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On bulk singularity structures and all order α' contact terms of BPS string amplitudes

Ehsan Hatefi

Institute for Theoretical Physics, TU Wien, Wiedner Hauptstrasse 8-10/136, A-1040 Vienna, Austria

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ABSTRACT

The entire form of the amplitude of three SYM (involving two transverse scalar fields, a gauge field) and a potential C_{n-1} Ramond–Ramond (RR) form field is found out. We first derive $< V_{C^{-2}} V_{A^0} V_{\phi^0} V_{\phi^0} >$ and then start constructing an infinite number of t, s channel bulk singularity structures by means of all order α' corrections to pull-back of brane in an Effective Field Theory (EFT). Due to presence of the complete form of S-matrix, several new contact interactions as well as new couplings are explored. It is also shown that these couplings can be verified at the level of EFT by either the combinations of Myers terms, pull-back, Taylor expanded of scalar fields or the mixed combination of the couplings of this paper as well as employed Bianchi identities. For the first time, we also derive the algebraic and the complete form of the integrations for some arbitrary combinations of Mandelstam variables and for the most general case $\int d^2z |1-z|^a |z|^b (z-\bar{z})^c (z+\bar{z})^3$ on upper half plane as well.

1. Introduction

D-branes are supposed to be the fundamental objects that do exist in type II string theory. Indeed their roles have been enormously appreciating over the last decade, most notably by high energy physicists and in particular widely by string theorists [1,2].

To have some sort of understanding the brane's dynamics, we start off addressing various effective actions of these branes. Basically, in the very stablished Dielectric effect the issue of multiple brane's effective action as well as the appearance of commutator of two massless transverse scalar fields (describing oscillations of branes) was clarified [3]. We have already applied the direct conformal field theory methods and also made use of the mixture of Ramond–Ramond (RR)-open string scattering amplitude computations to actually (up to some convincing field theory contents) gain the so called generalized Myers action.

It is worthwhile to point out this action within its all order α' higher derivative corrections have been investigated in [4]. On the other hand, already various anomalous D-branes' couplings as well as dissolving branes inside the branes have been verified in detail [5]. Given the potential of S-matrix, various new couplings in [6] are revealed and the important point is that these couplings can not be established by all the three standard ways of EFT, namely neither Taylor expansion, Myers terms nor pull-back formalism worked out. Ultimately as physical applications to those effective couplings, we first found out the so called N^3 phenomenon for particularly $M5$ branes and some other systems, such as M2–M5 and consequently dS solutions as well as realizing the growth of the entropy of diverse configurations [7].

The symmetrized action at non-Abelian level was given by [8], whereas originally the action for single bosonic brane had been introduced by Leigh in [9]. Almost a decade later the supersymmetric part of the effective action becomes known [10]. To complete the effective actions various people including the author have taken a step further and verified within tremendous details the D-brane anti D-brane string theory effective actions that are consistent with direct string amplitude calculations, while in this context duality does not seem to be promising any more, given the nature of tachyonic systems [11].

Let us elaborate on the fact that the couplings of RR with even non-supersymmetric branes at first glimpse investigated by the same prospective in [12] as we head off from now on. Further explanations about standard EFT couplings as well as almost all effective actions can be achieved in [13].

The paper is written as follows. In the next section we try to find out all the closed form of the correlators for two transverse scalar fields, one gauge field and a potential RR, C-field $< V_{C^{-2}} V_{A^0} V_{\phi^0} V_{\phi^0} >$ we then continue by just mentioning the

E-mail addresses: ehsan.hatefi@tuwien.ac.at, ehsan.hatefi@cern.ch, e.hatefi@qmul.ac.uk.

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final result of the same field contents but in symmetric picture. Although recently a method was given in [14], we would like to evidently keep considering all the terms including all momenta of RR and in particular its momenta within the bulk such as $p.\xi_1$ and $p.\xi_2$ terms inside the S-matrix. Because indeed various correlation functions such as $< e^{ip.x(z)}\partial_i x^i(x_1) >$ obviously have non-zero contributions to amplitude. We also clearly take into account the Bianchi identities to be able to produce new bulk singularities as well as all contact terms that are located in transverse directions.

For the first time, the explicit form of integrations on upper half plane for arbitrary combinations of Mandelstam variables and for generic case $\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^3$ is achieved. We also generate an infinite number of t, s channel bulk singularity structures by means of all order α' corrections to pull-back of brane and highlight the fact that unlike [15], neither there are u-channel bulk nor $(t+s+u)$-channel bulk singularity structures. Due to presence of complete form of the S-matrix of this paper, several new contact interaction couplings will be discovered and those couplings can just be verified at the level of Effective Field Theory (EFT) by either the combination of Myers terms, pull-back, Taylor expanded of scalar fields or the mixed combination of the desired couplings as we will point out later on. Notice to the extremely important point that by just carrying out direct S-matrix computations apart from exploring new couplings with distinguished structures, we are also able to precisely fix the coefficients of those new couplings to all orders in α'.

2. The $< C^{-2}A^0\phi^0\phi^0 >$ S-matrix

First of all let us clarify the notation. $\mu, \nu = 0, 1, ..., 9$ representing the whole space–time, world volume indices are $b, c = 0, 1, ..., p$ and eventually the transverse indices can be shown by $i, j = p+1, ..., 9$. In order to actually find out exact and all order α' contact terms as well as bulk singularity structures of BPS strings, one must apply direct CFT techniques to get to the complete form of the so called S-matrix elements. In this section we would like to investigate the closed and complete form of particular BPS string amplitudes, a world volume gauge field and two transverse scalar fields in the presence of a potential $(p+1)$-form field of C-term which is called potential of Ramond–Ramond (RR) in the whole space–time. This amplitude can be explored if one does all the CFT correlation functions of $< C^{-2}A^0\phi^0\phi^0 >$ S-matrix. The interested reader may find some partial results that have come out of the precise and direct string scattering amplitude calculations in [16]. Hence, one needs to entirely figure out all the correlators $\langle V_A^{(0)}(x_1)V_\phi^{(0)}(x_2)V_\phi^{(0)}(x_3)V_{RR}^{(-\frac{3}{2},-\frac{1}{2})}(z,\bar{z})\rangle$ as well.[1]

To do so, one simply separates all the bosonic and fermionic correlation functions and starts to explore each of them. For discovering explicitly all two spin operators with different numbers of fermion fields or currents, we also employ the so called generalized Wick-like rule, that is, the two point function of fermionic operators gets changed with a minus sign.[2]

Note that all the vertex operators, propagators and on-shell relations are given in section 2 of [15], where the RR vertex operator in asymmetric picture was first hinted in [17] and eventually its compact form is derived in [18]. In order to simplify all the entire analysis of the correlation functions, we would like to write down just the compact form of the S-matrix as follows.

$$\begin{aligned}
\mathcal{A}^{C^{-2}A^0\phi^0\phi^0} \sim & \operatorname{Tr}(\lambda_1\lambda_2\lambda_3)\int dx_1dx_2dx_3dx_4dx_5(P_-\not{C}_{(n-1)}M_p)^{\alpha\beta} \\
& \times I\xi_{1a}\xi_{2i}\xi_{3j}x_{45}^{-3/4} \\
& \times\Bigg((x_{45}^{-5/4}C^{-1}_{\alpha\beta})\Big[a_1^a a_2^i a_3^j - \eta^{ij}x_{23}^{-2}a_1^a\Big] \\
& + i\alpha' k_{2b}a_1^a a_3^j a_2^{ib} + i\alpha' k_{1d}a_3^{ad}(-\eta^{ij}x_{23}^{-2} + a_2^i a_3^j) \\
& - \alpha'^2 k_{1d}k_{2b}a_3^j a_4^{ibad} + i\alpha' k_{3c}a_1^a a_2^i a_5^{jc} \\
& - \alpha'^2 k_{3c}k_{2b}a_1^a a_6^{jcib} - \alpha'^2 k_{3c}k_{1d}a_2^i a_7^{jcad} \\
& - i\alpha'^3 k_{1d}k_{2b}k_{3c}a_8^{jcibad}\Bigg), \qquad (1)
\end{aligned}$$

where

$$\begin{aligned}
I = & |x_{12}|^{\alpha'^2 k_1.k_2}|x_{13}|^{\alpha'^2 k_1.k_3}|x_{14}x_{15}|^{\frac{\alpha'^2}{2}k_1.p}|x_{23}|^{\alpha'^2 k_2.k_3}| \\
& \times x_{24}x_{25}|^{\frac{\alpha'^2}{2}k_2.p}|x_{34}x_{35}|^{\frac{\alpha'^2}{2}k_3.p}|x_{45}|^{\frac{\alpha'^2}{4}p.D.p},
\end{aligned}$$

$$a_2^i = ip^i\left(\frac{x_{54}}{x_{24}x_{25}}\right)$$

$$a_1^a = ik_2^a\left(\frac{x_{42}}{x_{14}x_{12}} + \frac{x_{52}}{x_{15}x_{12}}\right) + ik_3^a\left(\frac{x_{43}}{x_{14}x_{13}} + \frac{x_{53}}{x_{15}x_{13}}\right),$$

$$a_3^j = ip^j\left(\frac{x_{54}}{x_{34}x_{35}}\right),$$

$$a_2^{ib} = 2^{-1}x_{45}^{-1/4}(x_{24}x_{25})^{-1}(\Gamma^{ib}C^{-1})_{\alpha\beta},$$

$$a_3^{ad} = 2^{-1}x_{45}^{-1/4}(x_{14}x_{15})^{-1}(\Gamma^{ad}C^{-1})_{\alpha\beta},$$

$$a_4^{ibad} = 2^{-2}x_{45}^{3/4}(x_{14}x_{15}x_{24}x_{25})^{-1}\Bigg\{(\Gamma^{ibad}C^{-1})_{\alpha\beta} + \alpha' p_1\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}}\Bigg\},$$

$$a_5^{jc} = 2^{-1}x_{45}^{-1/4}(x_{34}x_{35})^{-1}(\Gamma^{jc}C^{-1})_{\alpha\beta},$$

$$a_6^{jcib} = 2^{-2}x_{45}^{3/4}(x_{34}x_{35}x_{24}x_{25})^{-1}\Bigg\{(\Gamma^{jcib}C^{-1})_{\alpha\beta} + \alpha' p_2\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}} + \alpha'^2 p_3\left(\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}}\right)^2\Bigg\},$$

$$a_7^{jcad} = 2^{-2}x_{45}^{3/4}(x_{34}x_{35}x_{14}x_{15})^{-1}\Bigg\{(\Gamma^{jcad}C^{-1})_{\alpha\beta} + \alpha' p_4\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}}\Bigg\}$$

with the following expressions for the above correlators

$$p_1 = \left(\eta^{db}(\Gamma^{ia}C^{-1})_{\alpha\beta} - \eta^{ab}(\Gamma^{id}C^{-1})_{\alpha\beta}\right),$$

$$p_2 = \left(\eta^{bc}(\Gamma^{ji}C^{-1})_{\alpha\beta} + \eta^{ij}(\Gamma^{cb}C^{-1})_{\alpha\beta}\right),$$

$$p_3 = (C^{-1})_{\alpha\beta}\left(-\eta^{bc}\eta^{ij}\right),$$

$$p_4 = \left(\eta^{dc}(\Gamma^{ja}C^{-1})_{\alpha\beta} - \eta^{ac}(\Gamma^{jd}C^{-1})_{\alpha\beta}\right).$$

The last fermionic correlation function is

[1] Having regarded all RR momenta in bulk directions and the fact that winding modes are not covered in the whole ten dimensional flat space, one would get to know that definitely not all the elements of $< V_{C^{-2}}V_{A^0}V_{\phi^0}V_{\phi^0} >$ S-matrix can be explored from the recent $< V_{C^{-2}}V_{\phi^0}V_{A^0}V_{A^0} >$ amplitude [15], where the other explanations are given in [19,20].

[2] $x_{ij} = x_i - x_j$, and $\alpha' = 2$.

$$a_8^{jcibad} = <: S_\alpha(x_4) : S_\beta(x_5) :: \psi^d \psi^a(x_1) : \psi^b \psi^i(x_2) : \psi^c \psi^j(x_3) > \tag{2}$$

which does have various terms and can be eventually found as follows

$$\begin{aligned}
a_8^{jcibad} = \Big\{ & (\Gamma^{jcibad} C^{-1})_{\alpha\beta} + \alpha' p_5 \frac{Re[x_{14}x_{25}]}{x_{12}x_{45}} + \alpha' p_6 \frac{Re[x_{14}x_{35}]}{x_{13}x_{45}} \\
& + \alpha' p_7 \frac{Re[x_{24}x_{35}]}{x_{23}x_{45}} \\
& + \alpha'^2 p_8 \left(\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}}\right)\left(\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}}\right) \\
& + \alpha'^2 p_9 \left(\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}}\right)\left(\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}}\right) \\
& + \alpha'^2 p_{10} \left(\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}}\right)\left(\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}}\right) \\
& + \alpha'^2 p_{11} \left(\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}}\right)^2 \\
& + \alpha'^3 p_{12} \left(\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}}\right)\left(\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}}\right)\left(\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}}\right)\Big\} \\
& \times 2^{-3} x_{45}^{7/4} (x_{14}x_{15}x_{24}x_{25}x_{34}x_{35})^{-1}
\end{aligned}$$

where

$$\begin{aligned}
p_5 &= \left(\eta^{db}(\Gamma^{jcia}C^{-1})_{\alpha\beta} - \eta^{ab}(\Gamma^{jcid}C^{-1})_{\alpha\beta}\right), \\
p_6 &= \left(\eta^{dc}(\Gamma^{jiba}C^{-1})_{\alpha\beta} - \eta^{ac}(\Gamma^{jibd}C^{-1})_{\alpha\beta}\right), \\
p_7 &= \left(\eta^{bc}(\Gamma^{jiad}C^{-1})_{\alpha\beta} + \eta^{ij}(\Gamma^{cbad}C^{-1})_{\alpha\beta}\right), \\
p_8 &= \left(\eta^{db}\eta^{ac}(\Gamma^{ji}C^{-1})_{\alpha\beta} - \eta^{dc}\eta^{ab}(\Gamma^{ji}C^{-1})_{\alpha\beta}\right), \\
p_9 &= \left(\eta^{db}\eta^{ij}(\Gamma^{ca}C^{-1})_{\alpha\beta} - \eta^{ab}\eta^{ij}(\Gamma^{cd}C^{-1})_{\alpha\beta}\right), \\
p_{10} &= \left(-\eta^{dc}\eta^{ij}(\Gamma^{ba}C^{-1})_{\alpha\beta} + \eta^{ac}\eta^{ij}(\Gamma^{bd}C^{-1})_{\alpha\beta}\right) \\
p_{11} &= \left(-\eta^{bc}\eta^{ij}(\Gamma^{ad}C^{-1})_{\alpha\beta}\right), \\
p_{12} &= (C^{-1})_{\alpha\beta}\left(\eta^{ij}\eta^{ab}\eta^{dc} - \eta^{ij}\eta^{ac}\eta^{db}\right).
\end{aligned} \tag{3}$$

Note that we wrote the amplitude in a manifest way so that, one is able to explicitly check that the amplitude is invariant under SL(2, R) transformation and volume of conformal killing group can be cancelled by fixing three positions of space–time, where we choose just to fix all the locations of open strings.[3] Thus, one needs to integrate out the remaining moduli space which is upper half plane and indeed all the integrations related to RR location [21]. Note that all the details of integrations are explained in Appendix B of [13]. However, in order to explore the integrations for p_3 of a_6^{jcib} and p_{12} of a_8^{jcibad}, one must find the algebraic solution of the following integrals $\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^3$ so that all a, b, c are written down in terms of any arbitrary Mandelstam variables. Once we are dealing with $(z+\bar{z})$ the result is explored in [21], meanwhile for $(z+\bar{z})^2$ one derives the entire result from [13].[4] For the first time, one finds the algebraic solution for the above integrals for $d = 3$ as follows:

$$\begin{aligned}
&\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^3 \\
&= (2i)^c 2^3 \pi \frac{K_1 + K_2}{\Gamma(\frac{-a}{2})\Gamma(\frac{-b}{2})\Gamma(\frac{(a+b)}{2}+c+5)}
\end{aligned} \tag{4}$$

where the functions K_1, K_2 are

$$\begin{aligned}
K_1 &= \Gamma(1+\frac{(a+c)}{2})\Gamma(4+\frac{(b+c)}{2})\Gamma(-1-\frac{(a+b+c)}{2}) \\
&\quad \times \Gamma(\frac{1+c}{2}), \\
K_2 &= \frac{3}{2}\Gamma(2+\frac{(a+c)}{2})\Gamma(3+\frac{(b+c)}{2})\Gamma(-2-\frac{(a+b+c)}{2}) \\
&\quad \times \Gamma(\frac{1+c}{2}).
\end{aligned} \tag{5}$$

Having set the solution for the new integrals, one would be able to obtain the final result for the S-matrix element in an asymmetric picture as follows

$$\begin{aligned}
\mathcal{A}^{C^{-2}A^0\phi^0\phi^0} = {} & \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_{41} + \mathcal{A}_{42} + \mathcal{A}_5 \\
& \mathcal{A}_{61} + \mathcal{A}_{62} + \mathcal{A}_{63} + \mathcal{A}_{64} + \mathcal{A}_{71} + \mathcal{A}_{72} + \mathcal{A}_{81} + \mathcal{A}_{82} \\
& \mathcal{A}_{83} + \mathcal{A}_{84} + \mathcal{A}_{85} + \mathcal{A}_{86} + \mathcal{A}_{87} + \mathcal{A}_{88} + \mathcal{A}_{89}
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
\mathcal{A}_1 \sim {} & i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p)\Big[-2suk_2.\xi_1 p.\xi_2 p.\xi_3 L_1 \\
& + 2tuk_3.\xi_1 p.\xi_2 p.\xi_3 L_1 + 2sk_2.\xi_1\xi_3.\xi_2 L_2(-s-u)(-t-u) \\
& - 2tk_3.\xi_1\xi_3.\xi_2 L_2(-s-u)(-t-u)\Big], \\
\mathcal{A}_2 \sim {} & ik_{2b}\xi_{2i}p.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ib})L_1\Big\{2usk_2.\xi_1 - 2utk_3.\xi_1\Big\} \\
\mathcal{A}_3 \sim {} & ik_{1d}\xi_{1a}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ad})\Big[-L_3\frac{(-s-u)(-t-u)}{(-u-\frac{1}{2})}\xi_3.\xi_2 \\
& - p.\xi_2 p.\xi_3 L_4\Big] \\
\mathcal{A}_{41} \sim {} & ip.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ibad})\xi_{2i}\xi_{1a}k_{2b}k_{1d}L_4 \\
\mathcal{A}_{42} \sim {} & ip.\xi_3\xi_{2i}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ia})L_1\Big\{tsu\xi_{1a} + 2suk_2.\xi_1 k_{1a}\Big\} \\
\mathcal{A}_5 \sim {} & ip.\xi_2 k_{3c}\xi_{3j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jc})L_1\Big\{2suk_2.\xi_1 - 2tuk_3.\xi_1\Big\} \\
\mathcal{A}_{61} \sim {} & i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jcib})\xi_{2i}\xi_{3j}k_{3c}k_{2b}L_1(-2suk_2.\xi_1 \\
& + 2tuk_3.\xi_1) \\
\mathcal{A}_{62} \sim {} & iL_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ji})\xi_{2i}\xi_{3j}\Big\{2suk_2.\xi_1 - 2tuk_3.\xi_1\Big\} \\
\mathcal{A}_{63} \sim {} & iL_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{cb})\xi_2.\xi_3(-k_{2b}k_{3c})\Big\{4sk_2.\xi_1 - 4tk_3.\xi_1\Big\} \\
\mathcal{A}_{64} \sim {} & -iu(2st-u)\xi_2.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p)L_2(-2sk_2.\xi_1 + 2tk_3.\xi_1)
\end{aligned}$$

[3] $x_1 = 0$, $x_2 = 1$, $x_3 \to \infty$.

[4] Where definitions are

$$s = \frac{-\alpha'}{2}(k_1+k_3)^2, \quad t = \frac{-\alpha'}{2}(k_1+k_2)^2, \quad u = \frac{-\alpha'}{2}(k_2+k_3)^2.$$

$$\mathcal{A}_{71} \sim ip.\xi_2 \mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{jcad})\xi_{1a}\xi_{3j}k_{3c}k_{1d}L_4$$
$$\mathcal{A}_{72} \sim ip.\xi_2 \mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{ja})L_1\xi_{3j}\bigg\{-uts\xi_{1a} - 2utk_3.\xi_1 k_{1a}\bigg\}$$
$$\mathcal{A}_{81} \sim -i\mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{jcibad})\xi_{2i}\xi_{1a}\xi_{3j}k_{1d}k_{2b}k_{3c}L_4$$
$$\mathcal{A}_{82} \sim -isu\mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{jcia})\xi_{2i}\xi_{3j}k_{3c}L_1\bigg\{t\xi_{1a} + 2k_2.\xi_1 k_{1a}\bigg\}$$
$$\mathcal{A}_{83} \sim -itu\mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{jiba})\xi_{3j}\xi_{2i}k_{2b}L_1\bigg\{-s\xi_{1a} - 2k_3.\xi_1 k_{1a}\bigg\}$$
$$\mathcal{A}_{84} \sim -istk_{1d}\xi_{1a}L_1\bigg\{u\xi_{2i}\xi_{3j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{jiad}) - 2\xi_2.\xi_3 k_{3c}k_{2b}\mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{cbad})\bigg\}$$
$$\mathcal{A}_{85} \sim -i\mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{ji})\xi_{2i}\xi_{3j}L_3\bigg\{-2tuk_3.\xi_1 + 2suk_2.\xi_1\bigg\}$$
$$\mathcal{A}_{86} \sim -i\xi_3.\xi_2 \mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{ca})L_3\bigg\{2tsk_{3c}\xi_{1a} + 4sk_2.\xi_1 k_{3c}k_{1a}\bigg\}$$
$$\mathcal{A}_{87} \sim -i\xi_3.\xi_2 \mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{ba})L_3\bigg\{2tsk_{2b}\xi_{1a} + 4tk_3.\xi_1 k_{2b}k_{1a}\bigg\}$$
$$\mathcal{A}_{88} \sim iL_3 \mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p \Gamma^{ad})\xi_{1a}k_{1d}(u\xi_2.\xi_3)\bigg\{\frac{-2st+u+s+t}{(-u-\frac{1}{2})}\bigg\}$$
$$\mathcal{A}_{89} \sim -i\xi_2.\xi_3 L_2 \mathrm{Tr}\,(P_-\not{C}_{(n-1)} M_p)(2sk_2.\xi_1 - 2tk_3.\xi_1)\bigg(tu + s(t + u + 2tu)\bigg) \tag{7}$$

with the definition for the functions L_1, L_2, L_3, L_4 as follows

$$L_1 = (2)^{-2(t+s+u)}\pi \times \frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s-u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}$$
$$L_2 = (2)^{-2(t+s+u+1)}\pi \times \frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s-u-\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}$$
$$L_3 = (2)^{-2(t+s+u)-1}\pi \times \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},$$
$$L_4 = (2)^{-2(t+s+u)+1}\pi \times \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u+1)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}. \tag{8}$$

Let's elaborate on the details. In fact the sum of the 3rd term of $\mathcal{A}_1$, 1st term of $\mathcal{A}_{64}$ and the 1st term of $\mathcal{A}_{89}$ is zero as well as the sum of the 4th term of $\mathcal{A}_1$, 2nd term of $\mathcal{A}_{64}$ and the 2nd term of $\mathcal{A}_{89}$. This obviously means that the last two terms of $\mathcal{A}_1$, the entire $\mathcal{A}_{64}$ and the whole $\mathcal{A}_{89}$ have no contribution to the asymmetric S-matrix at all.

Note that $\mathcal{A}_{62}$ is precisely cancelled with the entire terms inside $\mathcal{A}_{85}$, this also clarifies that $\mathcal{A}_{62}$, $\mathcal{A}_{85}$ will not contribute to our physical asymmetric amplitude either. On the other hand, if we just consider the RR in terms of its field strength we get to obtain the following [4]

$$\mathcal{A}^{<C^{-1}A^{-1}\phi^0\phi^0>} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 + \mathcal{A}_5 + \mathcal{A}_6 + \mathcal{A}_7 + \mathcal{A}_8 + \mathcal{A}_9 + \mathcal{A}_{10} \tag{9}$$

where

$$\mathcal{A}_1 \sim -2^{-1/2}\xi_{1a}\xi_{2i}\xi_{3j}\bigg[k_{3c}k_{2b}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jciba}) - k_{2b}p^j \mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{iba}) - k_{3c}p^i \mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jca}) + p^i p^j \mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \gamma^a)\bigg]4(-s-t-u)L_3,$$
$$\mathcal{A}_2 \sim 2^{-1/2}\bigg\{-2\xi_1.k_2 k_{3c}\xi_{3j}\xi_{2i}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jci})\bigg\}(us)L_1$$
$$\mathcal{A}_3 \sim 2^{-1/2}\bigg\{\xi_{1a}\xi_{2i}\xi_{3j}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jia})\bigg\}(-ust)L_1$$
$$\mathcal{A}_4 \sim 2^{-1/2}\bigg\{2k_3.\xi_1 k_{2b}\xi_{3j}\xi_{2i}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jib})\bigg\}(ut)L_1$$
$$\mathcal{A}_5 \sim 2^{-1/2}\bigg\{2\xi_3.\xi_2 k_{2b}k_{3c}\xi_{1a}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{cba})\bigg\}(st)L_1$$
$$\mathcal{A}_6 \sim 2^{1/2}(us)L_1\bigg\{p^j\xi_1.k_2\xi_{2i}\xi_{3j}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \gamma^i)\bigg\}$$
$$\mathcal{A}_7 \sim -2^{-1/2}(ut)L_1\bigg\{2k_3.\xi_1 p^i\xi_{3j}\xi_{2i}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \gamma^j)\bigg\}$$
$$\mathcal{A}_8 \sim 2^{1/2}L_3\bigg\{2k_2.\xi_1 k_{3c}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \gamma^c)(-s\xi_2.\xi_3)\bigg\}.$$
$$\mathcal{A}_9 \sim 2^{1/2}L_3\bigg\{2k_3.\xi_1 k_{2b}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \gamma^b)(-t\xi_2.\xi_3)\bigg\}$$
$$\mathcal{A}_{10} \sim 2^{1/2}L_3\bigg\{\xi_{1a}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \gamma^a)(ts\xi_3.\xi_2)\bigg\} \tag{10}$$

While the other symmetric amplitude has already been found in [6] to be

$$\mathcal{A}^{<C^{-1}A^0\phi^{-1}\phi^0>} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 + \mathcal{A}_5 + \mathcal{A}_6 \tag{11}$$

where

$$\mathcal{A}_1 \sim 2^{-1/2}\xi_{1a}\xi_{2i}\xi_{3j}p^j \mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \gamma^i)\bigg[-2k_3^a(ut) + 2k_2^a(us)\bigg]L_1$$
$$\mathcal{A}_2 \sim 2^{-1/2}k_{3c}\bigg\{-2k_2.\xi_1\xi_{2i}\xi_{3j}(us)L_1 \mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jci}) + 2k_3.\xi_1\xi_{2i}\xi_{3j}(ut)L_1 \mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jci}) + 4t\xi_2.\xi_3 k_3.\xi_1 L_3 \mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \gamma^c) - 4s\xi_2.\xi_3 k_2.\xi_1 L_3 \mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \gamma^c)\bigg\}$$
$$\mathcal{A}_3 \sim 2^{-1/2}k_{1b}\xi_{1a}\xi_{2i}\xi_{3j}4(-u-s-t)L_3\bigg(\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{iab})p^j - k_{3c}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jciab})\bigg)$$
$$\mathcal{A}_4 \sim 2^{-1/2}(ut)L_1\bigg\{-s\xi_{1a}\xi_{2i}\xi_{3j}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jia}) - 2k_3.\xi_1 k_{1b}\xi_{2i}\xi_{3j}\mathrm{Tr}\,(P_-\not{H}_{(n)} M_p \Gamma^{jib})\bigg\}$$

$$\mathcal{A}_5 \sim 2^{1/2}(st)L_1\xi_2.\xi_3\xi_{1a}k_{1b}k_{3c}\mathrm{Tr}\,(P_-H\!\!\!/_{(n)}M_p\Gamma^{cab})$$

$$\mathcal{A}_6 \sim 2^{1/2}\xi_3.\xi_2\bigg(ts\mathrm{Tr}\,(P_-H\!\!\!/_{(n)}M_p\gamma^a)\xi_{1a} + 2tk_3.\xi_1\mathrm{Tr}\,(P_-H\!\!\!/_{(n)}M_p\gamma^b)k_{1b}\bigg)L_3 \tag{12}$$

where the functions L_1, L_2 are given in (8). In the next section we are going to compare within details all the singularity structures of asymmetric with symmetric analysis and then start producing an infinite number of t,s-channel bulk singularity structures in an EFT as well.

3. Singularity comparisons

In this section we are going to provide precise analysis of all singularity structures involving even bulk singularities that are about to be found in this paper. To do so, we first try to regenerate singularities that have been already derived in symmetric analysis.

In order to produce all infinite t-channel poles of symmetric result, one needs to start adding up the first term of $\mathcal{A}_{61}$ with the second term of $\mathcal{A}_{82}$ and apply momentum conservation along the brane to obtain

$$2isuk_2.\xi_1L_1\mathrm{Tr}\,(P_-C\!\!\!/_{(n-1)}M_p\Gamma^{jcid})\xi_{2i}\xi_{3j}k_{3c}(p+k_3)_d \tag{13}$$

obviously the 2nd term in above equation has no contribution to S-matrix, because it is symmetric under interchanging k_{3c}, k_{3d} but also is antisymmetric as it involves ϵ tensor so the result for the 2nd term is zero, meanwhile its first term ($pC\!\!\!/ = H\!\!\!/$) does generate $\mathcal{A}_2$ of (9) (which is the fifth term of S-matrix elements in symmetric picture). One can do the same procedure, namely by adding the 2nd terms of $\mathcal{A}_{83}$, $\mathcal{A}_{61}$ and using momentum conservation, we gain all infinite s-channel poles or $\mathcal{A}_4$ of (9) as follows

$$-2ituk_3.\xi_1L_1\mathrm{Tr}\,(P_-C\!\!\!/_{(n-1)}M_p\Gamma^{jibd})\xi_{2i}\xi_{3j}k_{2b}(p+k_2)_d \tag{14}$$

Note that, making use of momentum conservation and ($pC\!\!\!/ = H\!\!\!/$), one reveals that the 2nd term of $\mathcal{A}_{84}$ exactly constructs all infinite u-channel poles or $\mathcal{A}_5$ of (9) as well.

Indeed for this particular $< C^{-2}A^0\phi^0\phi^0 >$ S-matrix, we have evidently shown that there are no u-channel Bulk singularity structures at all. The physical explanation for this is as follows. Suppose, we take into account the following rule in effective field theory side,

$$\mathcal{A} = V^a_\alpha(C_{p-3},A_1,A)G^{ab}_{\alpha\beta}(A)V^b_\beta(A,\phi_2,\phi_3), \tag{15}$$

we then may clarify that the vertex of $V^a_\alpha(C_{p-3},A_1,A)$ must be derived from Chern–Simons coupling as $(2\pi\alpha')^2\int_{\Sigma_{p+1}}C_{p-3}\wedge F\wedge F$ and in fact all $(p+1)$ indices have been considered and there are no leftover indices to be compensated by transverse directions (we have no external scalar field for this part of the sub field theory amplitude), which is why we no longer have any u-channel bulk singularity structures.

All u-channel gauge field poles can be written as

$$\mu_p(2\pi\alpha')^2 2k_{2b}k_{3c}p_d\xi_2.\xi_3\frac{1}{(p-3)!u}\epsilon^{a_0\cdots a_{p-4}cbad}C_{a_0\cdots a_{p-4}}\xi_{1a} \times\sum_{n=-1}^{\infty}b_n\left(\frac{\alpha'}{2}\right)^{n+1}(s+t)^{n+1} \tag{16}$$

Considering (15), taking the fixed scalar fields's kinetic term in the action (it receives no correction at all) as $T_p\frac{(2\pi\alpha')^2}{2}\mathrm{Tr}\,(D^a\phi^iD_a\phi_i)$, one finds out the $V^b_\beta(A,\phi_2,\phi_3)$ and gauge field propagator.[5] By taking the higher derivative corrections to Chern–Simons coupling as follows

$$i(2\pi\alpha')^2\mu_p\int d^{p+1}\sigma\sum_{n=-1}^{\infty}b_n(\alpha')^{n+1} \times C_{p-3}\wedge D_{a_0}\cdots D_{a_n}F\wedge D^{a_0}\cdots D^{a_n}F \tag{18}$$

we would be able to exactly generate the extension of the $V^a_\alpha(C_{p-3},A_1,A)$ vertex operator to all order α' as below

$$V^a_\alpha(C_{p-3},A_1,A) = \frac{(2\pi\alpha')^2\mu_p}{(p-3)!}\epsilon^{a_0\cdots a_{p-1}a} \times C_{a_0\cdots a_{p-4}}\xi_{1a_{p-3}}k_{a_{p-2}}p_{a_{p-1}}\sum_{n=-1}^{\infty}b_n(t+s)^{n+1} \tag{19}$$

where $k=(k_2+k_3)$ is employed and it now becomes clear that if we substitute (19) and (17) into field theory amplitude, then all order u-channel singularities of string amplitude in (16) can be explored in EFT as well.

Adding the 1st term of $\mathcal{A}_{63}$ with the 2nd term of $\mathcal{A}_{86}$ and applying momentum conservation, one explores

$$4isk_2.\xi_1\xi_3.\xi_2L_3\mathrm{Tr}\,(P_-C\!\!\!/_{(n-1)}M_p\Gamma^{cd})k_{3c}(p+k_3)_d \tag{20}$$

where the second term in above equation has no contribution to S-matrix, while its first term does generate precisely $\mathcal{A}_8$ of symmetric result in (9) (of course with a different sign, which is over all factor at the end).

Having added up the 2nd terms of $\mathcal{A}_{87}$, $\mathcal{A}_{63}$, we were able to obtain the following term

$$4itk_3.\xi_1\xi_3.\xi_2L_3\mathrm{Tr}\,(P_-C\!\!\!/_{(n-1)}M_p\Gamma^{bd})k_{2b}(p+k_2)_d \tag{21}$$

which is exactly $\mathcal{A}_9$ of symmetric result of (9) (with a different sign).

Eventually if we add up all the first terms of $\mathcal{A}_3$, $\mathcal{A}_{86}$, $\mathcal{A}_{87}$ with the entire $\mathcal{A}_{88}$, we derive

$$-2its\xi_{1a}\xi_3.\xi_2L_3\mathrm{Tr}\,(P_-C\!\!\!/_{(n-1)}M_p\Gamma^{da})(k_1+k_2+k_3)_d \tag{22}$$

Now using momentum conservation and ($pC\!\!\!/ = H\!\!\!/$), one is able to regenerate precisely $\mathcal{A}_{10}$ of symmetric result of (9). Thus all the infinite $(t+s+u)$ channel poles have also been reconstructed. Note to the following important point.

Indeed here for this particular $< C^{-2}A^0\phi^0\phi^0 >$ S-matrix (unlike $< C^{-2}\phi^0A^0A^0 >$ S-matrix), we have clearly shown that there is not even one $(t+s+u)$-channel Bulk singularity structure. The physical explanation for that is as follows. Suppose, we consider the following rule in effective field theory side,

$$\mathcal{A} = V^a_\alpha(C_{p-1},A)G^{ab}_{\alpha\beta}(A)V^b_\beta(A,A_1,\phi_2,\phi_3)$$

then one observes that the vertex of $V^a_\alpha(C_{p-1},A)$ must be derived from Chern–Simons coupling as $(2\pi\alpha')\int_{\Sigma_{p+1}}C_{p-1}\wedge F$ and in fact all $(p+1)$ indices have been taken into account and there are no leftover indices to be compensated by transverse directions either, that is why we have no $(t+s+u)$-channel bulk singularity structures any more. It is worth noting that the universal conjecture of

[5]
$$V^b_\beta(A,\phi_2,\phi_3) = i(2\pi\alpha')^2T_p\xi_2.\xi_3(k_2-k_3)^b\mathrm{Tr}\,(\lambda_2\lambda_3\lambda_\beta)$$
$$G^{ab}_{\alpha\beta}(A) = \frac{-i}{(2\pi\alpha')^2T_p}\frac{\delta^{ab}\delta_{\alpha\beta}}{k^2}\ . \tag{17}$$

all order α' corrections in [22] has played the significant role in matching all Supersymmetric Yang–Mills couplings at both string and EFT levels.

Now if we consider two gauge field two scalar couplings to all order in α' (appeared in [19]) and construct $V^b_\beta(A, A_1, \phi_2, \phi_3)$, then we will be able to precisely generate all infinite gauge field of $(t+s+u)$ channels. These poles have already been constructed out in [4], where we advise the reader to explore them directly in section four of [4]. Furthermore, for the same reasons, one immediately expects not to have u-channel Bulk singularities either.

Considering the 1st term of $\mathcal{A}_2$ of asymmetric amplitude and the 2nd term of $\mathcal{A}_{42}$, applying momentum conservation and taking $(p\not{C} = \not{H})$, not only we obtain the $\mathcal{A}_6$ of symmetric result in (9)

$$-2isuL_1k_2.\xi_1 p.\xi_3\xi_{2i}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{id})(p+k_3)_d \tag{23}$$

but also we generate a new kind of bulk pole. Indeed the 2nd term in (23) is related to an infinite number of t-channel extra bulk poles, which will be taken care of.

Finally, by adding the 2nd terms of $\mathcal{A}_5$ and $\mathcal{A}_{72}$ and making use of momentum conservation, we produce the following terms

$$2iutk_3.\xi_1 p.\xi_2 L_1\xi_{3j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jc})(p+k_2)_c \tag{24}$$

where the first term in (24) does produce $\mathcal{A}_7$ of symmetric result in (9), while the 2nd term in (24) is exactly an infinite number of s-channel extra bulk poles for which remain to be explored. Notice that the first terms of $\mathcal{A}_5$, $\mathcal{A}_1$, also the second terms of $\mathcal{A}_2$, $\mathcal{A}_1$ of asymmetric S-matrix (6) do generate an infinite number of bulk t, s-channel singularities accordingly, where we consider them in the next sections as well.

4. All order t, s-channel bulk singularity structures

As we have explicitly shown in the previous section, we could precisely produce all the singularities of (9) by using some (but definitely not all) of the singularities of asymmetric S-matrix. Indeed unlike the previous section, here not all the indices of Wess–Zumino action can be covered by world volume indices and in fact due to presence of external scalar field states as well as all non-zero $p.\xi_1, p.\xi_2$ terms, one expects to have bulk t, s channel singularities as well, for which we discuss from now on.

All infinite massless scalar t, s channel singularities (not Bulk t, s-channel singularities) have already been generated in section 4.1 of [4] but the aim of this section is to find out all order t, s channel Bulk singularity structures.

Performing careful comparisons of all singularities in both symmetric and antisymmetric amplitudes, as well as extracting all the related traces, one would be able to write down all order t-channel Bulk singularity structures that do exist just in asymmetric S-matrix (6) as follows:

$$2iusk_2.\xi_1\xi_{2i}\xi_{3j}\frac{16L_1}{(p+1)!}\bigg\{\epsilon^{a_0\ldots a_p}\Big(-p^ip^jC_{a_0\ldots a_p}\Big) + k_{3c}\epsilon^{a_0\ldots a_{p-1}c}\Big(p^iC_{ja_0\ldots a_{p-1}} - p^jC_{ia_0\ldots a_{p-1}}\Big)\bigg\} \tag{25}$$

and also all order s-channel bulk singularities as follows

$$2iutk_3.\xi_1\xi_{2i}\xi_{3j}\frac{16L_1}{(p+1)!}\bigg\{\epsilon^{a_0\ldots a_p}\Big(p^ip^jC_{a_0\ldots a_p}\Big) + k_{2b}\epsilon^{a_0\ldots a_{p-1}b}\Big(p^iC_{ja_0\ldots a_{p-1}} - p^jC_{ia_0\ldots a_{p-1}}\Big)\bigg\} \tag{26}$$

Note that all infinite t, s channel bulk singularities of (25) and (26), are needed as we are going to produce them in an EFT by introducing various couplings as follows.

Here we just produce all the infinite t-channel bulk singularities of (25) and then according to symmetries and by exchanging the scalar fields's momenta $k_2 \leftrightarrow k_3$ and interchanging the scalar fields polarizations $\xi_2 \leftrightarrow \xi_3$ one also will be able to explore all the infinite s-channel bulk singularities in an EFT as well.

Let us apply usL_1 expansion to (25) to generate all infinite t-channel Bulk singularity singularities as follows

$$\begin{aligned}&2ik_2.\xi_1\frac{16\pi^2\mu_p}{(p+1)!}\sum_{n=-1}^{\infty}b_n\frac{1}{t}(u+s)^{n+1}\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3)\\&\times\bigg\{\Big(-\epsilon^{a_0\ldots a_p}p.\xi_2p.\xi_3C_{a_0\ldots a_p}\Big)\\&+k_{3c}\epsilon^{a_0\ldots a_{p-1}c}\xi_{2i}\xi_{3j}\Big(p^iC_{ja_0\ldots a_{p-1}} - p^jC_{ia_0\ldots a_{p-1}}\Big)\bigg\}\end{aligned} \tag{27}$$

First we would like to reconstruct the bulk poles that are mentioned in the first two lines of (27), where we need to actually consider the following sub-amplitude in an effective field theory

$$\mathcal{A} = V^i_\alpha(C_{p+1},\phi_3,\phi)G^{ij}_{\alpha\beta}(\phi)V^j_\beta(\phi, A_1,\phi_2), \tag{28}$$

$V^j_\beta(\phi, A_1, \phi_2)$ must be re-constructed by means of the standard scalar fields's kinetic term in DBI action that has no correction and has already been fixed in the effective action as $\frac{(2\pi\alpha')^2}{2}\mathrm{Tr}\,(D_a\phi^iD^a\phi_i)$ and the other vertices are

$$\begin{aligned}V^j_\beta(\phi, A_1,\phi_2) &= -2i(2\pi\alpha')^2T_pk_2.\xi_1\xi_2^j\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_\beta)\\ G^{ij}_{\alpha\beta}(\phi) &= \frac{-i}{(2\pi\alpha')^2T_p}\frac{\delta^{ij}\delta_{\alpha\beta}}{k^2}\ ,\end{aligned} \tag{29}$$

so that $k^2 = -(k_2+k_1)^2 = t$ is replaced in the propagator.

To explore the vertex of $V^i_\alpha(C_{p+1},\phi_3,\phi)$ at leading order, one needs to keep in mind the following vertex $\frac{\mu_p(2\pi\alpha')^2}{2(p+1)!}\int d^{p+1}\sigma\epsilon^{a_0\cdots a_p}\ \mathrm{Tr}\,(\phi^j\phi^i)\partial_i\partial_jC_{a_0\cdots a_p}$ to be able to extract the vertex of an on-shell scalar and an off-shell scalar field as well as the potential C-term as follows

$$V^i_\alpha(C_{p+1},\phi_3,\phi) = \frac{\mu_p(2\pi\alpha')^2}{(p+1)!}p^ip.\xi_3\epsilon^{a_0\cdots a_p}C_{a_0\cdots a_p}\mathrm{Tr}\,(\lambda_3\lambda_\alpha) \tag{30}$$

Substituting (30) and (29) into (28), we find out the first t-channel bulk singularity of string amplitude. The propagator is fixed and there is no correction to $V^j_\beta(\phi, A_1, \phi_2)$, given the fact that it is obtained from kinetic term, therefore we conclude that there is no way of producing all the other bulk t-channel singularities, except one inserts all order α' higher derivative corrections to the following coupling

$$\frac{\mu_p(2\pi\alpha')^2}{2(p+1)!}\int d^{p+1}\sigma\epsilon^{a_0\cdots a_p}\sum_{n=-1}^{\infty}b_n(\alpha')^n \times\Big(\mathrm{Tr}\,(D_{a_1}\ldots D_{a_n}\phi^iD^{a_1}\ldots D^{a_n}\phi^j)\Big)\partial_i\partial_jC_{a_0\cdots a_p} \tag{31}$$

to actually derive the all order extended vertex operator of $V^i_\alpha(C_{p+1},\phi_3,\phi)$ as below

$$\begin{aligned}&V^i_\alpha(C_{p+1},\phi_3,\phi)\\&=\frac{\mu_p(2\pi\alpha')^2}{(p+1)!}\mathrm{Tr}\,(\lambda_3\lambda_\alpha)\epsilon^{a_0\cdots a_p}\sum_{n=-1}^{\infty}b_n(k_3.k)^{n+1}p.\xi_3p^iC_{a_0\cdots a_p}\end{aligned} \tag{32}$$

where $\sum_{n=-1}^{\infty} b_n (k_3.k)^{n+1} = \sum_{n=-1}^{\infty} b_n (s+u)^{n+1}$ should be used. Now if one inserts (32) into (28) and keeps fixed (29) then all order t-channel bulk singularities in the EFT are found out to be

$$\sum_{n=-1}^{\infty} b_n \frac{1}{t}(u+s)^{n+1} \frac{16\pi^2\mu_p}{(p+1)!} 2k_2.\xi_1 \epsilon^{a_0\cdots a_p} C_{a_0\cdots a_p} p.\xi_2 p.\xi_3 \tag{33}$$

which are precisely all the t-channel bulk poles of the string amplitude that appeared in the first two lines of (27), so we could regenerate them in an EFT as promised.

Eventually we would like to produce the rest of the t-channel bulk poles as follows

$$2ik_2.\xi_1 \frac{16\pi^2\mu_p}{(p+1)!} \sum_{n=-1}^{\infty} b_n \frac{1}{t}(u+s)^{n+1} \mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3) k_{3c} \epsilon^{a_0\ldots a_{p-1}c} \times \xi_{2i}\xi_{3j} \left(p^i C_{ja_0\ldots a_{p-1}} - p^j C_{ia_0\ldots a_{p-1}} \right) \tag{34}$$

where the same field theory amplitude (28) is needed. Note that due to $(-p^j C_{ia_0\ldots a_{p-1}})$ term in string amplitude, one might think that we just need to employ one scalar field from Taylor expansion and the other external scalar field from Pull-Back of brane in an EFT but as we can see from string amplitude we need to produce the other term $(p^i C_{ja_0\ldots a_{p-1}})$ in an EFT as well, so that the proper combination of terms in EFT is needed.

Suppose both external scalar fields come from pull-back of brane as

$$\frac{(2\pi\alpha')^2\mu_p}{2} \int d^{p+1}\sigma \frac{1}{(p-1)!} \epsilon^{a_0\cdots a_p} \mathrm{Tr}\left(D_{a_0}\phi^i D_{a_1}\phi^j \right) C_{ija_2\cdots a_p} \tag{35}$$

More significantly, consider the following Bianchi identity

$$\epsilon^{a_0\cdots a_p} \left(-p_{a_p}(p+1) H^{ij}_{a_0\cdots a_{p-1}} - p^j H^i_{a_0\cdots a_p} + p^i H^j_{a_0\cdots a_p} \right) = dH^{p+2} = 0 \tag{36}$$

extract the momentum of RR to make it just in terms of the potential of RR as below

$$p_{a_0}\epsilon^{a_0\cdots a_p} \left(-p_{a_p} p(p+1) C_{ija_1\cdots a_{p-1}} - p^j C_{ia_1\cdots a_p} + p^i C_{ja_1\cdots a_p} \right) = 0 \tag{37}$$

Now if we extract the vertex of an on-shell scalar, an off-shell scalar field and a potential C-field from (35) and bear in mind the fact that the covariant derivative D_{a_0} can act just on C-field (also taking integration by parts), expecting to obtain the following vertex operator

$$V_{i\alpha}(C_{p+1},\phi_3,\phi) = \frac{\mu_p(2\pi\alpha')^2 p(p+1)}{(p+1)!} \mathrm{Tr}\,(\lambda_3\lambda_\alpha) \epsilon^{a_0\cdots a_p} C_{ija_1\cdots a_{p-1}} k_{3a_0} p_{pa_p} \xi_{3j} \tag{38}$$

Indeed we now can use (37) to be able to replace in (38) $p_{a_p} p(p+1) C_{ija_1\cdots a_{p-1}}$ in terms of $(-p^j C_{ia_1\cdots a_p} + p^i C_{ja_1\cdots a_p})$. By doing so and taking into account (29), replacing (37) inside (38) as well as holding (28), we are able to construct out just the first t-channel bulk singularity structure of (34) in an EFT.

Given the previous clarifications and in order to regenerate an infinite number of t-channel bulk singularity structures in EFT, one has to apply the correct higher derivative corrections to pull-back as follows

$$\frac{(2\pi\alpha')^2\mu_p}{2} \int d^{p+1}\sigma \frac{1}{(p-1)!} \epsilon^{a_0\cdots a_p} \times \sum_{n=-1}^{\infty} b_n (\alpha')^n \mathrm{Tr}\left(D_{a_0} D_{a_1} \ldots D_{a_n} \phi^i D_{a_1} D^{a_1} \ldots D^{a_n} \phi^j \right) C_{ija_2\cdots a_p} \tag{39}$$

so that the all order extension of the above vertex operator would be gained as follows

$$V_{i\alpha}(C_{p+1},\phi_3,\phi) = \mathrm{Tr}\,(\lambda_3\lambda_\alpha) \frac{\mu_p(2\pi\alpha')^2}{(p-1)!} \epsilon^{a_0\cdots a_p} \times \sum_{n=-1}^{\infty} b_n (s+u)^{n+1} C_{ija_1\cdots a_{p-1}} k_{3a_0} p_{pa_p} \xi_{3j} \tag{40}$$

Once more the contributions of (29), replacement (37) inside (40) as well as the sub field theory amplitude (28), are taken. Having carried it out, we would be able to precisely produce all order t-channel bulk singularity structures of (34) in an effective field theory as well.

This ends our goal of producing an infinite number of t,s-channel bulk singularity structures of BPS branes. It is worth mentioning that, there is another way of producing t,s-channel bulk poles in such a way that one needs to relate combination of certain terms in the effective actions of BPS branes, let's devote the rest of this section to it.

Consider the action where an scalar comes from Taylor expansion and the other scalar comes from pull-back as follows

$$(2\pi\alpha')^2\mu_p \int d^{p+1}\sigma \frac{1}{(p)!} \epsilon^{a_0\cdots a_p} \mathrm{Tr}\left(\phi^j D_{a_0}\phi^i \right) \partial_j C_{ia_1\cdots a_p} \tag{41}$$

and add (35) with (41) as well as the terms that have the same order in α' such as Myers terms

$$\frac{i}{4}(2\pi\alpha')^2\mu_p \int d^{p+1}\sigma \frac{1}{(p-1)!} \epsilon^{a_0\cdots a_p} \mathrm{Tr}\left(F_{a_0a_1}[\phi^j,\phi^i] \right) C_{ija_2\cdots a_p}, \tag{42}$$

so that after having taken into account the integrations by parts, we would have left with the desired action as

$$(2\pi\alpha')^2\mu_p \int d^{p+1}\sigma \frac{1}{p!} \epsilon^{a_0\cdots a_p} \mathrm{Tr}\left(D_{a_0}\phi^j\phi^i \right) p^i C_{ja_1\cdots a_p} \tag{43}$$

One may use (43) to extract $V^i_\alpha(C_{p+1},\phi_3,\phi)$ as

$$V^i_\alpha(C_{p+1},\phi_3,\phi) = \frac{\mu_p(2\pi\alpha')^2}{p!} \mathrm{Tr}\,(\lambda_3\lambda_\alpha) \epsilon^{a_0\cdots a_p} p^i C_{ja_1\cdots a_p} k_{3a_0} \xi_{3j} \tag{44}$$

Furthermore, we might consider the other term so that this turn ϕ^i comes from pull-back and ϕ^j comes from Taylor expansion as follows

$$-(2\pi\alpha')^2\mu_p \int d^{p+1}\sigma \frac{1}{p!} \epsilon^{a_0\cdots a_p} \mathrm{Tr}\left(D_{a_0}\phi^i\phi^j \right) p^j C_{ia_1\cdots a_p} \tag{45}$$

keeping in mind (45) and extracting the rest of the terms, $V^i_\alpha(C_{p+1},\phi_3,\phi)$ vertex is got to be

$$V^i_\alpha(C_{p+1},\phi_3,\phi) = -\frac{\mu_p(2\pi\alpha')^2}{p!} \mathrm{Tr}\,(\lambda_3\lambda_\alpha) \epsilon^{a_0\cdots a_p} p^j C_{ia_1\cdots a_p} (k_3+p)_{a_0} \xi_{3j} \tag{46}$$

Now we may want to add (44) with (46) and use the Bianchi identity $p_{a_0}\epsilon^{a_0\cdots a_p}=0$ to produce the leading order of the vertex operator in an EFT so that the first t-channel bulk singularity structure of (34) is produced. We could apply the correct higher derivative corrections to (43) and (45) such as

$$\frac{\mu_p(2\pi\alpha')^2}{p!}\int d^{p+1}\sigma\epsilon^{a_0\cdots a_p}\times\sum_{n=-1}^{\infty}b_n(\alpha')^n\mathrm{Tr}\left(D_{a_0}D_{a_1}...D_{a_n}\phi^jD^{a_1}...D^{a_n}\phi^i\right)p^iC_{ja_1\cdots a_p} \tag{47}$$

to indeed get to all order t,s-channel bulk poles in an EFT. Finally, let us turn to all contact interactions as well.

5. All order α' contact interaction analysis

Let us construct the complete and all order contact terms of this S-matrix. If we start to add all the first terms of $\mathcal{A}_{82}$, $\mathcal{A}_{83}$, $\mathcal{A}_{84}$ of asymmetric amplitude in (6) we then derive

$$isut\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jiba})\xi_{2i}\xi_{3j}\xi_{1a}(k_3+k_2+k_1)_bL_1 \tag{48}$$

where by using momentum conservation along the brane, the above terms exactly produce $\mathcal{A}_3$ of symmetric amplitude in (9).

Considering the 1st terms $\mathcal{A}_{42}$, $\mathcal{A}_{72}$ and extracting the trace, one finds out the following terms

$$-iustL_1\xi_{1a}\xi_{2i}\xi_{3j}\frac{16}{(p+1)!}\epsilon^{a_0\ldots a_{p-1}}(p^iC_{ja_0\ldots a_{p-1}}-p^jC_{ia_0\ldots a_{p-1}}) \tag{49}$$

where we consider these new terms later on.

Note that $\mathcal{A}_{81}$ precisely does produce the 1st term of $\mathcal{A}_1$ of symmetric result of (9). On the other hand, using momentum conservation, $\mathcal{A}_{71}$ can be written down as

$$ip.\xi_2\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jcad})\xi_{1a}\xi_{3j}k_{3c}L_4(-p-k_3-k_2)_d \tag{50}$$

where using the anti symmetric property of ϵ, one reveals that the 2nd term in (50) has no contribution. Making use of ($p\not{C}=\not{H}$), the 1st term of (50) generates the 3rd term of $\mathcal{A}_1$ of symmetric amplitude in (9) (which is contact interaction term), while the last term in (50) is an extra contact interaction that we take it into account in a moment. Likewise, the same analysis holds for $\mathcal{A}_{41}$ as follows

$$ip.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ibad})\xi_{1a}\xi_{2i}k_{2b}L_4(-p-k_2-k_3)_d \tag{51}$$

where the 1st term in (51) does reconstruct the 2nd term of $\mathcal{A}_1$ of symmetric amplitude in (9) (its second term has zero contribution), meanwhile the last term in (51) is an extra contact interaction that we regard it in the next sections as well.

Ultimately, the 2nd term $\mathcal{A}_3$ of asymmetric amplitude in (6) is written down as

$$-ip.\xi_3p.\xi_2\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ad})\xi_{1a}L_4(-p-k_2-k_3)_d \tag{52}$$

indeed the first term in above equation regenerates the 4th contact term $\mathcal{A}_1$ of symmetric amplitude in (9).

Hence, we are able to produce all the contact interactions of (9) by using the elements of asymmetric S-matrix. However, the last two terms of (52) are extra contact terms in asymmetric S-matrix (6) and we claim their contribution is needed to our actual S-matrix as we demonstrate it right now.

Let us just end this section by adding all the extra contact interactions, extracting all the traces and using the antisymmetric property of ϵ tensor to be able to essentially obtain the following new contact terms to all orders

$$\begin{aligned}i\xi_{1a}\xi_{2i}\xi_{3j}\frac{16}{(p-1)!}\Bigg\{&L_4\Bigg((k_2+k_3)_dp^ip^j\epsilon^{a_0\ldots a_{p-2}ad}C_{a_0\ldots a_{p-2}}\\&+k_{3d}k_{2b}\epsilon^{a_0\ldots a_{p-3}bad}(p^iC_{ja_0\ldots a_{p-3}}-p^jC_{ia_0\ldots a_{p-3}})\Bigg)\\-ustL_1\frac{1}{p(p+1)}&\epsilon^{a_0\ldots a_{p-1}a}\left(p^iC_{ja_0\ldots a_{p-1}}-p^jC_{ia_0\ldots a_{p-1}}\right)\Bigg\}\end{aligned} \tag{53}$$

The first term in (53) is symmetric under interchanging both scalar fields and is needed in the string theory amplitude as it can be explored by means of Taylor expansions of the Effective field theory couplings, whereas its infinite higher derivative corrections can also be explored by applying appropriate higher derivative corrections to either Wess–Zumino or Chern–Simons couplings. As the method for extracting all order α' corrections to BPS contact interactions has been comprehensively explained in section five of [4] and [6] accordingly. Note also, as we explained earlier on, by combining the couplings of (43) and (45) and inserting the correct higher derivative corrections to them, one immediately starts to generate all order α' corrections to all new BPS contact terms that have been released in (53).

Notice that, since we have found all these terms by direct S-matrix analysis, one assured that the coefficients of the corrections are also exact and have no ambiguity any more. Ultimately, it is worth to point out that several new couplings within new structures have also been explored in section 9 of [6].

By explicit computations, it was also revealed that in an effective field theory, most of the super gravity field contents in the actions should be various functions of SYM. Because it is evidently realized that either Taylor expanded of transverse scalar fields (for the background fields) or some combinations of pull-back, Taylor expansion employed and this has been first regarded in the so called Dielectric effect [3].

One may have some hopes in figuring out the importance of the above new couplings, results for the S-matrices to construct not only future research areas in theoretical high energy physics, most notably in D-branes area but also in discovering new sort of Myers terms as well as constructing higher point functions or mathematical results (symmetries) behind the scattering amplitude prospectives. We intend to investigate and go through some of unanswered open questions in near future.

6. Conclusion

In this paper we started exploring the complete form of the S-matrix of two transverse scalar fields, a gauge field and a potential RR form-field in type IIA, IIB superstring theory, namely among other contents, we have derived even the terms that explicitly carry $p.\xi_1$ and $p.\xi_2$ elements in the string amplitude. For the last part of the S-matrix we needed to find out the explicit form of integrations on upper half plane for arbitrary combinations of Mandelstam variables including the terms that do clearly involve $\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^3$, where this was derived.

We also generated an infinite number of t,s channel bulk singularity structures by means of all order α' corrections to pull-back of brane and highlighted the fact that unlike [15], neither there are u-channel bulk nor $(t+s+u)$-channel bulk singularity structures. Due to presence of the complete form of S-matrix, several new contact interaction couplings in (53) have been discovered, whereas these terms can be verified at the level of effective field theory by either the combinations of Myers terms, pull-back, Taylor expanded of scalar fields or the mixed combination of the couplings of (43) and (45). Given the method that is explained in section five of [4] and [6], one is able to constantly apply the higher derivative corrections on contact terms and immediately explores their generalization to all orders in α'.

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