Physics Letters B 753 (2016) 670–676

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Dual condensates at finite isospin chemical potential

Zhao Zhang *, Qing Miao

School of Mathematics and Physics, North China Electric Power University, Beijing 102206, China

A R T I C L E   I N F O

Article history:
Received 31 July 2015
Received in revised form 24 December 2015
Accepted 3 January 2016
Available online 5 January 2016
Editor: J.-P. Blaizot

A B S T R A C T

The dual observables as order parameters for center symmetry are tested at finite isospin chemical potential μ_I in a Polyakov-loop enhanced chiral model of QCD with physical quark masses. As a counterpart of the dressed Polyakov-loop, the first Fourier moment of pion condensate is introduced for $\mu_I > m_\pi/2$ under the temporal twisted boundary conditions for quarks. We demonstrate that this dual condensate exhibits the similar temperature dependence as the conventional Polyakov-loop. We confirm that its rapid increase with T is driven by the evaporating of pion condensation. On the other hand, the dressed Polyakov-loop shows abnormal thermal behavior, which even decreases with T at low temperatures due to the influence of pion condensate. We also find that the dressed Polyakov-loop always rises most steeply at the chiral transition temperature, which is consistent with the previous results in Nambu–Jona-Lasinio (NJL) model and its variants without considering the center symmetry. Since both quantities are strongly affected by the chiral symmetry and pion condensation, we conclude that it is difficult to clarify the deconfinement transition from the dual condensates in this situation within this model.

1. Introduction

The main phenomena in QCD at finite temperature and density are the chiral restoration and deconfining phase transitions. In the chiral limit, the standard order parameter for chiral transition is the quark condensate. However, it is conceptually difficult to define an order parameter for deconfinement in QCD. Usually, the expectation value of the Polyakov-loop (PL) is adopted as the indicator for quark deconfining transition. This quantity is a true order parameter for center symmetry in pure Yang–Mills theory. Nevertheless, this symmetry is badly broken by the light dynamical quarks in QCD. These two order parameters had been extensively studied in lattice QCD. It suggests that both phase transitions are smooth crossovers at finite T and zero density and two pseudo-critical temperatures are very close to each other [1,2].

It is well known that the nonzero Dirac zero-mode density is responsible for the dynamical chiral symmetry breaking in QCD, according to the celebrated Banks–Casher relation [3]. An interesting question is to what extent the spectral properties of the Dirac operator contain the confinement information. Recently, some authors have tried to link the Dirac spectral modes to the PL or its equivalent quantities with the same winding number in the time direction [4–10]. In these studies, some dual observables are introduced as the new order parameters for center symmetry by using the twisted boundary conditions for quarks. Especially, it is demonstrated in the formalism of lattice QCD [5–7] that the dressed Polyakov-loop (DPL) interpolates between the chiral condensate and the thin PL. The studies from the functional methods [11–14] and effective models [15–17] also suggest the DPL shows the order parameter-like behavior like the thin PL.

In principle, one can construct many dual observables which transform in the same way as the thin PL under the center transformation [7,10]. They are true order parameters for deconfinement in the static limit $m \to \infty$, where m is the mass of dynamical quarks. However, the center symmetry is seriously broken in QCD since the light quark masses are very small. Then, a question naturally arises: to what extent do these quantities still contain the confinement information? Recently, it is demonstrated in the NJL model that the rapid rise of DPL near T_c is totally driven by the chiral transition [18]. The author attributes the reason to the lacking confinement of NJL. However, the following study based on several variants of NJL with (possible) confining elements sug-

* Corresponding author.
E-mail address: zhaozhang@pku.org.cn (Z. Zhang).

http://dx.doi.org/10.1016/j.physletb.2016.01.001
0370-2693/

gests that the rapid rise of DPL with T still happens in the chiral transition region and no effect related to the change of the confining properties of the propagator is observed [19]. The authors thus conclude that the DPL obtained in these models is not an appropriate order parameter for deconfinement. Note that the gluon degree with center symmetry is not considered in these model studies.

To further test weather the dual observables can be used as order parameters, we extend the previous study to finite isospin chemical potential μ_I by simultaneously considering the pion condensation and the twisted boundary conditions in this paper. We mainly concentrate on the thermal properties of two simple dual observables for $\mu_I > m_\pi/2$[1]: the DPL and the first Fourier moment of the generalized pion condensate. Here we refer to the later as the dual pion condensate (DPC), which is the counterpart of the DPL at finite μ_I. Due to the influence of pion condensate, the thermal property of DPL may change explicitly for $\mu_I > m_\pi/2$. Thus it is interesting to check whether the DPL still behaves like an order parameter in this situation. Second, similar to the DPL, the DPC transforms in the same manner as the thin PL under the center transformation. So it is also interesting to explore whether this simple dual quantity can be used to indicate the deconfinement transition at finite isospin density.

We employ the PL enhanced NJL model (PNJL) in our investigation by adopting the U(1)-valued boundary conditions. Compared to [17,18], the advantage of PNJL is that the PL dynamics is included to partially mimic the confinement, which is directly related to the center symmetry. Moreover, the pion condensate and PL obtained in lattice simulations [21,22] for $\mu_I > m_\pi/2$ can be well reproduced in this model [23]. The paper is organized as follows. In Sec. 2 the dual pion condensate is defined and the PNJL model with the twisted boundary conditions for $\mu_I > m_\pi/2$ is introduced. The numerical results and discussion are given in Sec. 3. In Sec. 4 we summarize.

2. Dual pion condensate and PNJL model with twisted boundary condition for $\mu_I > m_\pi/2$

2.1. Dual pion condensates for $\mu_I > m_\pi/2$

According to [5], the dual quark condensates are defined as

$$\Sigma_\sigma^{(n)} = -\int_0^{2\pi} \frac{d\phi}{2\pi} e^{-in\phi}\sigma(\phi), \tag{1}$$

where n is an integer and $\sigma(\phi)$ is the generalized quark condensate

$$\sigma(\phi) = \langle\bar\psi\psi\rangle_\phi = -\frac{1}{V}\langle Tr[(m+D_\phi)^{-1}]\rangle, \tag{2}$$

which is obtained with the twisted boundary condition in the time direction

$$\psi(x,\beta=1/T) = e^{i\phi}\psi(x,0). \tag{3}$$

The D_ϕ in (2) is the Dirac operator without the quark mass for the twisted angle ϕ. Note that $\phi=\pi$ corresponds to the physical boundary condition. The dressed PL is defined as the first Fourier moment of $\sigma(\phi)$, namely

$$\Sigma_\sigma^{(1)} = -\int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi}\sigma(\phi). \tag{4}$$

[1] It is well known that the charged pion condensation appears at zero and low temperature for $\mu_I > m_\pi/2$ in QCD [20].

In the lattice language, this quantity only includes the contributions of (infinite) closed loops with the winding number one in the compact time direction [5]. So it belongs to the same class as the thin PL under the center transformation.

In the same way, we can introduce the dual pion condensates at finite μ_I. The charged pion condensates for $\mu_I > m_\pi/2$ are defined as

$$\langle\bar\psi i\gamma_5\tau_+\psi\rangle = \pi^+ = \frac{\pi}{\sqrt2}e^{i\theta}, \quad \langle\bar\psi i\gamma_5\tau_-\psi\rangle = \pi^- = \frac{\pi}{\sqrt2}e^{-i\theta}, \tag{5}$$

where $\tau_\pm = (\tau_1 \pm \tau_2)/\sqrt2$ and τ_i is the Pauli matrix in quark flavor space. In (5) nonzero π indicates the spontaneous breaking of the isospin I_3 symmetry and the breaking direction is described by the phase factor θ. Without loss of generality, we adopt $\theta = 0$ in the following and the pion condensate is expressed as

$$\langle\bar\psi i\gamma_5\tau_1\psi\rangle = \pi. \tag{6}$$

Similar to (1), we can define the dual pion condensates

$$\Sigma_\pi^{(n)} = -\int_0^{2\pi} \frac{d\phi}{2\pi} e^{-in\phi}\pi(\phi), \tag{7}$$

where $\pi(\phi)$ is the generalized pion condensate under the boundary condition (3), which takes the form

$$\pi(\phi) = -\frac{1}{V}\langle Tr[i\gamma_5\tau_1(m+D_\phi)^{-1}]\rangle. \tag{8}$$

As mentioned, the DPC is defined as the first Fourier moment of $\pi(\phi)$, namely

$$\Sigma_\pi^{(1)} = -\int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi}\pi(\phi). \tag{9}$$

Analogous to the DPL (and also the dual density proposed in [10]), $\Sigma_\pi^{(1)}$ is gauge invariant which merely includes the contributions of closed loops with wingding number one. Thus it is another simple dual observable transforming in the same manner as the thin PL under the Z(3) center transformation. It is interesting to check whether this quantity also exhibits an order parameter-like behavior with increasing T at finite isospin density.

The previous studies [22,23] suggest that under the physical boundary condition, the pion condensate competes with the quark condensate for $\mu_I > m_\pi/2$. Or in other words, the quark condensate partially rotates into the pion condensate when the isospin chemical potential surpasses the half of the pion mass at zero T and their competition becomes more involved at finite T. We can expect that there may exist the similar interplay between these two condensates for other twisted boundary angles. This implies $\pi(\phi)$ affects $\sigma(\phi)$, and vice versa. So the thermal behavior of DPL at $\mu_I > m_\pi/2$ may deviate significantly from that at zero μ_I due to the influence of $\pi(\phi)$. We will test whether such a deviation still supports the DPL as an indicator for quark deconfinement transition with physical quark masses.

2.2. PNJL model for $\mu_I > m_\pi/2$ with twisted boundary condition

We adopt the following Lagrangian of two-flavor PNJL model:

$$\begin{aligned}\mathcal{L} = {} & \bar\psi\left(i\gamma_\mu D^\mu + \gamma_0\hat\mu - \hat m_0 - i\lambda\gamma_5\tau_1\right)\psi \\ & + g_s\left[\left(\bar\psi\psi\right)^2 + \left(\bar\psi i\gamma_5\vec\tau\psi\right)^2\right] - g_v^s\left(\bar\psi\gamma_\mu\psi\right)^2 \\ & - g_v^v\left(\bar\psi\vec\tau\gamma_\mu\psi\right)^2 - \mathcal{U}\left(\Phi,\bar\Phi,T\right),\end{aligned} \tag{10}$$

where the last term is the effective PL potential. This type of model has been used to study the DPL at zero density [15]. Compared to [15], we ignore the eight-quark interaction but include four-quark vector interactions with two different couplings. It is demonstrated in [24] that the mismatch between g_v^v and g_v^s can lead to non-anomaly flavor mixing at finite baryon and isospin densities.

The $\hat{m_0}$ is the matrix of current quark masses

$$\hat{m_0} = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}, \tag{11}$$

and we choose $m_u = m_d \equiv m$. The $\hat{\mu}$ is the matrix of quark chemical potentials

$$\hat{\mu} = \begin{pmatrix} \mu_u & \\ & \mu_d \end{pmatrix} = \begin{pmatrix} \mu + \mu_I & \\ & \mu - \mu_I \end{pmatrix}, \tag{12}$$

with

$$\mu = \frac{\mu_u + \mu_d}{2} = \frac{\mu_B}{3} \quad \text{and} \quad \mu_I = \frac{\mu_u - \mu_d}{2}. \tag{13}$$

The μ_B and μ_I in (13) are the baryon and isospin chemical potentials, which correspond to the conserved baryon and isospin charges, respectively. Following [22,23], we introduce a small parameter λ in (10), which explicitly breaks the I_3 symmetry.

The mean field thermal potential of PNJL model for $\mu_I > m_\pi/2$ under the physical boundary condition has been given in [23], where vector interactions are ignored. Its form is slightly modified when considering the vector interactions

$$\begin{aligned}\Omega = &\,\mathcal{U}(\Phi, \bar{\Phi}, T) - 2N_c \int \frac{d^3p}{(2\pi)^3}\left[E_p^- + E_p^+\right]\theta(\Lambda^2 - \vec{p}^2) \\ &- 2T \int \frac{d^3p}{(2\pi)^3}\Big\{ \ln\Big[1 + 3\big(\Phi + \bar{\Phi}e^{-(E_p^- - \mu')\beta}\big)e^{-(E_p^- - \mu')\beta} \\ &+ e^{-3(E_p^- - \mu')\beta}\Big] + \ln\Big[1 + 3\big(\bar{\Phi} + \Phi e^{-(E_p^- + \mu')\beta}\big)e^{-(E_p^- + \mu')\beta} \\ &+ e^{-3(E_p^- + \mu'))\beta}\Big] + \ln\Big[1 + 3\big(\Phi + \bar{\Phi}e^{-(E_p^+ - \mu')\beta}\big)e^{-(E_p^+ - \mu')\beta} \\ &+ e^{-3(E_p^+ - \mu')\beta}\Big] + \ln\Big[1 + 3\big(\bar{\Phi} + \Phi e^{-(E_p^+ + \mu')\beta}\big)e^{-(E_p^+ + \mu')\beta} \\ &+ e^{-3(E_p^+ + \mu')\beta}\Big]\Big\} + g_s(\sigma^2 + \pi^2) - g_v^s(\rho_u + \rho_d)^2 \\ &- g_v^v(\rho_u - \rho_d)^2, \end{aligned} \tag{14}$$

with the quasi-particle energy $E_p^\pm = \sqrt{(E_p \pm \mu_I')^2 + N^2}$ and $E_p = \sqrt{\vec{p}^2 + M^2}$ in which the two energy gaps are defined as

$$M = m - 2g_s\sigma, \tag{15}$$

$$N = \lambda - 2g_s\pi. \tag{16}$$

The μ' and μ_I' are the shifted quark and isospin chemical potentials

$$\mu' = \mu - 2g_v^s(\rho_u + \rho_d), \quad \mu_I' = \mu_I - 2g_v^v(\rho_u - \rho_d), \tag{17}$$

where $\rho_{u(d)}$ is the u (d) quark density.

In the following, we only consider the situation with finite μ_I and zero μ. In this case, the baryon number density is zero for $\phi = \pi$ and Φ equals to $\bar{\Phi}$ strictly. Minimizing the thermal dynamical potential (14), the motion equations for the mean fields σ, π, Φ and the density ρ_I are determined through the coupled equations

$$\frac{\partial\Omega}{\partial\sigma} = 0, \quad \frac{\partial\Omega}{\partial\pi} = 0, \quad \frac{\partial\Omega}{\partial\Phi} = 0, \quad \frac{\partial\Omega}{\partial\rho_I} = 0. \tag{18}$$

This set of equations is then solved for the fields σ, π, Φ and ρ_I as functions of T and μ_I.

Under the generalized boundary condition, the modified quark chemical potential μ' in (14) should be replaced by $iT(\phi - \pi)$ [5,10,15], which is nothing but an effective imaginary chemical potential. Strictly speaking, the μ' for $\phi \neq \pi$ should also contain the density-related contribution $2g_v^s(\rho_u + \rho_d)$ even if the real μ is zero. This is because the imaginary chemical potential also leads to a nonzero baryon number density. It has been shown in [15] that the coupling g_v^s only has significant effect on $\Sigma_\sigma^{(1)}$ for $T > 1.5T_c$ in PNJL. Since we are only interested in the thermal behavior of dual observables near and below T_c, the correction $2g_v^s(\rho_u + \rho_d)$ is ignored in our calculation. Note that the μ_I' is still real and keeps the form as (17). The reason is that it is the difference between μ_u' and μ_d' and their imaginary parts cancel each other out for $\phi \neq \pi$.

According to the definition of DPL (and also the DPC), the twisted boundary condition is imposed on the Dirac operator D_ϕ, and the bracket $\langle\cdots\rangle$ still keeps the antiperiodic condition with $\phi = \pi$ [5,6]. So in our calculation, the Φ as a function of T and μ_I is first obtained by solving (18) using the physical boundary condition. The other quantities, such as $\sigma(\phi)$, $\pi(\phi)$ and $\rho(\phi)$, are then determined by the following coupled equations:

$$\frac{\partial\Omega}{\partial\sigma(\phi)} = 0, \quad \frac{\partial\Omega}{\partial\pi(\phi)} = 0, \quad \frac{\partial\Omega}{\partial\rho_I(\phi)} = 0, \tag{19}$$

with Φ keeping its value for $\phi = \pi$. Such a treatment is consistent with [15].

2.3. Model parameters

In our calculation, the model parameters related to the NJL sector, such as the current quark mass m, the momentum cutoff Λ and the scalar coupling g_s, are all adopted from [23], which take the values

$$m = 5.5 \text{ MeV}, \ \Lambda = 0.651 \text{ GeV}, \ \text{and } g_s = 5.04 \text{ GeV}^{-2}, \tag{20}$$

respectively. The vector coupling g_v^v is fixed as $0.25g_s$, which is supported by the instanton liquid molecular model.[2]

As for the PL potential, we employ the logarithm form [25]. It has been reported that this type of $\mathcal{U}$ can reproduce the LQCD data at finite imaginary chemical potential, but the polynomial one (which is used in [23]) does not [15]. Following [15], the parameter T_0 in the logarithm potential is fitted as 200 MeV to reproduce the lattice pseudo-critical temperature T_c at zero density.

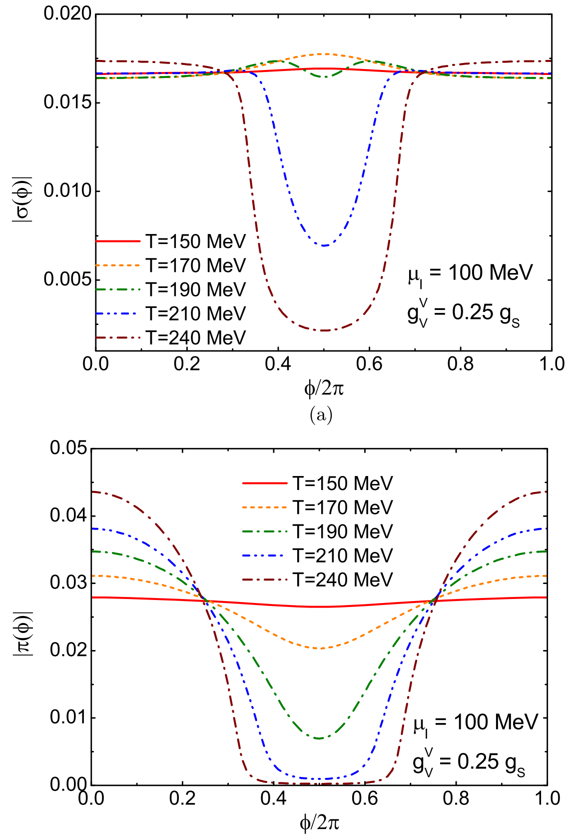
For computational convenience, a small λ with value $m/10$ is used. It is confirmed that the deviation of our main results from zero λ is negligible.

3. Numerical results and discussions

3.1. ϕ-dependence of quark and pion condensates

The generalized quark and pion condensates as functions of ϕ for $\mu_I = 100$ MeV at different temperatures are shown in Fig. 1. Fig. 1.a indicates that the shapes of $|\sigma(\phi)|$ for temperatures below and above T_c^χ are quite different: for $T = 210$ MeV and

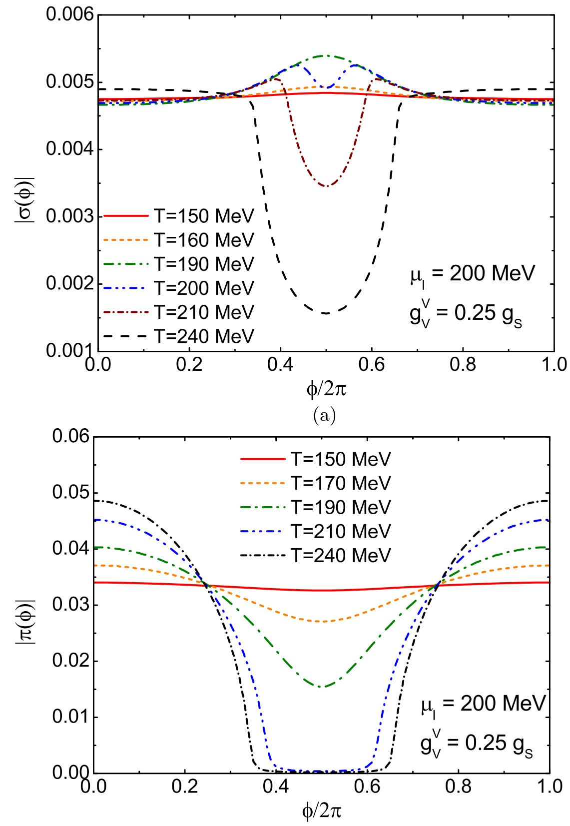
[2] In two flavor case, the acceptable theoretical value of g_v^v may be in the range $0.25g_s$–$0.5g_s$, where the lower and upper limits are determined by the instanton induced interaction and the Fierz transformation of the one gluon exchange interaction, respectively.



(a)

(b)

Fig. 1. The twisted angle dependences of the quark condensate $\sigma(\phi)$ and pion condensate $\pi(\phi)$ at $\mu_I = 100$ MeV for different temperatures.



(a)

(b)

Fig. 2. The twisted angle dependences of the quark condensate $\sigma(\phi)$ and pion condensate $\pi(\phi)$ at $\mu_I = 200$ MeV for different temperatures.

240 MeV (or $T > T_c^\chi$), the quark condensates are concave lines with $|\sigma(\pi)| < |\sigma(0)|$; but for $T = 150$ MeV and 170 MeV (or $T < T_c^\chi$), they are convex ones with $|\sigma(\pi)| > |\sigma(0)|$. The transition line between the concave and convex ones takes the wavy shape, as displayed in Fig. 1.a for $T = 190$ MeV (or $T \sim T_c^\chi$). All this is quite different from what is obtained in [5,12,15], where only concave curves emerge for vanishing μ and μ_I. Fig. 1.a also shows that $|\sigma(\phi \sim \pi)|$ first increases and then decreases with T due to the impact of pion condensate[3]; but in [5,12,15], $|\sigma(\phi \sim \pi)|$ always decreases with T.

In contrast, Fig. 1.b shows that all lines of $|\pi(\phi)|$ at different fixed T are concave curves. We see that in the fermionic-like region (namely the area for $\phi \sim \pi$), $|\pi(\phi)|$ decreases with T but it increases with T in the bosonic-like region (namely the area for ϕ near zero or 2π). In addition, the curve of $|\pi(\phi)|$ becomes more flat with decreasing T. All this is very similar to the ϕ-dependence of the quark condensate obtained for zero μ and μ_I [5,12,15]. The similarity can be understood in the following way: for $\mu_I > m_\pi/2$, the quark condensate partially turns into the pion condensate, and thus the later inherits some properties of the former. However, the ϕ-dependence of $|\sigma(\phi)|$ changes obviously due to such a transformation, as shown in Fig. 1.a.

Fig. 2 shows the ϕ-dependence of quark and pion condensates at different temperatures for $\mu_I = 200$ MeV. Compared with Fig. 1, we see that $\sigma(\phi)$ is suppressed and $\pi(\phi)$ is enhanced (suppressed) in the fermion-like (boson-like) region. But the ϕ- and T-dependences of these two condensates are still qualitatively consistent with that displayed in Fig. 1.

3.2. Thermal behaviors of dual condensates

Two dual condensates and other three (pseudo-) order parameters as functions of T for $\mu_I = 100$ MeV and 200 MeV are shown in Figs. 3–4, respectively. The quark and pion condensates are obtained with physical boundary condition $\phi = \pi$, which are normalized by $\sigma_0 = \sigma(T = 0, \mu = 0, \mu_I = 0)$; the dual condensates and PL are normalized by their corresponding values at $T = 240$ MeV. We mainly focus on the thermal behaviors of these quantities near the phase transitions.

Fig. 3 shows that $|\pi(\phi = \pi)|$ (Φ) decreases (increases) monotonically with T, but $|\sigma(\phi = \pi)|$ first increases (slowly) up to $T \sim 180$ MeV and then decreases. The similar T-dependences of these quantities are also observed in Fig. 4. These results are qualitatively in agreement with what is obtained in [23] by using the polynomial PL potential. As mentioned above, the increase of $|\sigma(\phi = \pi)|$ with T is due to the competition between the quark and pion condensates.

Consistent with Fig. 1.b, Fig. 3 indicates that the normalized DPC really behaves like an order parameter for center symmetry: analogous to the DPL obtained in [5,12,15], it keeps rather small value in low temperature region and gradually becomes larger with T. Like the thin PL, the DPC increases monotonically with T. However, the normalized DPL in Fig. 3 shows abnormal thermal behavior, which first reduces with T (up to $T \sim 180$ MeV) and then raises. Fig. 3 also shows that the DPL even becomes negative near and below $T \sim 190$ MeV. The similar T-dependences of DPL and DPC are also observed in Fig. 4.

[3] This is also observed in [23] and in other chiral model studies [26]. The reason for such an anomaly is that the quantity $\sqrt{\sigma^2 + \pi^2}$ always decreases with T but $|\pi|$ drops more quickly near the phase transition since λ is zero but m is finite.

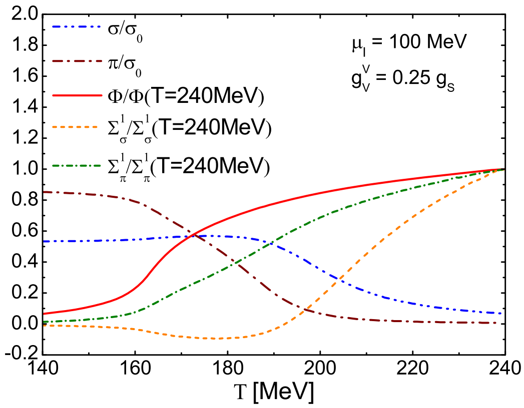


Fig. 3. The temperature dependences of the normalized conventional Polyakov-loop, quark and pion condensates and their corresponding dual parters at $\mu_I = 100$ MeV.

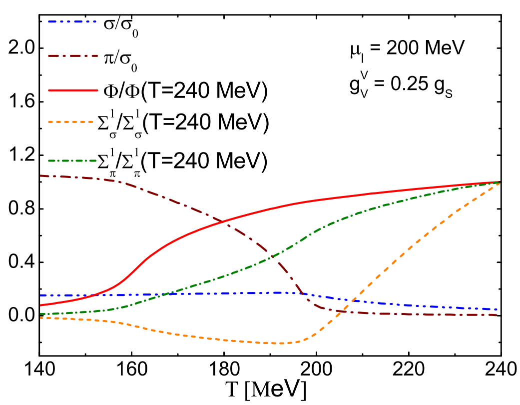


Fig. 4. The temperature dependences of the normalized conventional Polyakov-loop, quark and pion condensates and their corresponding dual parters at $\mu_I = 200$ MeV.

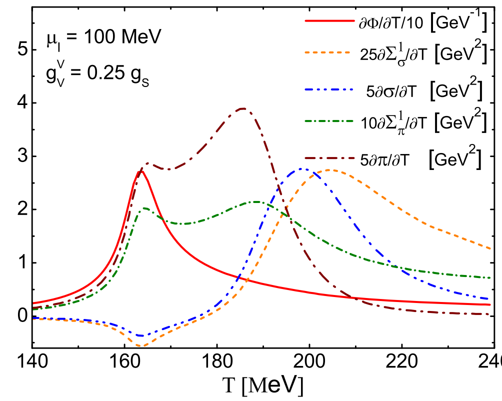


Fig. 5. The temperature dependences of the T-derivatives of the conventional Polyakov-loop, quark and pion condensates and their corresponding dual parters at $\mu_I = 100$ MeV.

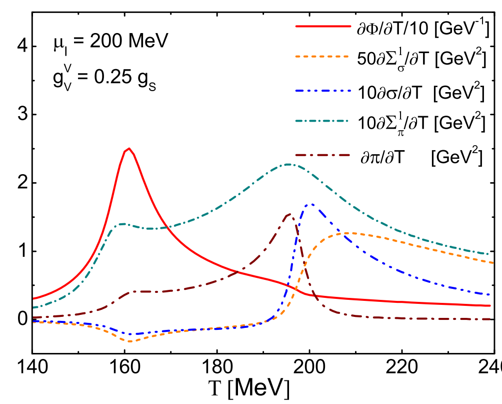


Fig. 6. The temperature dependences of the T-derivatives of the conventional Polyakov-loop, quark and pion condensates and their corresponding dual parters at $\mu_I = 200$ MeV.

The abnormal thermal behavior of DPL can be traced back to the non-concave lines of $\sigma(\phi)$ displayed in Figs. 1(a)–2(a). As mentioned, due to the influence of $\pi(\phi)$, $|\sigma(\phi)|$ increases with T below T_c^π (the critical temperature for the I_3 symmetry restoration) for $\phi \sim \pi$. This results in the DPL not always raising with T. Actually, Figs. 3–4 clearly show that when $|\sigma(\phi = \pi)|$ increases with T, the DPL decreases, and vice versa. So the DPL is quite sensitive to the T-dependence of the quark condensate. In contrast, Figs. 3–4 indicate that the DPL is insensitive to the thin PL, at least at low temperature region. All this suggests that the DPL obtained with physical quark masses mainly reflects chiral transition rather than deconfinement. Such a conclusion is in agreement with the claim given in [18] that the rapid change of DPL near T_c^σ in NJL is totally driven by the chiral restoration.

Following Ref. [18], we also calculate several susceptibilities which are defined as the T-derivatives of the quantities displayed in Figs. 3–4. As in [18], the peak of a susceptibility is used to locate the critical temperature. The susceptibilities as functions of T for $\mu_I = 100$ MeV and 200 MeV are shown in Fig. 5 and Fig. 6, respectively.

Fig. 5 shows that the PL susceptibility has only one peak, which indicates $T_c^P = 163$ MeV. But the other susceptibilities all have double peaks, one of which coincides with T_c^P due to the coupling between the PL and quark/pion condensate in PNJL. We see that the highest peaks of $\partial\Sigma_\sigma{}^1/\partial T$ and $\partial|\sigma|/\partial T$ in Fig. 5 are very close to each other, and the corresponding critical temperatures $T_c^{d\sigma}$ and T_c^σ are about 40 MeV lager than T_c^P. The coincidence of $T_c^{d\sigma}$ and T_c^σ is consistent with [18] even if the PL dynamics and pion condensate are considered in our calculations. In addition, Fig. 5 also shows that the critical temperatures $T_c^{d\pi}$ and T_c^π (extracting from $\partial\Sigma_\pi{}^1/\partial T$ and $\partial|\pi|/\partial T$, respectively) almost coincide, which are about 25 MeV lager than T_c^P. The slight difference between T_c^π and $T_c^{d\pi}$ can be traced back to a small $\lambda = 0.1m$ used in our numerical calculation. All these coincidences are also observed in Fig. 6. Actually, the accordance of T_c^π and $T_c^{d\pi}$ shown in Figs. 3–4 is completely analogous to $T_c^\sigma = T_c^{d\sigma}$ obtained in [18] for zero m. We thus get the similar conclusion that the rapid change of DPC near T_c^π is driven by the restoration of I_3 symmetry, even if it exhibits an order parameter-like thermal behavior as the PL.

Note that we also perform the similar calculations by varying g_V^v in this model. We confirm that the thermal properties of DPL and DPC shown in Figs. 1–6 do not change qualitatively.

3.3. The case in the chiral limit

Here we only show the results with physical quark masses. In the chiral limit with finite μ_I, the pion condensate appears at low temperatures but the quark condensate vanishes. Or in other words, the quark condensate totally turns into the pion condensate due to the nonzero μ_I. Correspondingly, the dual quark condensate is replaced by the dual pion condensate. In this case, the approach used in [18] for analyzing the DPL, such as the Ginzburg–Landau method, can be borrowed directly to study the DPC. We have checked that without the quark condensate, the T-dependence of DPC for finite μ_I is much more similar to that of DPL obtained in the chiral limit at zero μ_I [18]. Actually, this case is also analogous to the situation with physical quark masses and large μ_I, in which the pion condensate dominates and the suppressed quark condensate can be ignored.

3.4. Discussions

So beyond [18,19], we give further evidences that the dual observable may not really reflect deconfinement transition in the model studies, even if the center symmetry is considered. Note that in [18], the author still insists that the DPL calculated in other methods, such as the truncated Dyson–Schwinger Equation (tDSE) [13], can be used as an order parameter for deconfinement. However, the dual quark condensates obtained in NJL [18] or its nonlocal variants [19] and tDSE [13] are qualitatively consistent with each other. In addition, our result and Ref. [15] suggest that the T-dependence of DPL obtained in PNJL with center symmetry is also quite similar to that calculated in NJL type models without center symmetry. So if the rapid change of DPL with T merely indicates the chiral transition in NJL, one should be cautious to interpret it as the deconfinement transition in other effective theories or models of QCD. We thus argue that the so-called coincidence of chiral and deconfinement transitions obtained in tDSE using the DPL as the order parameter may also be problematic, just as that in NJL [17].

Our investigation suggests that the DPL is strongly affected by both the chiral and pion condensates. This implies that it is difficult to clarify the deconfinement transition from this quantity, at least in the model studies. Of course, PNJL is just a simple model and including PL in NJL may only partially reflect the connection between the (dynamically) center symmetry breaking and the DPL existing in QCD. Actually, as pointed out in [19], such a relation is totally ignored in the NJL model. So the DPL mainly indicates the chiral transition in [18,19] and our conclusions at finite isospin chemical potential may not really happen in QCD.

However, the results from PNJL may be still indicative for QCD. First, the center symmetry is severely violated by the light quarks. So it is very likely that the dual observables, such as the DPL, may not be so sensitive to the deconfinement transition, unless the dynamical quarks are heavy enough.[4] Second, formally, the definitions of the DPL and DPC involve quark fields which are naturally related to the quark and pion condensates, respectively. So it is not strange that the rapid change of the former mainly or even totally reflects the chiral transition in the chiral limit and that of the later just indicates the restoration of I_3 symmetry.

4. Conclusion

The dual observables as possible order parameters for center symmetry are tested at finite temperature and isospin density with physical quark masses. Besides the dressed Polyakov-loop, another simple dual condensate, namely the dual pion condensate is proposed for $\mu_I > m_\pi/2$. We investigate the thermal behaviors of these two quantities in the PL enhanced NJL model of QCD by considering the pion condensation. Our model study suggests that both dual observables contain little or very limited information on quark deconfinement transition. On the other hand, the fast variations of both quantities are quite sensitive to the T-dependence of the quark and pion condensates.

First, we find that the twisted angle dependence of pion condensate is quite analogous to that of quark condensate obtained at zero μ and μ_I. Correspondingly, the DPC exhibits the similar T-dependence as the conventional PL for $\mu_I > m_\pi/2$. We demonstrate that the derivative of DPC with respect to T peaks exactly at T_c^π at which the pion condensation evaporates. This is very similar to the coincidence verified in [18,19], where the critical temperature extracting from the DPL equals exactly to T_c^σ in the chiral limit. Thus, we get the analogous conclusion that the rapid change of DPC near T_c^π is driven by the restoration of I_3 symmetry. So even if the DPC shows order parameter-like behavior, the critical temperature extracted from it has nothing to do with the deconfinement transition.

Second, we find that the DPL displays abnormal thermal property for $\mu_I > m_\pi/2$, which even decreases with T for $T \le T_c^\sigma$. This is quite different from the thin PL, which always increases with T. The anomaly arises due to the interplay between the quark and pion condensates. We verify that the DPL increases with T if the quark condensate (its absolute value) decreases, and vice versa. This implies the variation of DPL with T is mainly determined by the chiral dynamics rather than the confinement in this situation. In addition, we confirm that the maximum slope of DPL is very close to T_c^σ rather than T_c^P extracted from the PL. Actually, the critical temperature determined by the DPL is just T_c^σ in the chiral limit, which is in agreement with [18,19].

We thus conclude that both dual condensates are not appropriate order parameters for deconfinement in PNJL, even if the center symmetry is considered. We stress that whether such a conclusion also holds true in QCD with physical quark masses is unclear and needs further investigation. Our study suggests that the DPL might be also quite sensitive to the chiral transition in QCD. This raises an interesting question: is the critical temperature from DPL also exactly T_c^σ in QCD for the chiral limit? This can be checked in lattice calculations by reducing the dynamical quark mass.

Since there is no sign problem at finite μ_I, our study can be performed in the lattice simulation. In addition, the pion condensation has been investigated in the tDSE formalism [28] and other effective models of QCD, such as the quark–meson model. It is also interesting to investigate the thermal properties of dual condensates at finite μ_I within these methods.

Acknowledgement

Z.Z. was supported by the NSFC (No. 11275069).

References

[1] Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S.D. Katz, S. Krieg, K.K. Szabo, J. High Energy Phys. 0906 (2009) 088.
[2] S. Borsanyi, et al., Wuppertal–Budapest Collaboration, J. High Energy Phys. 1009 (2010) 073.
[3] T. Banks, A. Casher, Nucl. Phys. B 169 (1980) 103.
[4] C. Gattringer, Phys. Rev. Lett. 97 (2006) 032003.
[5] E. Bilgici, F. Bruckmann, C. Gattringer, C. Hagen, Phys. Rev. D 77 (2008) 094007.
[6] E. Bilgici, F. Bruckmann, J. Danzer, C. Gattringer, C. Hagen, E.M. Ilgenfritz, A. Maas, Few-Body Syst. 47 (2010) 125.
[7] B. Zhang, F. Bruckmann, C. Gattringer, Z. Fodor, K.K. Szabo, AIP Conf. Proc. 1343 (2011) 170, arXiv:1012.2314 [hep-lat].
[8] F. Synatschke, A. Wipf, C. Wozar, Phys. Rev. D 75 (2007) 114003.
[9] F. Synatschke, A. Wipf, K. Langfeld, Phys. Rev. D 77 (2008) 114018.
[10] J. Braun, L.M. Haas, F. Marhauser, J.M. Pawlowski, Phys. Rev. Lett. 106 (2011) 022002.
[11] C.S. Fischer, Phys. Rev. Lett. 103 (2009) 052003.
[12] C.S. Fischer, J.A. Mueller, Phys. Rev. D 80 (2009) 074029.
[13] C.S. Fischer, J. Luecker, J.A. Mueller, Phys. Lett. B 702 (2011) 438.
[14] C.S. Fischer, A. Maas, J.A. Muller, Eur. Phys. J. C 68 (2010) 165.
[15] K. Kashiwa, H. Kouno, M. Yahiro, Phys. Rev. D 80 (2009) 117901.
[16] R. Gatto, M. Ruggieri, Phys. Rev. D 82 (2010) 054027.
[17] T.K. Mukherjee, H. Chen, M. Huang, Phys. Rev. D 82 (2010) 034015.
[18] S. Benic, Phys. Rev. D 88 (2013) 077501.
[19] F. Marquez, A. Ahmad, M. Buballa, A. Raya, Phys. Lett. B 747 (2015) 529, arXiv:1504.06730 [nucl-th].
[20] D.T. Son, M.A. Stephanov, Phys. Rev. Lett. 86 (2001) 592;
D.T. Son, M.A. Stephanov, Phys. At. Nucl. 64 (2001) 834.
[21] J.B. Kogut, D.K. Sinclair, Phys. Rev. D 66 (2002) 034505.

[4] Actually, even to what extent does the PL contain the information of quark deconfinement transition is also a subtle problem if the quark mass is very small [27].

[22] J.B. Kogut, D.K. Sinclair, Phys. Rev. D 70 (2004) 094501.
[23] Z. Zhang, Y.-X. Liu, Phys. Rev. C 75 (2007) 064910.
[24] Z. Zhang, H.P. Su, Phys. Rev. D 89 (2014) 054020.
[25] S. Roessner, C. Ratti, W. Weise, Phys. Rev. D 75 (2007) 034007.
[26] L.y. He, M. Jin, P.f. Zhuang, Phys. Rev. D 71 (2005) 116001.
[27] Kohsuke Yagi, Tetsuo Hatsuda, Yasuo Miake, Quark–Gluon Plasma, Cambridge University Press, 2005.
[28] Z. Zhang, Y.x. Liu, Phys. Rev. C 75 (2007) 035201.