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# Modelling of cutting force and deflection of medical needles with different tip geometries

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## Abstract

Insertion of needle into tissue is influenced by the cutting edges and the needle tip geometry. In this paper, we aimed at investigating the relationship between the insertion forces and cutting edge geometry and general mathematical models of included and inclination angles for two different needle configurations. Two new geometries, one with parabolic cut and other with parabolic surface, cut with two bevel planes have been proposed along with the corresponding mathematical models to improve the cutting edge properties. Results show that parabolic needles after cutting with bevel planes have significantly improved cutting edge properties and are better than the other common needle configurations such as bevel cut. Simulation studies predicted that the deformation and stresses are slightly higher for proposed needle tip geometries than currently available topologies. However, the advantage of included and inclination angles reduce insertion forces thus optimization of medical needle design for the selected tip geometries is a trade-off between insertion forces and deflection in needle.

*Keywords:* Needles, cutting edges, insertion forces

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## 1. Introduction

Medical needles are widely used in these days in percutaneous procedures such as biopsy, acupuncture, brachytherapy, etc. which mainly involve inserting a needle to a target inside the body for drug delivery or for the removal of a sample from the tissue. The success of these procedures depends on accuracy to reach the target which is affected by deformation of tissue, deflection of the needle and the length of the sample collected (Abolhassani, et al., 2007). One way to control these placement errors is to minimize the needle insertion force which in turn reduces the deformation of tissue and deflection of needle. Studies also indicate that needle insertion forces are related with the intensity of pain and trauma experienced by the patients.

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Insertion of needle into a tissue is a cutting process and the force required to cut a tissue greatly depends on the geometry of the needle. Solid needles are generally used for diagnosis and treatment, one of such procedure is dry needling. Dry needling is the use of solid filiform needles for therapy of muscle pain, sometimes also known as intramuscular stimulation. Dry needling contrasts with the use of a hollow hypodermic needle to inject substances such as saline solution, Botox or corticosteroids to the same point. Such use of a solid needle has been found to be as effective as injection of substances in such cases as relief of pain in muscles and connective tissue. Analgesia produced by needling a pain spot has been called the needle effect (David et al. 2013).

The cutting edge geometry of a needle is defined by rake angle, inclination angle, included angle etc. Included angle is the angle between two surfaces which intersects to form main cutting edge and inclination angle is the angle which describes the angle of the cutting edge relative to the cutting direction (Peidong, et al., 2012). Let us consider a knife with included angle  $\theta$  maximal knife sharpness is obtained when  $\theta$  is very low which means a minimal cutting force. So smaller included angle result in minimum cutting force. The cutting force exerted by a sharp object depends on slice/push ratio given by  $k = \tan \lambda$ , which is given as the ratio of speed parallel to cutting edge/speed perpendicular to cutting edge (Atkins, et al., 2004) and  $\lambda$  is the inclination angle. So an increase in  $\lambda$  increases  $k$  which reduces cutting force. So we can conclude that smaller included angle and larger inclination angle are desirable for reducing insertion force.

While there is limited literature available on design aspects of medical needles, needle tips are usually generated by planes or other surfaces when they intersect each other. In fig.1, two surfaces  $F_1$  and  $F_2$  forms the tip of the needle, with the X-axis passing through the lowest point of the needle tip and Z-axis along the needle longitudinal axis.

The normal vector at a point  $(x_0, y_0, z_0)$  on a surface  $F=0$  is given by  $N = [ F_x \ F_y \ F_z ]$  where  $F_x, F_y, F_z$  are partial derivatives at  $(x_0, y_0, z_0)$ . At a point  $M_0(x_0, y_0, z_0)$  on the cutting edge the normal vector to the surface  $F_1$  is  $N_1 = [ (F_{1,x})_{M_0} \ (F_{1,y})_{M_0} \ (F_{1,z})_{M_0} ]$  to  $F_2$  is  $N_2 = N_1 = [ (F_{2,x})_{M_0} \ (F_{2,y})_{M_0} \ (F_{2,z})_{M_0} ]$ . The included angle  $\theta$  is given by the angle between the vectors  $N_1, N_2$  at  $M_0(x_0, y_0, z_0)$  by the definition of included angle (Peidong et al., 2012).

$$\begin{aligned} \cos(\pi - \theta) &= \frac{N_1 \cdot N_2}{\|N_1\| \|N_2\|} \\ \theta &= \pi - \cos^{-1} \left( \frac{(F_{1,x})_{M_0} (F_{2,x})_{M_0} + (F_{1,y})_{M_0} (F_{2,y})_{M_0} + (F_{1,z})_{M_0} (F_{2,z})_{M_0}}{\sqrt{(F_{1,x})_{M_0}^2 + (F_{1,y})_{M_0}^2 + (F_{1,z})_{M_0}^2} \sqrt{(F_{2,x})_{M_0}^2 + (F_{2,y})_{M_0}^2 + (F_{2,z})_{M_0}^2}} \right) \\ &= \frac{(F_{1,x})_{M_0} (F_{2,x})_{M_0} + (F_{1,y})_{M_0} (F_{2,y})_{M_0} + (F_{1,z})_{M_0} (F_{2,z})_{M_0}}{\sqrt{(F_{1,x})_{M_0}^2 + (F_{1,y})_{M_0}^2 + (F_{1,z})_{M_0}^2} \sqrt{(F_{2,x})_{M_0}^2 + (F_{2,y})_{M_0}^2 + (F_{2,z})_{M_0}^2}} \end{aligned} \tag{1}$$

For insertion of needle the penetration direction vector coincides with needle longitudinal axis  $v = [0 \ 0 \ 1]$ . The tangent vector to the cutting edge can be given by cross product of  $t = N_1 \times N_2$  and  $\lambda$  is the angle between vector  $t$  and XY plane.

$$\lambda = \sin^{-1} \frac{|t \cdot v|}{\|t\| \|v\|} \tag{2}$$

In this paper, the above formulation has been applied over two novel needle tip configurations proposed in this study. These needle tips are generated by cutting a cylindrical needle with a parabolic surface instead of the regular planes or cylinders.

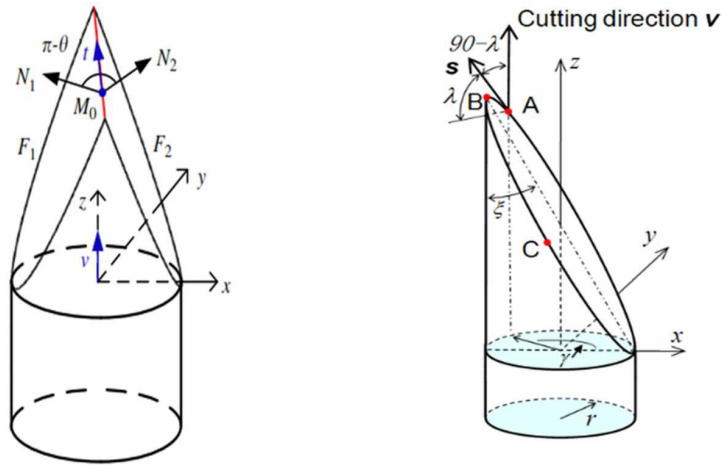


Fig. 1. Conventional tip geometries of solid medical needles: (a) Needle tip formed by two surfaces  $F_1$  and  $F_2$  (Peidong Han et al., 2012) (b) Inclination and included angles in bevel tip needle (Moore et al., 2010).

**2. Mathematical model for included and inclination angles**

*2.1 Parabolic tip needle*

Fig. 2 shows the needle tip geometry with parabolic configuration and the tip point of the needle. Initially, a parabola  $z = ax^2 + bx + c$ , in the ZX plane and extended in y direction to form a parabolic surface is considered. Now this parabolic surface is used to cut a cylindrical needle and the cutting edge is formed by the intersection of the needle surface and parabolic surface. Let the radius of the cylinder be ‘r’ and the angular position of a point on the cutting edge is given by  $\gamma$ . So the parametric equations of a point on the needle cutting edge are given by

$$x = r \cos \gamma ; y = r \sin \gamma \text{ and } z = a(r \cos \gamma)^2 + b(r \sin \gamma)^2 + c$$

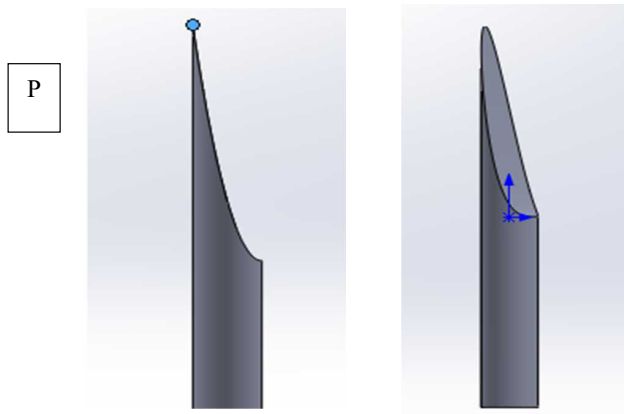


Fig. 2. Cylindrical needle cut with a parabolic surface

The angle between needle side surface and the parabola surface is the included angle  $\theta$ . The unit normal vector to the needle side surface at a point on the cutting edge is given by  $c = [\cos \gamma \quad \sin \gamma \quad 0]$  and the unit normal vector to parabolic surface is given by

$$b = [-(2ar\cos\gamma + b), 0, 1]$$

$$\cos(\pi - \theta) = \frac{b \cdot c}{\|b\| \|c\|} = \frac{-(2ar\cos\gamma + b)\cos\gamma}{\sqrt{1 + (2ar\cos\gamma + b)^2}}$$

$$\theta = \pi - \arccos\left(\frac{-(2ar\cos\gamma + b)\cos\gamma}{\sqrt{1 + (2ar\cos\gamma + b)^2}}\right) \tag{3}$$

The tangent vector of the cutting edge at that point is  $s = [-r\sin\gamma, r\cos\gamma, -(2ar\sin 2\gamma + b\sin\gamma)]$ . The penetration direction vector  $v = [0 \ 0 \ 1]$ . From equation (2) the inclination angle  $\lambda$  for this needle is given by

$$\sin \lambda = \frac{|s \cdot v|}{\|s\| \|v\|} = \frac{|ar\sin 2\gamma + b\sin\gamma|}{\sqrt{1 + (ar\sin 2\gamma + b\sin\gamma)^2}}$$

$$\lambda = \arcsin\left(\frac{|ar\sin 2\gamma + b\sin\gamma|}{\sqrt{1 + (ar\sin 2\gamma + b\sin\gamma)^2}}\right) = \arctan |ar\sin 2\gamma + b\sin\gamma| \tag{4}$$

In this needle at  $\gamma = 180^\circ$  that is at the needle tip point the value of the inclination angle from the above equation equals to zero.  $\lambda = 0^\circ$  at the tip point of the needle which is not desired from the viewpoint of minimized penetration force.

2.2 Parabolic tip needle cut with bevel planes

In the second case a new configuration has been designed where the parabolic tip of the needle has been modified by incorporating two bevel planes  $F_1$  and  $F_2$  as shown in Fig. 3.  $F_1$  and  $F_2$  make an angle  $\xi$  with respect to needle longitudinal axis. The bevel plane  $F_1$  is obtained by rotating a plane which is making an angle  $\xi$  with z-axis by a counter clockwise angle of  $\phi$  and similarly  $F_2$  is obtained by a clockwise angle of  $\phi$ . The intersection of the parabolic surface and these planes forms the cutting edges. The unit normal vector ( $b_1$ ) to the plane  $F_1$  is given by transforming ‘b’ by an angle  $\phi$  about z-axis.

$$b_1 = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\xi \\ 0 \\ \sin\xi \end{bmatrix} = \begin{bmatrix} \cos\xi \cos\phi \\ \cos\xi \sin\phi \\ \sin\xi \end{bmatrix}$$

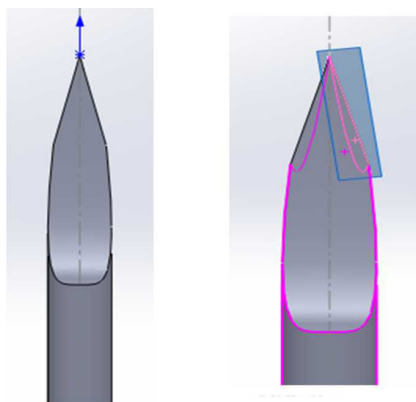


Fig. 3. Needle with cutting edges formed by cutting with parabolic surface and bevel planes

If the value of  $\varphi$  is less than  $90^\circ$  then the cutting edge is the one that is formed by the intersection of plane and the needle surface. So the values of  $\varphi$  are taken in between  $90^\circ$  and  $180^\circ$ . The included angle is the angle between the bevel plane and the parabolic surface. The normal to the parabola is

$$p = [-(2 \arccos \gamma + b), 0, 1]$$

From the equation (1),

$$\cos(\pi - \theta) = \frac{p \cdot b_1}{\|p\| \|b_1\|} = \frac{-(2 \arccos \gamma + b) \cos \xi \cos \phi + \sin \xi}{\sqrt{1 + (2 \arccos \gamma + b)^2}}$$

$$\theta = \pi - \arccos \left( \frac{-(2 \arccos \gamma + b) \cos \xi \cos \phi + \sin \xi}{\sqrt{1 + (2 \arccos \gamma + b)^2}} \right); \quad (90^\circ \leq \varphi \leq 180^\circ) \quad (5)$$

The tangent vector of the cutting edge is given by the cross product of the normal vectors  $p$  and  $b_1$ .  $t = p \times b_1$ .

$$t = [-\cos \xi \sin \phi (2 \arccos \gamma + b) \sin \xi + \cos \xi \cos \phi - (2 \arccos \gamma + b) \cos \xi \sin \phi]$$

The cutting direction vector is  $v = [0 \ 0 \ 1]$ . From the equation (2) the inclination angle of the cutting edge is given by

$$\lambda = \arcsin \frac{|t \cdot v|}{\|t\| \|v\|}$$

$$\lambda = \arcsin \left( \frac{|(2 \arccos \gamma + b) \cos \xi \sin \phi|}{\sqrt{\cos^2 \xi \sin^2 \phi (1 + (2 \arccos \gamma + b)^2) + ((2 \arccos \gamma + b) \sin \xi + \cos \xi \cos \phi)^2}} \right); \quad (6)$$

$$0 \leq \xi \leq \pi/2; \quad \pi/2 \leq \varphi \leq \pi$$

At  $\gamma = 180^\circ$  that is at the tip point the value of the inclination angle is not zero and it changes with the parabolic surface used to cut the needle.

### 3. Results and Discussion

The expressions for included angle and inclination angle for the new configurations are derived and now we compare these results with previously defined configurations such as single plane bevel needle and symmetric multi-plane needle.

#### 3.1. Single plane bevel needle

As we can see that at  $\gamma = 180^\circ$  the value of inclination angle  $\lambda$  is zero irrespective to whatever the value of  $\xi$  is, which means the worst possible cutting configuration. At needle tip point cutting edge has  $\theta = \xi$ , the smallest possible included angle however the inclination angle is  $0^\circ$ . At  $\gamma = 90^\circ$  and  $\gamma = 270^\circ$  the cutting edge has  $\lambda = (90^\circ - \xi)$ , the largest possible inclination angle however the cutting edge has  $\theta = 90^\circ$ . Remembering that smaller included angle and larger inclination angle are desirable for reducing needle penetration force the cutting edge of a bevel tip needle is undesirable.

### 3.2. Symmetric multi-plane needle

It can be seen that a smaller bevel angle  $\xi$  leads to a smaller included angle and the lowest value of  $\theta$  limited to  $(p-2) \pi/p$ . The included angle increases with increase in  $p$  for a given  $\xi$  which means that the lowest value of  $\theta$  is achieved when  $p = 2$ . However the inclination angle along the cutting edge is equal to zero which makes three plane symmetrical needles more desirable. An increase in  $p$  beyond 3 greatly increases the included angle, thereby making the needle less effective.

Smaller  $\xi$  leads to larger inclination angles with  $p > 2$  and increases with increase in  $p$ . It has been reported that  $p = 3$  gives the best combination of included and inclination angles. For instance when  $\xi = 15^\circ$ , the included angle for a three plane needle is  $66^\circ$  and inclination angle is  $61.8^\circ$ . We can see that the included angle of the cutting edge is very large which is not desirable to reduce the insertion force.

### 3.3. Parabolic tip needle

From equations (3) and (4), the values of included angle and inclination angle are dependent on the radius ‘r’ of the needle, parametric position of a point on the needle cutting edge, and the parabolic surface used (‘a’ and ‘b’). The radius of the needle is taken as  $r = 0.5$  mm as it is a cylindrical needle. Now we can vary the angles by varying the parabolic surface used to cut the cylindrical needle i.e., ‘a’ and ‘b’. The leading cutting edge of this needle vary from  $\gamma = 90^\circ$  to  $\gamma = 270^\circ$ . So the graphs are plotted for inclination angle and included angle from  $90^\circ$  to  $270^\circ$  degrees for four different parabolas. Radius for all the needles is constant and is equal to 0.5 mm. Fig.4 shows the variation of included angle and inclination angle with  $\gamma$ .

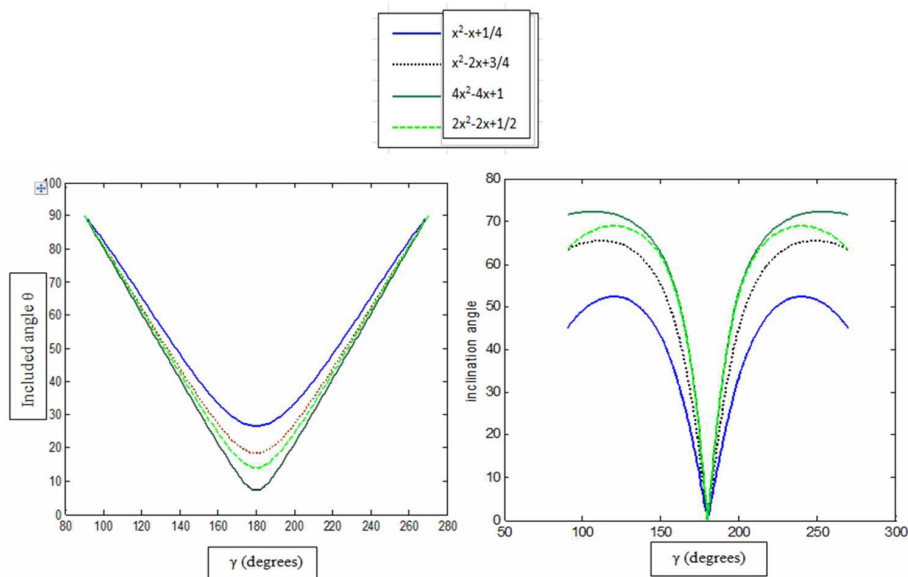


Fig.4. Included angle and inclination angle for a needle cut with a parabolic surface

The included angle for parabolic surfaced needle shown in the above figure indicates that the value of included angle decreases with increase in  $\gamma$  until it reaches  $180^\circ$  and again increases symmetrically from  $\gamma = 180^\circ - 270^\circ$ . Also the angle decreases with change in the parabola’s curvature.  $x^2-x+1/4$  has the largest included angle at  $\gamma = 180^\circ$  and the angle decreases for  $x^2-2x+3/4$  and the least value is obtained for  $4x^2-4x+1$  out of the four parabolas. Now the inclination angle variation is considered with  $\gamma$  and we can see that the angle tends to zero at  $\gamma = 180^\circ$  irrespective to the parabola. So like one plane bevel tip needle this configuration is not suitable for reducing insertion forces.

### 3.4. Parabolic tip needle cut with bevel planes

As discussed earlier the inclination angle at the needle tip of the parabolic surface needle that is at  $\gamma = 180^\circ$  now the included angle and inclination angle after cutting the parabolic surfaced needle with bevel planes are plotted and the results are compared. The radius of the needle  $r = 0.5$  mm, and the values for the same parabolas used above are plotted to compare the results. The main cutting edge of the needle ends at the  $\gamma_m \geq 90^\circ$ , and from  $90^\circ \leq \gamma \leq \gamma_m$  the cutting edge is same as that of simple parabolic surfaced needle. The equations for included and inclination angle from (5) and (6) are used to plot the graphs.

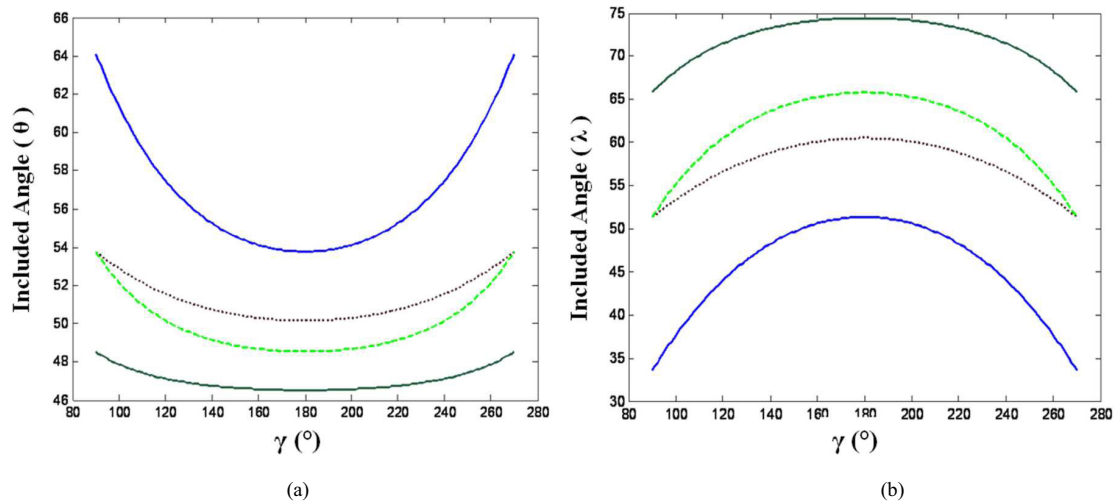


Fig.5. Included angle and inclination angle for varying value of  $\gamma$

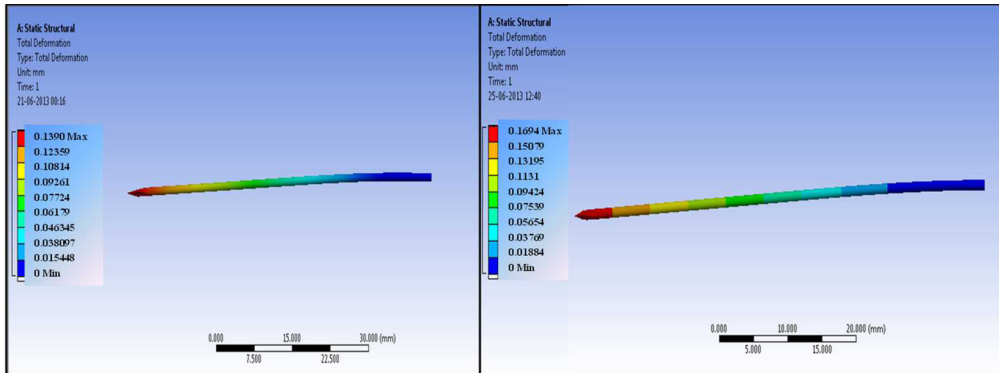
The graphs plotted here are for  $\xi = 5^\circ$  and  $\phi = 135^\circ$ . As we can see that the inclination angle at the needle tip point is not equal to zero and  $\lambda$  increases with increase in  $\gamma$  till  $180^\circ$  and then decreases symmetrically. Higher inclination angle is achieved for  $4x^2-4x+1$  and lower for  $x^2-x+1/4$ . In case of included angle, there is a rise in the values when compared with the simple parabolic surfaced needle. Considering the overall values of included and inclination angles this configuration is found to be suitable to reduce insertion forces. Maximum inclination angle is  $74.44^\circ$  and minimum included angle is  $46.51^\circ$  which are better than the other configurations.

## 4. Simulation of deformation and stresses in solid needles

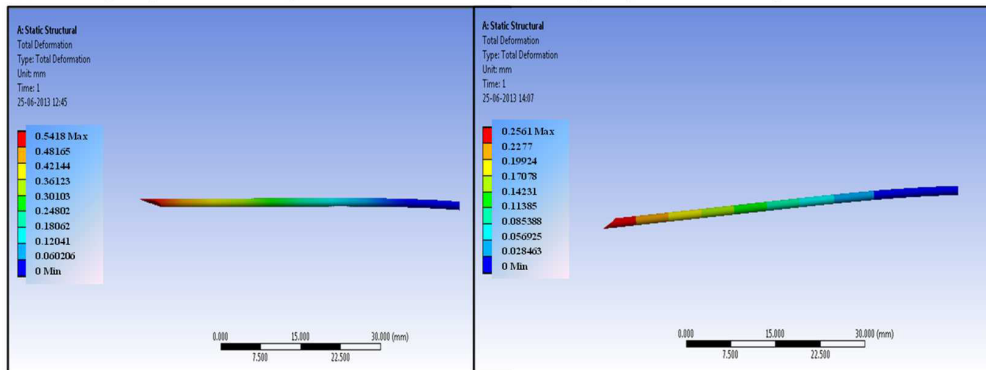
These needles are inserted into tissues to reach a specific target in the tissues for diagnosis. So obviously there will be some placement errors associated during insertion, which might be due to manual errors, higher insertion forces as discussed earlier, geometry of the needle tip etc. After inserting the needle into the tissue it will exert some pressure on the surface of the needle which develops stresses and bending in the needle. This bending and stresses in the needle vary with the geometry of the needle tip configuration used and we can evaluate the performance of a needle by observing the results of bending and stresses for various tip geometries that are discussed above. Since the geometry of the needle tips are a little complex, the results of bending and stresses were analysed through simulation in ANSYS.

For convenience all the needles are considered with a radius of  $r = 0.5$  mm and 60 mm long. A pressure of 1.6MPa, which is the internal pressure of a normal tissue (Priyanka et al., 2004) up to a certain length, had been considered. The true deformation and the equivalent stresses are inserted in the solution column which shows the result of bending and stresses developed. Maximum stress occurs at the fixed support of the needle and minimum at the needle tip. It has been observed that both the bending and stresses for a parabolic surfaced needle are higher than the symmetric three plane needle and cylindrical curved surface needle. The reason for this is due to lack of axis

symmetry that is about the needle longitudinal axis in case of parabolic needles. This problem can be accounted for to some extent by changing the parabola used to cut the needle by the loss of advantage of inclination and included angles.

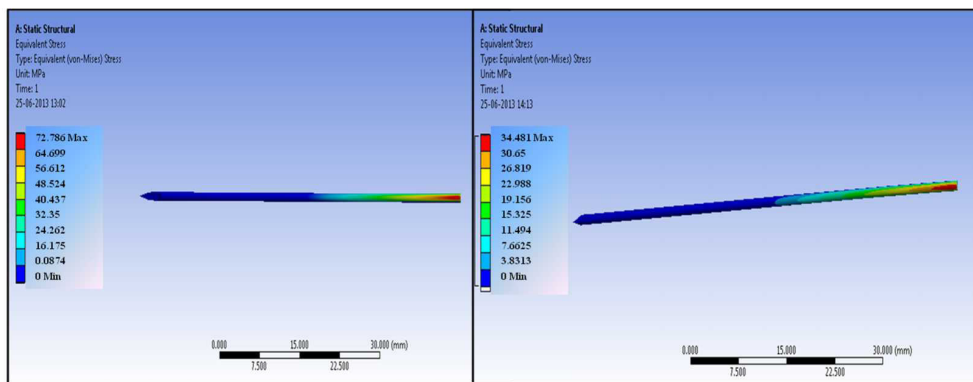


(a) Bending in symmetric three plane bevel needle (b) Bending in symmetric three cylinder surfaced needle



(c) Bending in parabolic surface needle after cutting with bevel planes  $(4x^2-4x+1)$  (d) Bending in parabolic surface needle after cutting with bevel planes  $(2x^2-2x+1/2)$

Fig.6. Deflection of 1mm (diameter) x 60mm long needle



(a) Stresses in parabolic surface needle after cutting with bevel planes  $(4x^2-4x+1)$  (b) Stresses in parabolic surface needle after cutting with bevel planes  $(2x^2-2x+1/2)$

Fig.7. Stress in parabolic surfaced needle cut with two bevel planes



## 5. Conclusions

This study includes understanding of mathematical models of included angle and inclination angle for currently available needles and proposing two novel needle geometries. It has been observed that the parabolic surface needle has undesirable configuration like that of bevel plane needle because the value of inclination angle at the needle tip point is zero irrespective of the parabola used. A new model generated by incorporating two bevel planes to the needle with parabolic cut created a sharp cutting tip and is proved to be better needle tip geometry, especially suitable when the principal aim is to reduce insertion force. Simulation results on bending and stresses in the needles show that proposed needle had undergone higher deflection and stresses are comparable to other earlier symmetric needle tip geometries. The main reason is due to the lack of axis symmetry in the proposed model. Altering the parabolic surface used to cut the cylindrical needle changes the deflection and stresses in the needle significantly. It further reduces the needle cutting edge length, which means reduction in the area of needle tip that pierces into tissue that aids in reducing the frictional forces while the needle cuts into the tissue. While the modeling and simulation results suggest that the tip of parabolic cut surface with two bevel planes reduces the insertion forces, deflection of the needle needs to be fixed. In addition, authors are currently conducting experimental investigations to assess the effect of this needle on its positioning accuracy and insertion force. Further, some other aspects like stiffness and frictional force are also being considered along with the tissue elastic properties.

## 6. References

- [1] Abolhassani N, Patel R, Moallem M., 2007. Needle insertion into soft tissue: a survey. *Medical Engineering Physics* 29, 413–31.
- [2] Atkins AG, Xu X, Jeronimidis G., 2004. Cutting, by pressing and slicing of thin floppy slices of materials illustrated by experiments on cheddar cheese and salami. *Journal of Material Science* 39, 2761–6.
- [3] David G. Simons, Janet G. Travel, Lois S. Simons., *Myofascial Pain and Dysfunction: The Trigger Point Manual; Vol. 1. The Upper Half of Body*. Lippincott Williams & Wilkins, 2<sup>nd</sup> Edition, November 1, 1998 ISBN-13: 978-0683083637
- [4] Moore JZ, Zhang QH, McGill CS, Zheng HJ, McLaughlin PW, Shih AJ., 2010. Modeling of the plane needle cutting edge rake and inclination angles for biopsy. *Journal of Manufacturing Science and Engineering-Trans. of ASME* 132. 051005-1:8.
- [5] Peidong Han, Demeng Che, Kumar Pallav, Kornel Ehmann, 2012. Models of the cutting edge geometry of medical needles with applications to needle design. *International Journal of Mechanical Sciences* 65, 157–167.
- [6] Priyanka Aggarwal, C. R. Johnston., 2004. Geometrical Effects in Mechanical Characterizing of Microneedle for Biomedical Applications. *NSTI - Nanotech* , ISBN 0-9728422-7-6.