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Two-charged non-extremal rotating black holes in seven-dimensional gauged supergravity: The single-rotation case

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ABSTRACT

We construct the solution for non-extremal charged rotating black holes in seven-dimensional gauged supergravity, in the case with only one rotation parameter and two independent charges. Using the Boyer–Lindquist coordinates, the metric is expressed in a generalized form of the ansatz previously presented in [S.Q. Wu, Phys. Rev. D 83 (2011) 121502(R)], which may be helpful to find the most general non-extremal two-charged rotating black hole with three unequal rotation parameters. The conserved charges for thermodynamics are also computed.

1. Introduction

The discovery of the AdS/CFT correspondence stimulates a great deal of interest in constructing rotating charged black holes in gauged supergravities during the last few years. Of particular interest are those non-extremal black hole solutions in spacetime dimensions $D = 4, 5, 7$, which respectively correspond to the maximal $D = 4$, $\mathcal{N} = 8$, SO(8); $D = 5$, $\mathcal{N} = 8$, SO(6); and $D = 7$, $\mathcal{N} = 4$, SO(5) gauged supergravities with respective Cartan subgroups $U(1)^4$, $U(1)^3$ and $U(1)^2$. In recent years, there has been much progress in obtaining new, non-extremal, asymptotically AdS black hole solutions of gauged supergravity theories in four, five, six and seven dimensions. For a comprehensive discussion of these solutions, see for example [1]. However, almost all of these solutions can be classified into three catalogues: either setting all of the angular momenta equal; or setting certain U(1) charges equal; or restricting to supersymmetric solutions. One main reason is that, for non-extremal solutions of gauged supergravity theories, there is no known solution-generating technique that can charge up a neutral solution, instead one must rely on inspired guesswork and apply these strategies to greatly simplify the problem in finding exact solutions. So far, there is no universal method to derive the above-mentioned solutions, namely those listed in [1].

In a recent paper [2] that extends an interesting work [3] in $D = 4$ to all dimensions, we have presented in a unified fashion the general non-extremal rotating, charged Kaluza–Klein AdS black holes with only one electric charge and with arbitrary angular momenta in all higher dimensions. The solutions in $D = 4, 5, 6, 7$ dimensions can be embedded into corresponding gauged supergravities and the scalar potential in the Lagrangian can be rigorously deduced via the Killing spinor equation [4]. What is more, it has been shown that the general solutions in all dimensions share a common and universal metric structure which can not only naturally reduce to the famous Kerr–Schild ansatz in the uncharged case, but also be generalized to all of already-known black hole solutions with multiple pure electric charges, both in the cases of rotating charged black holes in ungauged supergravity and in the cases of nonrotating AdS black holes in gauged supergravity theories. This means that all previously-known supergravity black hole solutions with multiple different electric charges can be recast into a unified metric ansatz, regardless they belong to ungauged theories or gauged ones. In other words, supergravity black hole solutions in gauged theory inherit the same underlying metric structure as their ungauged counterparts. This significant feature of supergravity black hole solutions had not been exploited in any other previous work. Although that work [2] only dealt with the single-charge case in Kaluza–Klein supergravity, it put forward a universal method to construct the most general rotating charged AdS black hole solutions with multiple pure electric charges in gauged supergravity theory. For example, guided by the generalized form of that ansatz, the most general charged rotating AdS_5 solution with three unequal charges and with two independent rotation parameters has been successfully constructed [5] within five-dimensional $U(1)^3$ gauged supergravity.

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In this Letter, we shall concentrate on constructing new non-extremal rotating charged AdS black hole solutions in the dimension $D=7$ which are relevant for the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence in M-theory. The $D=7$ case singles it out a unique role since there appears a first-order "odd-dimensional self-duality" for the 4-form field strength, not seen previously in lower-dimensional examples. This feature makes it quite complicated for finding an exact solution. The currently-known charged non-extremal black hole solutions in the seven-dimensional gauged supergravity are as follows. Nonrotating static charged AdS_7 black hole solutions were known [6–8] for a decade because of the AdS/CFT correspondence. The first non-extremal rotating charged AdS black holes with two different electric charges in seven-dimensional gauged supergravity were obtained in [9], in the special case where the three rotation parameters are set equal. Inspired by a special case [9] where both U(1) charges are set equal, Chow [10] found a new rotating charged AdS black hole solution with three independent angular momenta and two equal U(1) charges. The single-charge case within $D=7$ Kaluza–Klein supergravity theory was recently found in [2]. However, the most general non-extremal rotating charged AdS black hole solution with all three unequal rotation parameter and two different U(1) charges is still not yet known.

In the present Letter, we shall construct new solution for non-extremal charged rotating black holes in seven-dimensional gauged supergravity, in the case with two independent charges but with only one rotation parameter. This is helped by the metric structure found for the two-charged Cvetič–Youm solution [11] in arbitrary dimensions, which is presented in Appendix A. It should also be pointed out that the recently-found two-charge rotating black holes [12] in four-dimensional gauged supergravity can be cast into the same ansatz. We present the solution in Section 2 and then examine its thermodynamics in Section 3. The conclusion section and the remaining two Appendices B and C summarize our results and present the clue toward constructing the most general solution with two unequal charges and with three unequal rotation parameters in seven-dimensional gauged supergravity.

2. Non-extremal rotating charged solution with one angular momentum

The $\mathcal{N}=4$ seven-dimensional gauged supergravity theory is a consistent reduction of eleven-dimensional supergravity on S^4. It is capable of supporting black holes with two independent electric charges, carried by gauge fields in the $U(1)\times U(1)$ Abelian subgroup of the full SO(5) gauge group. After a further consistent truncation, in which all except the $U(1)\times U(1)$ subgroup of gauge fields are set to zero, the fields in the final theory comprise the metric, two dilatons, two U(1) gauge fields and a 4-form field strength that satisfies an odd-dimensional self-duality equation. For the purpose of this Letter, we will consider the bosonic sector of the seven-dimensional gauged supergravity that consist of a graviton, two dilaton scalars, two Abelian gauge potentials and a 3-form potential, whose Lagrangian is given by [9]

$$\begin{aligned}\mathcal{L} &= R\star\mathbb{1} - {}^\star d\varphi_1\wedge d\varphi_1 - 5\,{}^\star d\varphi_2\wedge d\varphi_2\\ &\quad -\frac{1}{2}X_1^{-2}\,{}^\star F_1\wedge F_1 - \frac{1}{2}X_2^{-2}\,{}^\star F_2\wedge F_2 - \frac{1}{2}(X_1X_2)^2\,{}^\star\mathcal{F}\wedge\mathcal{F}\\ &\quad +(F_1\wedge F_2 - g\mathcal{F})\wedge\mathcal{C} + 2g^2\big[8X_1X_2\\ &\quad +4X_1^{-1}X_2^{-2} + 4X_1^{-2}X_2^{-1} - (X_1X_2)^{-4}\big]\star\mathbb{1},\end{aligned}\tag{1}$$

where

$$\begin{aligned}&F_1 = dA_1,\qquad F_2 = dA_2,\qquad \mathcal{F} = d\mathcal{C},\\ &X_1 = e^{-\varphi_1-\varphi_2},\qquad X_2 = e^{\varphi_1-\varphi_2},\end{aligned}\tag{2}$$

together with a first-order odd-dimensional self-duality equation

$$(X_1X_2)^2\,{}^\star\mathcal{F} = -2g\mathcal{C} - \mathcal{H},\tag{3}$$

satisfied by the 4-form field strength, which is conveniently stated by introducing an additional 2-form potential $\mathcal{B}$

$$\mathcal{H} = d\mathcal{B} - (A_1\wedge F_2 + A_2\wedge F_1)/2.\tag{4}$$

Now we present the exact solution for non-extremal charged rotating black holes in the above theory, in the case with only one rotation parameter and with two independent charges. In terms of the generalized Boyer–Lindquist coordinates, the metric is written in a generalized form of the ansatz given in a previous work [2], which sheds light on how to find the most general non-extremal two-charged rotating black hole with three unequal rotation parameters. The metric and two U(1) Abelian gauge potentials have the following exquisite form

$$\begin{aligned}ds^2 &= (H_1H_2)^{1/5}\bigg[-\frac{(1+g^2r^2)\Delta_\theta}{\chi}dt^2 + \frac{\Sigma}{\Delta_r}dr^2 + \frac{\Sigma}{\Delta_\theta}d\theta^2\\ &\quad + \frac{(r^2+a^2)\sin^2\theta}{\chi}d\phi^2\\ &\quad + r^2\cos^2\theta\big(d\psi^2 + \cos^2\psi\, d\zeta^2 + \sin^2\psi\, d\xi^2\big)\\ &\quad + \frac{2ms_1^2}{r^2\Sigma H_1\chi^2(s_1^2-s_2^2)}k_1^2 + \frac{2ms_2^2}{r^2\Sigma H_2\chi^2(s_2^2-s_1^2)}k_2^2\bigg],\end{aligned}\tag{5}$$

$$A_i = \frac{2ms_i}{r^2\Sigma H_i\chi}k_i,\tag{6}$$

in which

$$\begin{aligned}k_1 &= c_1\sqrt{\Xi_2}\Delta_\theta\, dt - c_2\sqrt{\Xi_1}a\sin^2\theta\, d\phi,\\ k_2 &= c_2\sqrt{\Xi_1}\Delta_\theta\, dt - c_1\sqrt{\Xi_2}a\sin^2\theta\, d\phi,\end{aligned}\tag{7}$$

and

$$\begin{aligned}&\Delta_r = \big(r^2+a^2-2m/r^2\big)\big(1+g^2r^2-2mg^2s_1^2s_2^2/r^2\big) + 2mg^2c_1^2c_2^2,\\ &\Delta_\theta = 1-g^2a^2\cos^2\theta,\qquad \Sigma = r^2+a^2\cos^2\theta,\\ &H_i = 1+2ms_i^2/\big(r^2\Sigma\big),\qquad \Xi_i = c_i^2 - s_i^2\chi,\quad \chi = 1-g^2a^2,\end{aligned}$$

where the short notations $c_i=\cosh\delta_i$ and $s_i=\sinh\delta_i$ $(i=1,2)$ are used.

Two scalars are given by $X_i=(H_1H_2)^{2/5}/H_i$, while the non-vanishing components of the 2-form potential and 3-form potential are

$$\mathcal{B}_{t\phi} = \frac{ms_1s_2\Delta_\theta a\sin^2\theta}{r^2\Sigma\chi}\left(\frac{1}{H_1}+\frac{1}{H_2}\right),\tag{8}$$

$$\begin{aligned}\mathcal{C}_{t\theta\phi} &= g\frac{2ms_1s_2a\sin\theta\cos\theta}{r^2\chi},\\ \mathcal{C}_{\psi\zeta\xi} &= \frac{2ms_1s_2a\cos^4\theta}{\Sigma}\sin\psi\cos\psi.\end{aligned}\tag{9}$$

We have found this solution by firstly recasting the metric and two gauge fields into a generalized ansatz in which two vectors k_1 and k_2 can be easily written down. This solution ansatz is inspired from our observation that the general two-charged rotating solutions [11] in all dimensions in ungauged supergravity and a special two-charged rotating Ads_7 gauged supergravity solution [9] with equal rotation parameters can be recast into a similar form, which are respectively rewritten in Appendices A and B. Next, we can decide the radial function Δ_r by requiring the metric determinant to be the expected expression. After doing this, we further find the

3-form potential $\mathcal{C}$ via solving the self-duality equation (3), since the expression for the 2-form potential $\mathcal{B}$ can be easily conjectured according to our previous experience. The last thing is to mechanically verify that the solution indeed solves all the field equations derived from the Lagrangian (1).

3. Thermodynamics

The two-charged Ads$_7$ black holes have Killing horizons at $r = r_+$, the largest positive root of $\Delta_r = 0$. The entropy and the Hawking temperature of the outer horizon are easily evaluated as

$$S = \frac{\pi^3 r_+}{4\chi}\sqrt{r_+^2\left(r_+^2+a^2\right)+2ms_1^2}\sqrt{r_+^2\left(r_+^2+a^2\right)+2ms_2^2}, \tag{10}$$

$$T = \frac{r_+^2\Delta'_{r_+}}{4\pi\sqrt{r_+^2(r_+^2+a^2)+2ms_1^2}\sqrt{r_+^2(r_+^2+a^2)+2ms_2^2}}. \tag{11}$$

On the horizon, the angular velocity and the electrostatic potentials are given by

$$\begin{aligned}\Omega &= \frac{2mr_+^2ac_1c_2\sqrt{\Xi_1\Xi_2}}{[r_+^2(r_+^2+a^2)+2ms_1^2][r_+^2(r_+^2+a^2)+2ms_2^2]}\\ &= \frac{a}{2mr_+^2c_1c_2\sqrt{\Xi_1\Xi_2}}\left(r_+^2+g^2r_+^4+2mg^2s_1^2\right)\\ &\quad\times\left(r_+^2+g^2r_+^4+2mg^2s_2^2\right),\end{aligned} \tag{12}$$

$$\Phi_1 = \frac{2mc_1s_1\sqrt{\Xi_2}}{r_+^2(r_+^2+a^2)+2ms_1^2}, \qquad \Phi_2 = \frac{2mc_2s_2\sqrt{\Xi_1}}{r_+^2(r_+^2+a^2)+2ms_2^2}. \tag{13}$$

The metric is obviously asymptotic anti-de Sitter, at infinity it approaches to the conformal metric

$$\begin{aligned}\lim_{r\to\infty}\frac{ds^2}{r^2} &= -\frac{g^2\Delta_\theta}{\chi}dt^2+\frac{dr^2}{g^2r^4}+\frac{d\theta^2}{\Delta_\theta}+\frac{\sin^2\theta}{\chi}d\phi^2\\ &\quad+\cos^2\theta\left(d\psi^2+\cos^2\psi\,d\zeta^2+\sin^2\psi\,d\xi^2\right),\end{aligned} \tag{14}$$

with which we can choose two vectors

$$\hat{N}^a = \frac{\sqrt{\chi}}{g\sqrt{\Delta_\theta}}(\partial_t)^a, \qquad \hat{n}^a = -g^2r^2(\partial_r)^a,$$

to compute the conserved charges that obey thermodynamical first laws.

Using the formulae [13]

$$\begin{aligned}\mathcal{Q}[\xi] &= \frac{1}{32\pi}\int_{S^5} d^5x\,\frac{\sin\theta\cos^3\theta\sin\psi\cos\psi}{g^3\sqrt{\chi\Delta_\theta}}r^2C_{acbd}\xi^a\hat{N}^b\hat{n}^c\hat{n}^d\\ &= \frac{\pi^2}{8}\int_0^{\pi/2}d\theta\,\frac{\sin\theta\cos^3\theta}{\Delta_\theta}r^6C_{artr}\xi^a,\end{aligned} \tag{15}$$

one can compute the conserved mass and angular momentum as

$$M = -\mathcal{Q}[\partial_t] = \frac{\pi^2 m}{8\chi}\left[2c_1^2c_2^2\left(1+\frac{1}{\chi}\right)+1-s_1^2s_2^2(1+3\chi)\right], \tag{16}$$

$$J = \mathcal{Q}[\partial_\phi] = \frac{\pi^2 mac_1c_2\sqrt{\Xi_1\Xi_2}}{4\chi^2}, \tag{17}$$

while two electric charges can be easily evaluated as

$$Q_1 = \frac{1}{16\pi}\int_{S^5}\left(X_1^{-2}{}^\star F_1 - F_2\wedge\mathcal{C}\right) = \frac{\pi^2mc_1s_1\sqrt{\Xi_2}}{2\chi}, \tag{18}$$

$$Q_2 = \frac{1}{16\pi}\int_{S^5}\left(X_2^{-2}{}^\star F_2 - F_1\wedge\mathcal{C}\right) = \frac{\pi^2mc_2s_2\sqrt{\Xi_1}}{2\chi}. \tag{19}$$

These thermodynamical quantities satisfy the differential and integral first laws of thermodynamics

$$dM = T\,dS + \Omega\,dJ + \Phi_1\,dQ_1 + \Phi_2\,dQ_2 - P\,d\mathcal{V}, \tag{20}$$

$$\frac{4}{5}(M - \Phi_1Q_1 - \Phi_2Q_2) = TS + \Omega J - P\mathcal{V}, \tag{21}$$

where we have introduced the generalized pressure [14]

$$\begin{aligned}P &= \frac{g^5m}{20\pi\chi}\left\{c_1^2c_2^2\left[\frac{1}{\chi}-1+\frac{g^2r_+^2(5r_+^2-8mg^2s_1^2s_2^2)}{r_+^2(1+g^2r_+^2)-2mg^2s_1^2s_2^2}\right]\right.\\ &\quad\left.-2g^2r_+^2\left(s_1^2+s_2^2+2s_1^2s_2^2\right)\right\},\end{aligned} \tag{22}$$

which is conjugate to the volume $\mathcal{V} = \pi^3/g^5$ of the 5-sphere with the AdS radius $1/g$. The results presented above include that given in Ref. [2] as a special case when one charge and two rotation parameters are set to zero in $D = 7$ dimensions.

4. Conclusions

In this Letter, we have successfully constructed the non-extremal two-charged single-rotating black holes in seven-dimensional gauged supergravity and computed their conserved charges closely related to thermodynamical first laws. Our results are significant for testing the AdS$_7$/CFT$_6$ correspondence in M-theory. To find this exact two-charged solution, we are helped by generalizing the solution ansatz previously proposed in Ref. [2] to the case with two different electric charges. Along the same line, it seems a direct and simple thing to apply this ansatz to construct the most general non-extremal rotating charged black hole solutions with two different charges and three unequal rotation parameters in seven-dimensional gauged supergravity theory. However, a peculiar self-duality equation (3) in $D = 7$ makes the thing quite complicated because we must also simultaneously deal with the 2-form potential and the 3-form potential. What is more, peered from the expressions of two U(1) Abelian gauge potentials (B.2), (B.3) given in Appendix B for the case with three equal rotation parameters, we infer that the expressions for two U(1) gauge potentials may not be so simple to be conjectured for the expected solutions. Nevertheless, the ansatz presented here already sheds new light on how to construct the most general rotating charged black hole solutions with multiple different electric charges in gauged supergravity theory. It deserves more deep investigations of the solution ansatz in the future work.

In addition, the corresponding supersymmetric solution, the separability of Hamilton–Jacobi equation, massless Klein–Gordon equation and their related hidden symmetry of the solution remain to be further investigated.

Note added

After our work appeared, the solution was also independently found by using a non-universal ansatz and its properties were examined in Ref. [15].

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Appendix A. Two-charged Cvetič–Youm solution in arbitrary dimensions

The general non-extremal rotating charged black holes, with two different charges and with arbitrary angular momenta in all higher dimensions, within ungauged supergravity theory were obtained in Ref. [11] (see also [16,17]) via solution-generating technique. These solutions are relevant for toroidally compactified heterotic supergravity. Although we focus here on the special case for $D=7$, it is universal to recast the metric into an exquisite ansatz in all dimensions. The underlying metric structure of the solutions can be easily recognized as follows

$$ds^2 = (H_1H_2)^{\frac{1}{D-2}}\Bigg[-dt^2 + \sum_{i=1}^{N+\epsilon}(r^2+a_i^2)\,d\mu_i^2 + \sum_{i=1}^{N}(r^2+a_i^2)\mu_i^2\,d\phi_i^2 + \frac{U}{V-2m}\,dr^2 + \frac{2ms_1^2}{UH_1(s_1^2-s_2^2)}\left(c_1\,dt - c_2\sum_{i=1}^{N}a_i\mu_i^2\,d\phi_i\right)^2 + \frac{2ms_2^2}{UH_2(s_2^2-s_1^2)}\left(c_2\,dt - c_1\sum_{i=1}^{N}a_i\mu_i^2\,d\phi_i\right)^2\Bigg], \tag{A.1}$$

$$A_1 = \frac{2ms_1}{UH_1}\left(c_1\,dt - c_2\sum_{i=1}^{N}a_i\mu_i^2\,d\phi_i\right), \tag{A.2}$$

$$A_2 = \frac{2ms_2}{UH_2}\left(c_2\,dt - c_1\sum_{i=1}^{N}a_i\mu_i^2\,d\phi_i\right), \tag{A.3}$$

$$\mathcal{B} = -\frac{ms_1s_2}{U}\left(\frac{1}{H_1}+\frac{1}{H_2}\right)dt\wedge\sum_{i=1}^{N}a_i\mu_i^2\,d\phi_i, \tag{A.4}$$

where $H_i = 1+2ms_i^2/U$ and

$$U = r^{\epsilon}\sum_{i=1}^{N+\epsilon}\frac{\mu_i^2}{r^2+a_i^2}\prod_{j=1}^{N}(r^2+a_j^2), \qquad V = r^{\epsilon-2}\prod_{i=1}^{N}(r^2+a_i^2).$$

In the above, we have denoted the dimension of spacetime as $D = 2N+1+\epsilon \geqslant 4$, with $N = [(D-1)/2]$ being the number of rotation parameters a_i associated with the N azimuthal angles ϕ_i in the N orthogonal spatial 2-planes, and $2\epsilon = 1+(-1)^D$. The $N+\epsilon = [D/2]$ 'direction cosines' μ_i obey the constraint $\sum_{i=1}^{N+\epsilon}\mu_i^2 = 1$, where $0\leqslant\mu_i\leqslant 1$ for $1\leqslant i\leqslant N$, and $-1\leqslant\mu_{N+1}\leqslant 1$ for even D.

Appendix B. Two-charged solution [9] in $D=7$ dimensions

The two-charged seven-dimensional rotating black hole solution [9], with the three rotation parameters being set equal, can be rewritten as

$$ds_7^2 = (H_1H_2)^{1/5}\Bigg[-\frac{1+g^2r^2}{\chi\,\Xi_-^2}\,dt^2 + \frac{(r^2+a^2)^2r^2}{Y}\,dr^2 + \frac{r^2+a^2}{\chi}\,d\Sigma_2^2 + \frac{r^2+a^2}{\chi}\left(\sigma+\frac{g}{\Xi_-}\,dt\right)^2 + \frac{2ms_1^2}{(s_1^2-s_2^2)(r^2+a^2)^2H_1\chi^2}(\alpha_1\,dt-\alpha_2a\sigma)^2 + \frac{2ms_2^2}{(s_2^2-s_1^2)(r^2+a^2)^2H_2\chi^2}(\alpha_2\,dt-\alpha_1a\sigma)^2\Bigg], \tag{B.1}$$

$$A_1 = \frac{2ms_1}{(r^2+a^2)^2H_1\chi}(\alpha_1\,dt-\alpha_2a\sigma), \tag{B.2}$$

$$A_2 = \frac{2ms_2}{(r^2+a^2)^2H_2\chi}(\alpha_2\,dt-\alpha_1a\sigma), \tag{B.3}$$

$$\mathcal{B} = \frac{ms_1s_2}{(r^2+a^2)^2\,\Xi_-^2}\left(\frac{1}{H_1}+\frac{1}{H_2}\right)dt\wedge a\sigma,$$

$$\mathcal{C} = \frac{mas_1s_2}{(r^2+a^2)\chi\,\Xi_-}\sigma\wedge d\sigma, \tag{B.4}$$

where

$$\alpha_1 = c_1 - \left(1-\Xi_+^2\right)(c_1-c_2)/2,$$
$$\alpha_2 = c_2 - \left(1-\Xi_+^2\right)(c_2-c_1)/2,$$
$$\Xi_\pm = 1\pm ga, \qquad \chi = 1-g^2a^2.$$

The two scalars are $X_i = (H_1H_2)^{2/5}H_i^{-1}$ with $H_i = 1+2ms_i^2/(r^2+a^2)^2$, while the function Y is presented in Ref. [9] and is not given here. The unit 5-sphere is parameterized by the Fubini–Study metric on $\mathbb{CP}^2$, and the connection on the U(1) fibre over $\mathbb{CP}^2$

$$d\Sigma_2^2 = d\xi^2 + \frac{1}{4}\sin^2\xi\left(d\theta^2+\sin^2\theta\,d\phi^2\right) + \frac{1}{4}\sin^2\xi\cos^2\xi(d\psi+\cos\theta\,d\phi)^2,$$
$$\sigma = d\tau + \frac{1}{2}\sin^2\xi(d\psi+\cos\theta\,d\phi).$$

When the rotation parameter is set to zero, the ansatz presented above contains as a special case the nonrotating charged Ads_7 black hole solution found in Refs. [6–8].

It should be pointed out that although the above solution can be recast into the general ansatz, the forms for the two gauge potentials are rather complicated after make a re-scaling $t\to\Xi_-\bar{t}$ and a shift $\sigma\to\bar{\sigma}-g\,d\bar{t}$. Because the metric contains the similar expressions to those of two gauge potentials, thus it provides little useful information on how to generalize the solution to the general case with three unequal rotation parameters.

Appendix C. A different but simpler form for two-charged solution

In this appendix, we present a different but simpler form for the two-charged seven-dimensional rotating black hole solution with the three rotation parameters being set equal. Written in terms of the previous ansatz, it is given by

$$ds_7^2 = (H_1H_2)^{1/5}\Bigg[-\frac{1+g^2r^2}{\chi}\,dt^2 + \frac{(r^2+a^2)^2r^2}{\Delta_r}\,dr^2 + \frac{r^2+a^2}{\chi}\,d\Sigma_2^2 + \frac{r^2+a^2}{\chi}(\sigma-g\,dt)^2 + \frac{2ms_1^2}{(s_1^2-s_2^2)(r^2+a^2)^2H_1\chi^2}(\alpha\,dt-\beta\gamma a\sigma)^2 + \frac{2ms_2^2}{(s_2^2-s_1^2)(r^2+a^2)^2H_2\chi^2}(\beta\,dt-\alpha\gamma a\sigma)^2\Bigg], \tag{C.1}$$

$$A_1 = \frac{2ms_1}{(r^2+a^2)^2H_1\chi}(\alpha\,dt-\beta\gamma a\sigma), \tag{C.2}$$

$$A_2 = \frac{2ms_2}{(r^2+a^2)^2H_2\chi}(\beta\,dt-\alpha\gamma a\sigma), \tag{C.3}$$

$$\mathcal{B} = \gamma \frac{ms_1s_2}{(r^2+a^2)^2}\left(\frac{1}{H_1}+\frac{1}{H_2}\right) dt \wedge a\sigma\,,$$
$$\mathcal{C} = \gamma \frac{mas_1s_2}{(r^2+a^2)\chi}\sigma \wedge d\sigma\,, \tag{C.4}$$

where

$$\Delta_r = g^2\big(r^2+a^2\big)^4 H_1H_2 + \chi\big(r^2+a^2\big)^3 + \frac{4mag\gamma\alpha\beta}{\chi}\big(r^2+a^2\big) - \frac{2m(s_1^2\beta^2 - s_2^2\alpha^2)}{\chi(s_1^2-s_2^2)}\big(r^2+a^2\big) + \frac{2ma^2\gamma^2(s_1^2\beta^2-s_2^2\alpha^2)}{s_1^2-s_2^2},$$

with the constraint condition

$$\alpha^2-\beta^2 = \chi^2\big(s_1^2-s_2^2\big). \tag{C.5}$$

In the above, we have included three extra parameters (α, β, γ) for convenience. But they are subject to one constraint Eq. (C.5), so only two of them are independent, and related to the two charge parameters. They cannot be removed by a coordinate transformation, rather are determined by the requirement that the new form of the solution approach to all known solutions, as did in Ref. [9] and in the below.

A simple form for the solution is given by choosing

$$\alpha = \chi c_1, \qquad \beta = \chi c_2, \qquad \gamma = \frac{1}{1+ga} = \frac{1-ga}{\chi}, \tag{C.6}$$

so we have

$$\Delta_r = g^2\big(r^2+a^2\big)^4 H_1H_2 + \chi\big(r^2+a^2\big)^3 - 2m(1-ga)^2r^2 + 4mag(1-ga)(c_1c_2-1)\big(r^2+a^2\big).$$

When both charges disappear, the metric becomes the vacuum Kerr-Ads$_7$ spacetime

$$ds^2 = -\frac{1+g^2r^2}{\chi}dt^2 + \frac{(r^2+a^2)^2r^2\,dr^2}{(1+g^2r^2)(r^2+a^2)^3 - 2m(1-ga)^2r^2} + \frac{r^2+a^2}{\chi}d\Sigma_2^2 + \frac{r^2+a^2}{\chi}(\sigma - g\,dt)^2 + \frac{2m}{(r^2+a^2)^2}(dt-\gamma a\sigma)^2.$$

We hope that the new form for the solution could act as an useful guidance on constructing the most general solution with two unequal charges and with three unequal rotation parameters in seven-dimensional gauged supergravity.

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