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Performance analysis of Arithmetic Mean method in determining peak junction temperature of semiconductor device

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Abstract High reliability users of microelectronic devices have been derating junction temperature and other critical stress parameters to improve device reliability and extend operating life. The reliability of a semiconductor is determined by junction temperature. This paper gives a useful analysis on mathematical approach which can be implemented to predict temperature of a silicon die. The problem could be modeled as heat conduction equation. In this study, numerical approach based on implicit scheme and Arithmetic Mean (AM) iterative method will be applied to solve the governing heat conduction equation. Numerical results are also included in order to assert the effectiveness of the proposed technique.

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1. Introduction

High power is usually encountered in a power device application and it is important to make power devices reliable for their intended application. In order to achieve this goal, considerations have to be taken regarding reliability and performance. During the design phase, especially when a new platform for new technology is involved, thorough calculations and simulations are carried out to ensure the designed electrical parameters and other reliability characteristics are optimized. High reliability users of microelectronic devices have been derating junction temperature and other critical stress parameters for decades to improve device reliability and extend operating life [1]. It is in the first phase, i.e., design phase where semiconductor devices are stressed for reliability and performance [2] and it is very important to predict junction temperature at this phase. Consequently, corresponding electrical circuits as thermal modeling are widely applied because of their easy application in circuit simulators.

The present paper gives a performance analysis of the finite-difference method (FDM) with Arithmetic Mean (AM) iterative method in determining peak junction temperature of semiconductor device. Previously, the AM method has been applied extensively for solving various types of matrix equations problems. The effectiveness of the AM method and its variants were studied and tested on linear and nonlinear systems, refer [3–6] for recent papers.

The rest of this paper is organized in the following way. The mathematical modeling and numerical approach to determine peak junction temperature of semiconductor device will be elaborated in Sections 2 and 3 respectively. In Section 4, some simulation results are included. The discussions and concluding remark are given in Section 5.

2. Mathematical modeling

The following one-dimensional heat conduction equation is considered in modeling the thermal control system

$$K\frac{\partial^2 T(x,t)}{\partial x^2} = \rho c\frac{\partial T(x,t)}{\partial t} \tag{1}$$

since the thermal characteristics of silicon are assumed to be independent of temperature [7]. The T, K, ρ and c represent the absolute temperature, thermal conductivity of the semiconductor device (silicon), mass density of silicon and specific heat of silicon respectively. An Eq. (1) satisfies the following boundary conditions

$$\left.\begin{array}{l} SK\frac{\partial T}{\partial x}\Big|_{x=0} = -P_{in} \\ T(L,t) = T_{in} \end{array}\right\} \tag{2}$$

where S, P_{in}, L and T_{in} are surface of silicon, input power, thickness of vertical power device and input temperature respectively.

Heat is generated at the top surface of silicon and flows linearly along the x-axis which is perpendicular to the silicon surface, S. Thus, the top surface is considered to be a geometrical boundary of the device at $x = 0$ and the input power is assumed to be uniformly dissipated. Meanwhile, the lower surface i.e. at $x = L$ is considered to be the cooling boundary and the temperature is assumed to be equal to the input temperature, T_{in}. Also, the convection and radiation are assumed to be negligible.

3. Numerical approach

In this paper, numerical approach based on implicit scheme and AM iterative method will be considered. The following subsections will explain in detail the application of the numerical approach.

3.1. BTCS discretization scheme

As aforementioned, in this paper, FDM based on implicit scheme i.e. Backward Time, Centered Space (BTCS) is utilized in order to construct algebraic equations for problem (1). Now, let the solution domain be partitioned uniformly in both x and t. Thus, the discrete set of points of x and t, respectively, be given by $x_i = i\Delta x \quad (i = 0, 1, 2, \ldots, n-2, n-1, n)$ and $t_j = j\Delta t \ (j = 0, 1, 2, \ldots, m-2, m-1, m)$ where

$$\Delta x = \frac{L}{n} \tag{3}$$

and

$$\Delta t = \frac{t}{m}. \tag{4}$$

For simplicity, the following notation i.e., $T_{i,j} \equiv T(x_i, t_j)$ will be applied subsequently.

By using BTCS scheme

$$\frac{\partial T}{\partial t} = \frac{T_{i,j+1} - T_{i,j}}{\Delta t} + O(\Delta t) \tag{5}$$

and

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i-1,j+1} - 2T_{i,j+1} + T_{i+1,j+1}}{(\Delta x)^2} + O(\Delta x^2). \tag{6}$$

By substituting formulae (5) and (6) (by dropping the truncation error terms), an application of the BTCS scheme reduces problem (1) to

$$K\frac{T_{i-1,j+1} - 2T_{i,j+1} + T_{i+1,j+1}}{(\Delta x)^2} = \rho c\frac{T_{i,j+1} - T_{i,j}}{\Delta t} \tag{7}$$

which can be rewritten as follows

$$-\alpha T_{i-1,j+1} + \beta T_{i,j+1} - \alpha T_{i+1,j+1} = \gamma T_{i,j} \tag{8}$$

with $\alpha = \frac{K}{(\Delta x)^2}, \beta = \frac{2K}{(\Delta x)^2} + \frac{\rho c}{\Delta t}$ and $\gamma = \frac{\rho c}{\Delta t}$.

An implementation of the BTCS scheme requires solving a linear system at each time step.

Whereas first order discretization of the boundary condition gives

$$SK\frac{T_{1,j+1} - T_{0,j+1}}{\Delta x} = -P_{in} \tag{9}$$

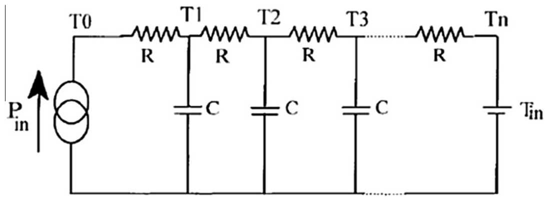


Figure 1 Equivalent thermal circuit networks obtained by using FDM. (*source* [7]).

and

$$T_{n,j+1} = T_{in}. \quad (10)$$

The equivalent circuit of one dimensional thermal model using FDM is shown in Fig. 1, where R and C are the elementary thermal resistance and capacitance, respectively.

Following the conventional process, the generated BTCS algebraic equations (refer Eq. (8)) with the boundary conditions (9) and (10) can be represented in matrix form as

$$AT = U \quad (11)$$

where

$$A = \begin{bmatrix} \sigma & -\alpha & & & & & \\ -\alpha & \sigma & -\alpha & & & 0 & \\ & -\alpha & \sigma & -\alpha & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & -\alpha & \sigma & -\alpha & \\ & 0 & & & -\alpha & \sigma & -\alpha \\ & & & & & -\alpha & \sigma \end{bmatrix},$$

$$T = \begin{bmatrix} T_{1,j+1} \\ T_{2,j+1} \\ T_{3,j+1} \\ \vdots \\ T_{n-3,j+1} \\ T_{n-2,j+1} \\ T_{n-1,j+1} \end{bmatrix} \text{ and } U = \begin{bmatrix} U_{1,j} \\ U_{2,j} \\ U_{3,j} \\ \vdots \\ U_{n-3,j} \\ U_{n-2,j} \\ U_{n-1,j} \end{bmatrix}$$

with

$$\sigma = \begin{cases} \beta - \alpha, & i = 1 \\ \beta, & i = 2, 3, \ldots, n-2, n-1 \end{cases}$$

and

$$U_{i,j} = \begin{cases} \gamma T_{i,j} + \alpha\left(\frac{\Delta x}{SK}\right) P_{in}, & i = 1 \\ \gamma T_{i,j}, & i = 2, 3, \ldots n-3, n-2. \\ \gamma T_{i,j} + \alpha T_{in}, & i = n-1 \end{cases}$$

3.2. Arithmetic Mean iterative method

In this section, the formulation and implementation of the AM method to solve resulting linear system (11) will be discussed. Fundamentally, each iteration of the AM method consists of solving two independent linear systems i.e. T^1 and T^2. Now, let us consider the following splitting

$$A = D - V - W \quad (12)$$

where $D, -V$ and $-W$ are diagonal, strictly lower triangular and strictly upper triangular matrices of A respectively. The general scheme of AM method can be written as follows

$$\left.\begin{aligned} (D - \omega V)T^1 &= [(1-\omega)D + \omega W]T^{(k)} + \omega U \\ (D - \omega W)T^2 &= [(1-\omega)D + \omega V]T^{(k)} + \omega U \\ T^{(k+1)} &= \tfrac{1}{2}(T^1 + T^2) \end{aligned}\right\} \quad (13)$$

where ω is an acceleration parameter. The performance of the AM method can be very often drastically improved with the proper choice of the ω. Based on the scheme (13), the iteration matrix of AM method, Y_{AM} is defined as

$$Y_{AM} = \frac{1}{2}\begin{bmatrix} (D - \omega V)^{-1}((1-\omega)D + \omega W) + \\ (D - \omega W)^{-1}((1-\omega)D + \omega V) \end{bmatrix}. \quad (14)$$

It is already noted that the AM method converges if and only if spectral radius of the iteration matrix is less than one i.e. $\zeta(Y_{AM}) < 1$ and $0 < \omega < 2$ [5]. By determining values of $D, -V$ and $-W$ as mentioned in Eq. (12), an algorithm of AM method to solve problem (1) would be generally described in Algorithm 1.

Algorithm 1. AM method

i. Set all the parameters
ii. Iteration cycle
 for $j = 1, 2, 3, \ldots, m-2, m-1, m$
  for $k = 0, 1, 2, \ldots$ until convergence **do**
  1. Sweep 1
  i) Level 1
  for $i = 1, 2, 3, \ldots, n-3,\ n-2,\ n-1$
   $T^1_{i,j+1} \leftarrow (1-\omega)T^{(k)}_{i,j+1} + \frac{\omega}{\sigma}\left[U_{i,j} + \alpha T^{(k+1)}_{i-1,j+1} + \alpha T^{(k)}_{i+1,j+1}\right]$
  ii) Level 2
  for $i = n-1, n-2, n-3, \ldots, 3, 2, 1$
  $T^2_{i,j+1} \leftarrow (1-\omega)T^{(k)}_{i,j+1} + \frac{\omega}{\sigma}\left[U_{i,j} + \alpha T^{(k)}_{i-1,j+1} + \alpha T^{(k+1)}_{i+1,j+1}\right]$
  2. Sweep 2
  for $i = 1, 2, 3, \ldots, n-3, n-2, n-1$
  $T^{(k+1)}_{i,j+1} \leftarrow \frac{1}{2}(T^1_{i,j+1} + T^2_{i,j+1})$
iii. Convergence test. If the convergence criterion is satisfied i.e. the maximum norm $\|T^{(k+1)} - T^{(k)}\| \leqslant \varepsilon$ (where ε is the convergence criterion) is satisfied, go to Step iv. Otherwise, go to Step ii.
iv. Stop.

4. Numerical simulations and discussions

It is important to define the initial and boundary conditions properly, as it will affect the outcome significantly. At time $t = 0$, it is assumed to be 294 Kelvin (which is the room temperature). Meanwhile, for boundary conditions (refer Eq. (2)), it defines the value at $x = 0$ and $x = L$. At lower boundary i.e. $x = 0$, Neumann condition is considered where the temperature gradient exists. For upper boundary, $x = L$ which is considered as the cooling boundary and assumed the temperature to be constant, $T_{in} = 300.15$ Kelvin. The other input parameters are $L = 0.055$ cm, $S = 0.1\ \text{cm}^2$, $\rho c = 1.63\ \text{J/K/cm}^3$, $K = 1.54$ W/cm/K and $P_{in} = 200$ W.

For the numerical simulations, parameters such as the number of iterations (k), computational time in seconds (CPU) and maximum temperature ($T_{\max}$) are measured for the comparative analysis and the value of initial datum, $T^{(0)}$, is set to zero. The optimal value of ω for AM method is chosen within ±0.1 by a trial and error process. All the simulations are performed on a personal computer with Intel(R) Core(TM) i3-2328 (2.20 GHz, 2.20 GHz) and 2.60 GB RAM, and the programs are compiled by using MatLab. In addition, numerical results of the conventional Gauss–Seidel (GS) method and built-in function in Matlab i.e. *pdepe* are also included in order to verify the performance of the AM method. In this study, the convergence criterion for GS and AM methods is $\varepsilon = 10^{-10}$ and three different elapsed time i.e. $t = 0.002$, $t = 0.006$ and $t = 0.010$ are considered. Numerical results from the simulations are presented in Tables 1–3. Meanwhile, temperature profile of each tested n for $t = 0.002$, $t = 0.006$ and $t = 0.010$ is illustrated in Figs. 2–4, respectively. Based on numerical results obtained (Tables 1–3), percentage gains in terms of number of iterations (%k) and computational time (%CPU) of the AM method compared with GS method are presented in Table 4.

Table 1 Numerical results for case $t = 0.002$.

n	Methods	k	CPU	$T_{\max}$
30	GS	12,155	0.88	355.7851
	AM	4322 (ω =1.6)	0.50	355.7851
	pdepe	–	–	357.5525
60	GS	45,725	1.85	356.5180
	AM	10,462 (ω =1.8)	1.30	356.5180
	pdepe	–	–	357.5523
90	GS	99,891	4.68	356.7612
	AM	18,881 (ω =1.8)	2.34	356.7612
	pdepe	–	–	357.5435
120	GS	173,960	10.14	356.8826
	AM	27,746 (ω =1.9)	3.59	356.8826
	pdepe	–	–	357.5460
150	GS	211,122	12.38	356.9554
	AM	36,920 (ω =1.9)	4.85	356.9554
	pdepe	–	–	357.5539

Table 2 Numerical results for case $t = 0.006$.

n	Methods	k	CPU	$T_{\max}$
30	GS	31,506	1.11	368.5538
	AM	8915 (ω =1.7)	0.85	368.5538
	pdepe	–	–	370.9317
60	GS	120,789	4.65	369.6896
	AM	22,635 (ω =1.9)	2.57	369.6896
	pdepe	–	–	370.9455
90	GS	264,487	12.88	370.0674
	AM	37,943 (ω =1.9)	4.64	370.0674
	pdepe	–	–	370.9221
120	GS	388,332	21.25	370.2562
	AM	55,476 (ω =1.9)	6.74	370.2562
	pdepe	–	–	370.9241
150	GS	573,252	27.36	370.3694
	AM	75,428 (ω =1.9)	9.77	370.3694
	pdepe	–	–	370.9451

Table 3 Numerical results for case $t = 0.010$.

n	Methods	k	CPU	$T_{\max}$
30	GS	46,424	1.55	369.1622
	AM	11,457 (ω =1.8)	1.28	369.1622
	pdepe	–	–	371.5474
60	GS	178,336	6.95	370.3479
	AM	29,839 (ω =1.9)	3.33	370.3479
	pdepe	–	–	371.5452
90	GS	329,889	18.32	370.7430
	AM	50,575 (ω =1.9)	6.15	370.7430
	pdepe	–	–	371.5507
120	GS	450,078	22.35	370.9406
	AM	75,013 (ω =1.9)	9.17	370.9406
	pdepe	–	–	371.5497
150	GS	506,660	29.02	371.0591
	AM	103,400 (ω =1.9)	13.35	371.0591
	pdepe	–	–	371.5454

Table 4 Percentage gains in terms of number of iterations and computational time.

Elapsed time	%k	%CPU
$t = 0.002$	64.44–84.06	29.72–64.60
$t = 0.006$	71.70–86.85	23.42–68.29
$t = 0.010$	75.32–84.67	17.41–66.44

5. Conclusion

In this paper, numerical approach based on BTCS scheme and AM method has been successfully implemented in determining peak junction temperature of semiconductor device. Based on the numerical results obtained, it clearly shows that an application of the AM method reduced the number of iterations and computational time compared to conventional GS method, refer Table 4. The numerical solutions obtained by using AM iterative method are in good agreement with the GS and *pdepe* methods. Overall, AM method is more superior

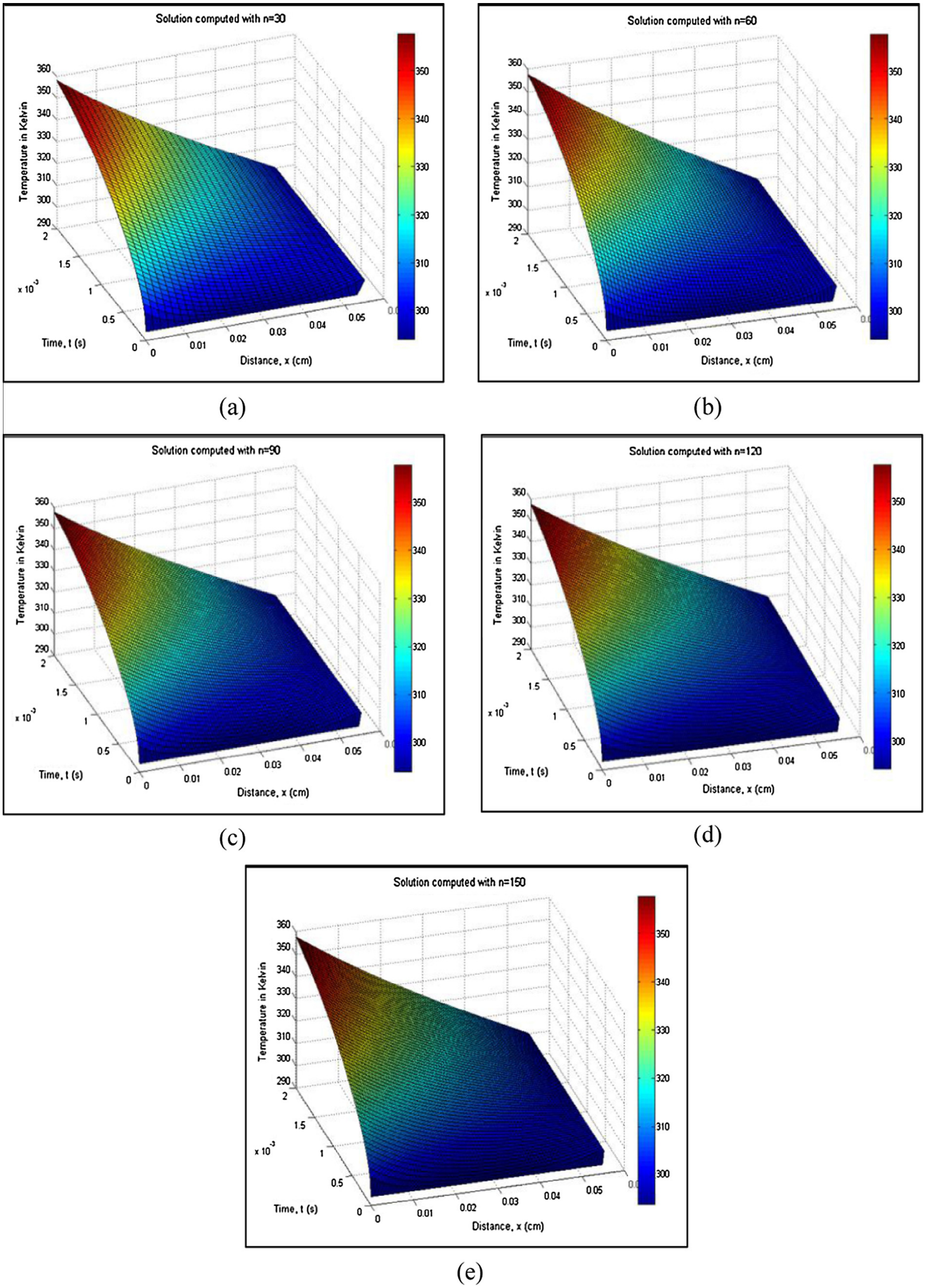


Figure 2 (a)–(e) Show the temperature profile for the case $t = 0.002$.

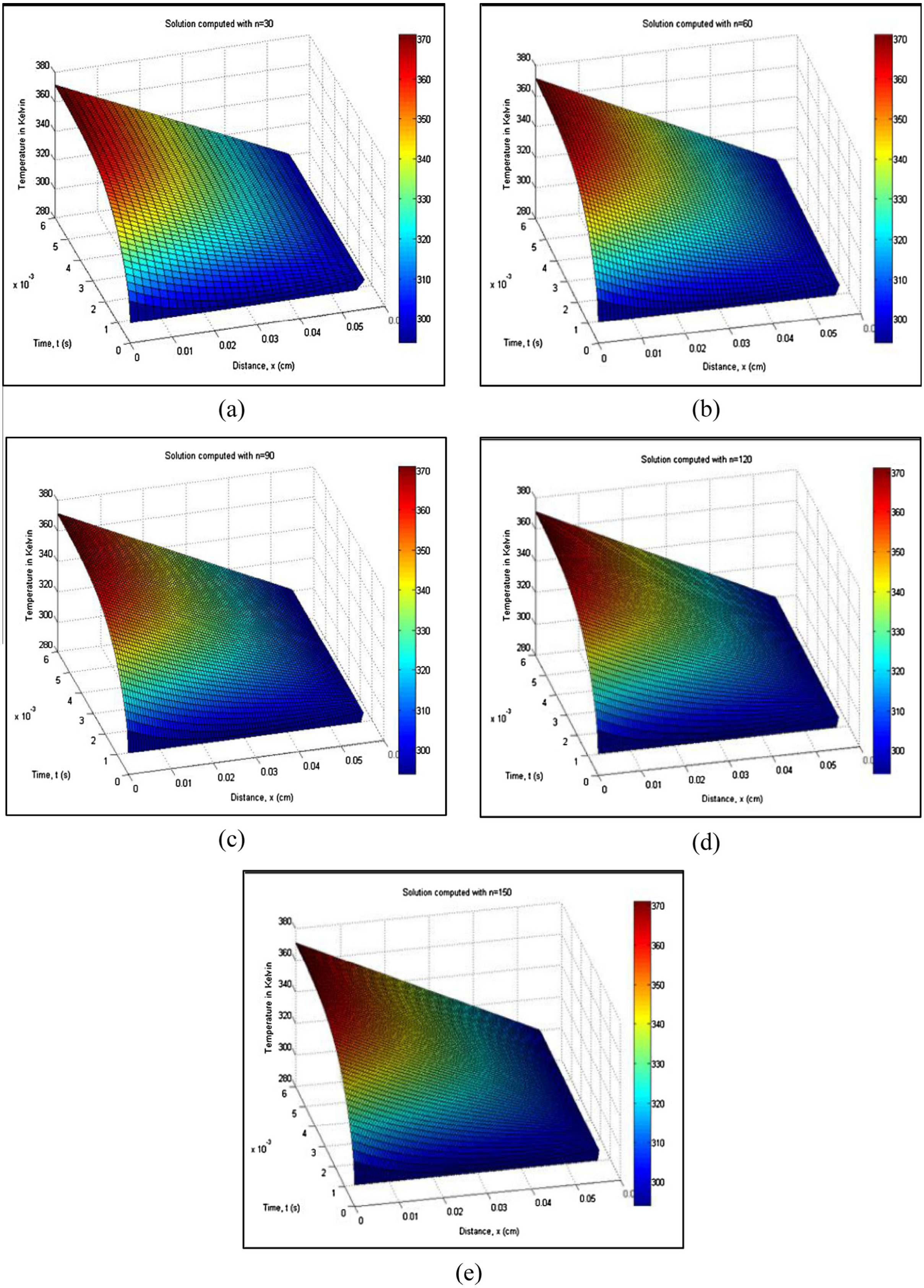
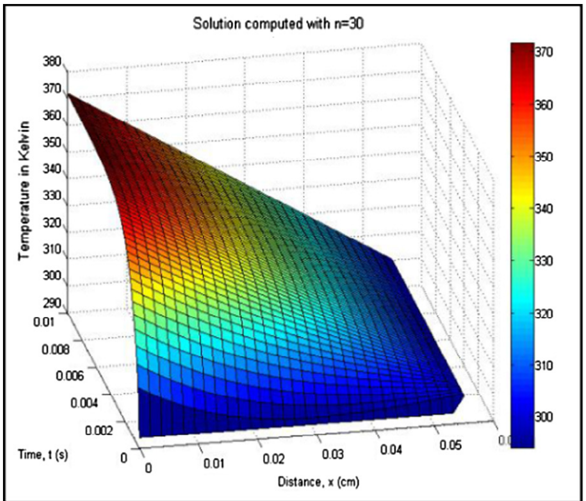
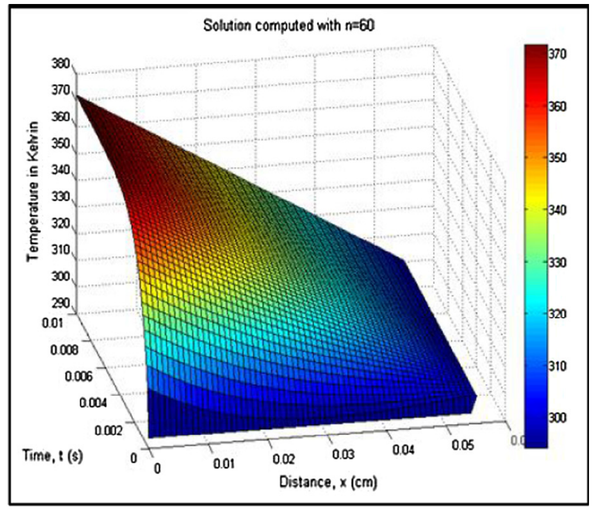


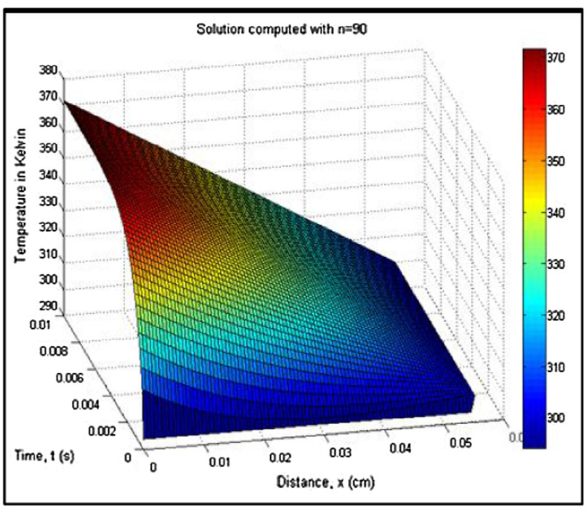
Figure 3 (a)–(e) Show the temperature profile for the case $t = 0.006$.



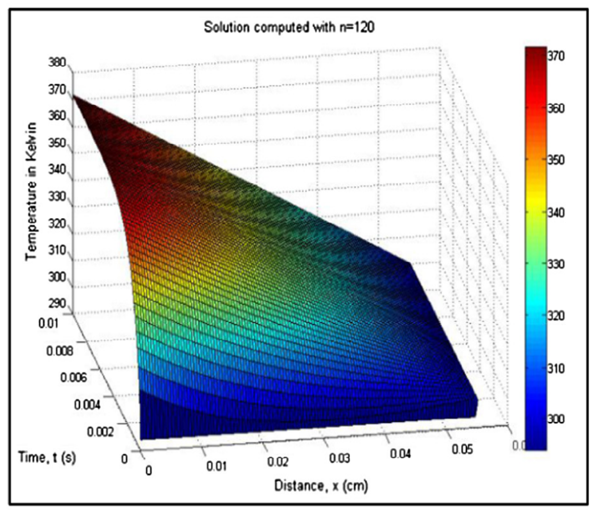
(a)



(b)



(c)



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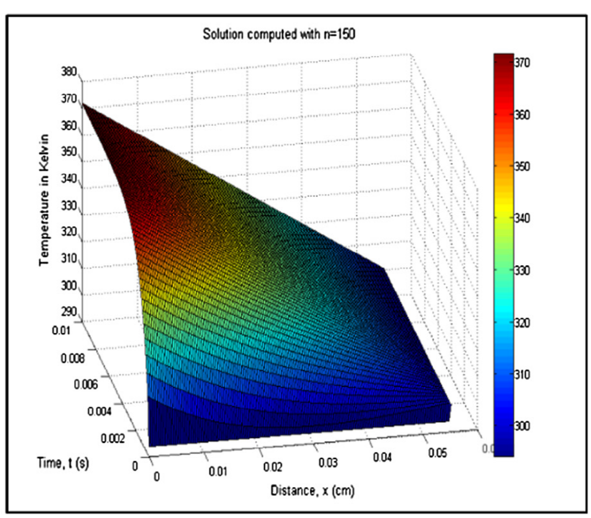

(e)

Figure 4 (a)–(e) Show the temperature profile for the case $t = 0.010$.

compared to GS method in determining the peak junction temperature. For the future works, this work can be extended to predict the actual IC die temperature of semiconductor device.

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