega_l} \sum_q g_\nu^2 \cdot G_4(q, \omega_l)$$

$$\cdot g(k + q, \varepsilon_n + \omega_l) \cdot [Dq^2 + |\omega_l|]^{-2}$$

$$-T \sum_{\omega_l} \sum_q g(-k + q, \varepsilon_n + \omega_l)$$

$$\cdot g_\nu^2 \cdot G_4(k' - k'', \omega_l)$$

$$\cdot \left[ \frac{1}{2\pi N(0)} \left( \frac{1}{\text{Im} \sum(\varepsilon_n, \omega_l)} \right)^2 \cdot \frac{1}{Dq^2 + |\omega_l|} \right]^2$$

$$\cdot g(k', \varepsilon_n) \cdot g(-k' + q, \varepsilon_n + \omega_l)$$

$$\cdot g(k'', \varepsilon_n + \omega_l) \cdot g(-k'' + q, \varepsilon_n).$$

(14)

Where $N(0)$ is the density of states per spin at the Fermi energy. If the interaction, $g_\nu^2 \cdot G_4(q, \omega_l)$, can be considered as independent of $q$ and $\omega_l$ in the region of the summation and integration and does not have very singular $q$-dependence at relatively large $q$, we can replace $g_\nu^2 \cdot G_4(q, \omega_l)$ and $g_\nu^2 \cdot G_4(k' - k'', \omega_l)$ by the constant, $g_\nu^2 \cdot G$ and the average over the scattering processes across the Fermi surface,

$$2N(0)g_\nu^2 < G_4(k' - k'', \omega_l) > \simeq 2N(0)g_\nu^2 < G_4(k' - k'', 0) > \equiv 2N(0)g^2 \bar{G},$$

respectively. Where,
implies the average over \( k' \) and \( k'' \) with \(|k'| = |k''| = k_F\) and the dynamics of the interaction is ignored since it plays minor roles for large momentum transfer. Thus the quantum correction to the Fock self-energy, \( \Sigma_F \), is given by

\[
\Sigma_F(k, \epsilon_n) \sim N(0) \cdot g^2 \cdot (G + \bar{G}) \cdot \frac{\text{Im} \Sigma(k, \epsilon_n)}{2\pi \epsilon_F} \cdot \frac{3\sqrt{3}\pi^2}{4} \cdot \left[ 2 + \sqrt{\frac{2\pi T}{\text{Im} \Sigma(k, \epsilon_n)}} \cdot \xi \left( \frac{1}{2}, \frac{\epsilon_n}{2\pi T} + \frac{1}{2} \right) \right] \cdot \left[ -\frac{i\text{Im} \Sigma(k, \epsilon_n)}{2} \cdot \text{sgn}(\epsilon_n) \right],
\]

where \( \xi(x, y) \) is the generalized zeta function. Through the similar process, we can get the quantum correction to the Hartree self-energy, \( \Sigma_H \), as follows,

\[
\Sigma_H(k, \epsilon_n) \sim 2N(0) \cdot g^2 \cdot G \cdot \frac{\text{Im} \Sigma(k, \epsilon_n)}{2\pi \epsilon_F} \cdot \frac{3\sqrt{3}\pi^2}{4} \cdot \left[ 2 + \sqrt{\frac{2\pi T}{\text{Im} \Sigma(k, \epsilon_n)}} \cdot \xi \left( \frac{1}{2}, \frac{\epsilon_n}{2\pi T} + \frac{1}{2} \right) \right] \cdot \left[ -\frac{i\text{Im} \Sigma(k, \epsilon_n)}{2} \cdot \text{sgn}(\epsilon_n) \right].
\]

We shall consider the transport property in the randomly distributed system of the hole-induced magnetic solitons in DMS, by using the effective Lagrangian of diffusion modes. In terms of the four-component supervector \( \psi[30] \), the Lagrangian in this system takes the form,

\[
L = i \int \bar{\psi}(r)(-H_0 - V(r) + \frac{1}{2}(\omega + i\delta)\Lambda)\psi(r) dr
\]

\[
H_0 = \varepsilon + \frac{1}{2m}\Delta + \mu.
\]

where \( V(r) \) is the random potential, and \( \Lambda \) is the diagonal supermatrix

\[
\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

with \( 1 \) the \( 4 \times 4 \) unity matrix. The potential will be regarded as a random quantity with a Gaussian \( \delta \)-correlation distribution

\[
\langle V(r)V(r') \rangle = \frac{\text{Im} \Sigma(k_F, \epsilon_F)}{2\pi \nu} \delta(r - r'),
\]

\[
\varepsilon = \frac{\mu}{2m}. 
\]
where \( \Sigma(k_F, \epsilon_F) \) of the self energy of the hedgehog-like solitons [20], and \( \nu \) is the state density per spin. After averaging, Eq. (17) can be rewritten in the form,

\[
L_{\text{eff}} = \int \left[ -i\bar{\psi}H_0\psi + \frac{\text{Im}\Sigma(k_F, \epsilon_F)}{4\pi \nu} (\bar{\psi}\psi)^2 \right. \\
\left. - \frac{i(\omega + i\delta)}{2} \bar{\psi}\Lambda\psi \right] dr \\
\sim \int \left[ -i\bar{\psi}(H_0 + \frac{i(\omega + i\delta)}{2} \Lambda + \frac{\text{Im}\Sigma(k_F, \epsilon_F)}{2} Q)\psi \right] dr \quad (22)
\]

with the \( 8 \times 8 \) supermatrix \( Q \) satisfying the following self-consistency equation,

\[
Q = \frac{2}{\pi \nu} \int \bar{\psi}\psi \exp(-L_{\text{eff}}) D\psi \equiv \frac{2}{\pi \nu} \langle \bar{\psi}\psi \rangle_{\text{eff}}. \quad (23)
\]

Then the free energy \( F(Q) \) can be written as

\[
F(Q) = \int \left[ -\frac{1}{2} \text{str} \ln(-iH_0 - \frac{i}{2} (\omega + i\delta)\Lambda + \frac{\text{Im}\Sigma(k_F, \epsilon_F)}{2} Q) \right. \\
\left. + \frac{\pi \nu \text{Im}\Sigma(k_F, \epsilon_F)}{8} \text{str} Q^2 \right] dr. \quad (24)
\]

In the case of low values of \( \omega \), the free energy for all \( Q \)'s satisfying the condition \( Q^2 = 1 \) differs little from the minimum value. All such zero-trace matrices can be written in the form, \( Q = V\Lambda V^\dagger \), where \( V \) is an arbitrary unitary supermatrix, \( VV^\dagger = 1 \). In order to discuss the spin dynamics and electron hopping, we envisage an effective hamiltonian, \( H \), for the magnetic-soliton, \( O(r_i) \), which is introduced in Eq. (2),

\[
H = -J \sum_{<i,j>} \left[ \cos(\theta_{ij}/2) \right]^2 O(r_i) \cdot O(r_j) \\
+ \frac{1}{2} K \sum_{i \neq j} \frac{O(r_i) \cdot O(r_j)}{|r_i - r_j|} \quad (25)
\]

and the first sum taken only over nearest neighbors the distance between each magnetic soliton is \( \leq 2R_c \) and the second taken over all pairs (the distance \( |r_i - r_j| \geq 2R_c \) [15]. \( S_i \equiv \sum_{i \in \{4/3\} \pi R_c^3} S_i \).

That is, \( S_i \) is the summation of the ferromagnetic spin, \( S_i \), of Mn within \( \sim (4/3)\pi R_c^3 \) around the photo-induced hole at the site \( r_i \). \( S_i \) represents the effective spin of the soliton \( O(r_i) \). \( \theta_{ij} \) is the angle between \( S_i \) and \( S_j \). The first term corresponds to short-range ferromagnetic ordering interaction and the second corresponds to long-range frustration. Although the first term of the effective Hamiltonian in Eq. (24) cannot be derived immediately from the effective Lagrangian in Eq. (1), this term can be introduced approximately as follows. When the magnetic soliton, \( O(r_i) \), with the effective spin \( S_i \) is located in the nearest neighbor of the magnetic soliton, \( O(r_j) \), with the effective spin \( S_j \), holes are hopping between two solitons \( O(r_i) \) and \( O(r_j) \). If \( S_i \) is parallel to \( S_j \), p-d exchange interaction induces much reduction of the
kinetic energy. This hopping term between the nearest neighbors of magnetic solitons (clusters) leads to an additional term in the $\sigma$-model describing a coupling of the supermatrices, $Q_i$, corresponding to different magnetic solitons (clusters). Approximately, we get the following free energy, using the formula of the model of a granulated metal [31,32].

$$\tilde{F}(Q) = \text{str}(- \sum_{<i,j>} J_{ij} \tilde{Q}_i \tilde{Q}_j + \frac{i}{4} (\omega + i\delta) \sum_i \Delta_i^{-1} \Lambda Q)$$

(26)

$J_{ij} = J[\cos(\theta_{ij}/2)]^2 \frac{1}{\Delta_i \Delta_j}$. Where $\Delta_i$ is the mean energy level spacing at the magnetic soliton (cluster) $O(r_i)$ and $J > 0$. In the limit $J_{ij} > 1$, only small variations of supermatrix $Q$ in space are important. Assuming that $\Delta$ is same for all granules of the diluted magnetic semiconductor, $J_{ij}$ depends on $|r_j - r_i|$, in this limit one comes to the continuous model of the free energy function as follows,

$$\text{Eq.(26)} \approx \frac{\pi}{4\Delta} \text{str}(\frac{1}{2} \sum_i \frac{4\Delta}{\pi} (\sum_j J_{ij} (r_i - r_j)^2 (\frac{dQ}{dx})^2)) + \text{str}(\frac{i}{4} (\omega + i\delta) \sum_i \frac{1}{\Delta} \Lambda Q)$$

$$\sim \frac{\pi}{8\Delta L} \text{str} \left( \sum_j J_{ij} (r_i - r_j)^2 (\frac{dQ}{dx})^2 \right) + (2i\omega \Lambda Q) dx$$

$$\sim \frac{\pi \nu S}{8} \text{str} \int [D_0 (\frac{dQ}{dx})^2 + 2i\omega \Lambda Q] dx$$

(27)

Where $\nu$ is the density of state of the carrier at the Fermi surface, and $S$ is $\pi R_s^2$. In addition, we used the equation $\frac{1}{\Delta L} = \nu S$. Then we get the effective diffusion coefficient $D_{eff}$ approximately as follows,

$$D_{eff} \sim p \cdot \exp[-s(\alpha - \alpha_c)^{1/2}] / (\alpha - \alpha_c)^{3/2}.$$ 

(28)

Where $\alpha = 8J_{ij}$, and $p$ and $s$ are constant parameters. $D_{eff}$ becomes to be zero when $\alpha$ decreases to $\alpha_c$. In quasi-two dimensional geometry, we can estimate $\alpha_c$ from $(\frac{\alpha_c}{\pi})^{1/2} \ln(\frac{\alpha_c}{\pi}) = \frac{1}{2d-1} |d=2$. When high magnetic field imposes on this system, $\theta_{ij}$ decreases and then $\alpha$ increases remarkably. As a result, the effective diffusion coefficient $D_{eff}$ increases.

2.1 Conclusions

The large magnetoresistance mechanism in diluted magnetic semiconductors in quasi-two dimensional geometry is presented, by using the magnetic solitons and the supersymmetric sigma formula. We have considered the interaction effects of the localization in DMSs. The lowest order contribution in the long-range interactions, which are mediated with massless gauge fields between the magnetic solitons, is given by Hartree and Fock processes for the self-energy. Then we have introduced approximately the effective diffusion coefficient of the carrier in quasi-two dimensional geometry.
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