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The Hagedorn temperature and open QCD-string tachyons in pure $\mathcal{N}=1$ super-Yang–Mills

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Abstract

We consider large-N confining gauge theories with a Hagedorn density of states. In such theories the potential between a pair of colour-singlet sources may diverge at a critical distance $r_c = 1/T_H$. We consider, in particular, pure $\mathcal{N}=1$ super-Yang–Mills theory and argue that when a domain wall and an anti-domain wall are brought to a distance near r_c the interaction potential is better described by an "open QCD-string channel". We interpret the divergence of the potential in terms of a tachyonic mode and relate its mass to the Hagedorn temperature. Finally we relate our result to a theorem of Kutasov and Seiberg and argue that the presence of an open string tachyonic mode in the annulus amplitude implies an exponential density of states in the UV of the closed string channel.
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1. Introduction

Gauge field theories with an asymptotic exponential density of states admit a "limiting temperature" called the Hagedorn temperature (see [1] for a recent review). It is believed that four dimensional gauge theories such as pure Yang–Mills (or pure $\mathcal{N}=1$ super-Yang–Mills) confine and that the colour-singlet states exhibit a Hagedorn behaviour. The goal of this note is to explore the implications of the Hagedorn behaviour on the short distance, $r < \Lambda_{\mathrm{QCD}}^{-1}$, dynamics of confining theories.

Our main focus is large-N pure $\mathcal{N}=1$ super-Yang–Mills, since in this theory the vacuum structure of the theory is well understood: the $U_R(1)$ symmetry is broken by the anomaly to a discrete $\mathbf{Z}_{2N}$ symmetry which is further broken down spontaneously to $\mathbf{Z}_2$ [2,3]. The gluino condensate is the order parameter for this breaking [4] which labels the N vacuum states. The fact that there are degenerate vacua means that there are domain walls in this theory which separate regions of different vacuum. What is particularly special is that planar domain walls are BPS objects that preserve one half of the supersymmetry of the theory [5]. These walls will be of prime interest to us.

$\mathcal{N}=1$ SYM is in some respects a good toy model for real QCD. The model consists of one quark flavor in the adjoint representation and its relation to ordinary one flavor QCD is not only qualitative. The model is expected to confine and the glueball spectrum is expected to have a mass gap. Understanding this model in full is an outstanding problem in its own right which will likely teach us much about real QCD. However, for present purposes we should note that QCD does not have domain walls.

Returning to the supersymmetric theory, we will consider a setup of a domain wall and an anti-domain wall and argue that as the walls are brought close to each other, the system develops a tachyonic QCD-string mode. This is in keeping with the interpretation of the domain wall as a D-brane [6]. Our main observation is a relation between the Hagedorn temperature and the mass of the tachyon

$$|M_0| = \frac{\sigma}{T_H}, \tag{1}$$

where M_0 is the tachyon mass, T_H is the Hagedorn temperature and σ is the QCD-string tension.

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The Letter is organized as follows: in Section 2 we discuss the general behaviour of a potential between colour-singlets in confining theories. In Section 3 we focus on domain walls in $\mathcal{N}=1$ SYM. Section 4 is devoted to a generalization of a theorem by Kutasov and Seiberg.

2. The potential between colour-singlets

Consider two colour singlets in a confining gauge theory, separated by a distance r. The colour singlets may be either glueballs, or QCD-strings (Wilson-loops) or domain walls.

The sources interact via the exchange of glueballs. If the source itself is a glueball or a QCD-string, the interaction is suppressed by $1/N$. If the source is a domain wall the interaction is not suppressed even at infinite N. At large enough N, the interaction is dominated by a tree-level exchange of glueballs. Therefore a general expression for the potential between the sources is

$$V(r) \sim \sum_n |C_n|^2 d_n \int d^3k \frac{e^{ikr}}{k^2+M_n^2} \sim \sum_n |C_n|^2 d_n e^{-M_n r}. \quad (2)$$

Here, the sum, is over glueballs states and d_n is the number of glueballs of mass M_n. C_n is the glueball-source coupling. Our discussion below will be valid in cases where C_n does not depend exponentially on n. An exponential dependence is not expected to occur in the wall-source case, as suggested by the type II annulus amplitude. On the other hand it does occur in the Veneziano amplitude.[1] Clearly, the interaction at large separation is controlled by the lowest states in the tower. If the system exhibits an asymptotic Hagedorn behaviour $d_n \to \exp(\gamma M_n)$, for a constant γ, the potential (2) then takes the form

$$V(r) \sim \sum_n e^{\gamma M_n} e^{-M_n r}. \quad (3)$$

We will ignore any sub-leading polynomial in M_n that can multiply $e^{\gamma M_n}$. Notice that (3) has the form of the canonical partition function of the system at a temperature $T=1/r$. In particular, the constant γ is identified with the inverse Hagedorn temperature, $\gamma = 1/T_H$. Now the canonical partition function is not well defined when $T \geqslant T_H$, and at the Hagedorn temperature $T=T_H$ the partition function diverges. This implies that the potential (3) diverges when r is reduced to a critical distance $r_c = 1/T_H$. What is the physical significance of this divergence? In general, it means that system is not stable and that there exists a better description of the interaction in terms of different degrees of freedom. When the sources are glueballs or QCD-strings, the answer is that the potential should be calculated by using the short distance degrees of freedom—the gluons, which interact with the constituents of the source. The subject of the next section is to discuss the meaning of the divergence in the potential when the sources are domain walls.

Before we continue, it is useful to replace the sum over the glueballs states in (3) by an integral over a mass density:

$$V(r) \sim \int dM\, e^{\gamma M} e^{-Mr}. \quad (4)$$

[1] We thank A. Schwimmer for bringing this example to our attention.

3. The domain wall–anti-domain wall system

$\mathcal{N}=1$ SYM $SU(N)$ theory admits N degenerate vacua. Hence there exist domain walls that separate those vacua [5]. The tension of the walls is given by the absolute value of the difference between the values of the gluino condensate [5]

$$T_k = \frac{N^2 \Lambda^3_{\mathrm{QCD}}}{4\pi^2} \sin\frac{\pi k}{N}. \quad (5)$$

The tension of a fundamental wall is $\sim N$. In string theory the tension of a D-branes is $T \sim 1/g_{\mathrm{st}}$. Together with the identification $g_{\mathrm{st}} \sim 1/N$, it is suggestive of the conjecture that the domain walls are D-branes for the $\mathcal{N}=1$ QCD string [6].

There is further evidence that domain walls are D-branes. First of all, the QCD string can end on a domain wall [6,7]. Domain walls interact via an exchange of glueballs [9] (states of the "QCD closed string"). Moreover, it has been argued that the collective dynamics of a stack of domain walls is described by a 3d gauge theory on the world volume [8]. By using this 3d world-volume gauge theory an explicit two-loops calculation of the interaction between two domain walls was made in [10,11]. This is a field theory example of open/closed string channel duality.

Let us briefly review how two domain walls interact. The domain walls carry a tension and a charge (in parallel to the NS–NS tension and R–R charge of D-branes in type II string theory). Even parity glueballs couple to the tension density F^2 and odd parity glueballs couple to the charge density $F\tilde{F}$. Supersymmetry implies the following identity [9]

$$0 = \int d^4x \left(\langle F^2(x), F^2(0)\rangle - \langle F\tilde{F}(x), F\tilde{F}(0)\rangle\right). \quad (6)$$

This is saturated at large-N, by the exchange of even and odd parity glueballs, namely

$$\langle F^2(x), F^2(0)\rangle - \langle F\tilde{F}(x), F\tilde{F}(0)\rangle = \int d^4q\, e^{iq\cdot x}\left(\sum_+ \frac{f_n^2}{q^2+M_n^2} - \sum_- \frac{f_n^2}{q^2+M_n^2}\right). \quad (7)$$

The vanishing of the right-hand side of this equation is due to supersymmetry: the couplings f_n and the masses M_n of the even and odd parity glueballs are degenerate. The interpretation is clear: if we place two parallel domain walls there will be no force between them, since this is a BPS configuration (at large-N). The microscopic reason for the vanishing of the force is a perfect cancellation between the two glueball towers.

Consider now the following set-up: a domain wall and an anti-domain wall, separated by a certain distance r. To the left and to the right of the configuration there exists the same vacuum state. However, in between the walls the vacuum is different. The walls are expected to attract each other and finally annihilate. Intuitively, there is a probability of a tube of the first vacuum state being formed at some local position on the domain walls. This tube would then expand. Going back to the expression (7). The only difference is a sign: both the even parity glueballs and the odd parity glueballs attract

$$\langle F^2(x), F^2(0)\rangle + \langle F\tilde{F}(x), F\tilde{F}(0)\rangle = \int d^4q\, e^{iq\cdot x}\left(\sum_+ \frac{f_n^2}{q^2+M_n^2} + \sum_- \frac{f_n^2}{q^2+M_n^2}\right). \quad (8)$$

The potential (4) between a wall and an anti-wall can be written as

$$V(r) \sim \int_0^\infty ds \int dk \int M^2\, dM\, e^{M/T_H} e^{-s(k^2+M^2)} e^{ikr}. \quad (9)$$

The integration over k and M yields

$$V(r) \sim \int ds\, e^{1/(4sT_H^2)} e^{-r^2/4s}. \quad (10)$$

Written in terms of the variable $t = 1/(4\sigma^2 s)$ the potential takes the form

$$V(r) \sim \int dt\, e^{t(\sigma^2/T_H^2-(\sigma r)^2)}. \quad (11)$$

Assuming that there is a field theory living on the wall–anti-wall system, then we can calculate the potential by the "open string channel", namely by calculating the Casimir energy of the system. The contribution of a single massive state to the Casimir energy is

$$E(r) = \int \frac{d^3k}{(2\pi)^3} \log\bigl(k^2+M^2\bigr) = \int \frac{dt}{t}\frac{1}{t^{3/2}} e^{-tM^2}. \quad (12)$$

The energy (12) can be matched with the potential (11) if

$$M^2 = -M_0^2 + (\sigma r)^2, \quad (13)$$

with

$$M_0^2 = \frac{\sigma^2}{T_H^2}. \quad (14)$$

Namely, there must exists a tachyonic mode in the "open QCD-string channel" whose mass respects (13) and (14).

Our result is an interesting UV/IR relation within field theory: we related the Hagedorn temperature—a property of the UV to a tachyonic mass (IR mode). Note that the above derivation assumes a stringy picture, but it does not assume a particular string theory.

4. A relation between open strings and closed strings

In this section we wish to comment on the relation between our field theory result and similar relations in string theory.

Kutasov and Seiberg proved a while ago [12], by using modular invariance, the following relation in any oriented closed string theory: if the density of states of bosons minus the density of states of fermions is exponential, it implies a tachyon. This is a remarkable UV/IR relation. The theorem was extended to open string theory by Niarchos [13] who showed by analyzing the annulus diagram that a closed string tachyon is related to an exponential density of states in the open string channel.

We wish to further extend the theorem to a relation between an open string tachyon and a Hagedorn density of NS–NS (or R–R) states. A related discussion for critical type II string theory can be found in Refs. [14,15].

Consider the annulus amplitude in open string theory. It has the following structure

$$\int \frac{dt}{t} A(t), \quad (15)$$

where $A(t)$ is the vacuum energy of the open string tower

$$A(t) = \int \frac{d^Dp}{(2\pi)^D} \sum_n e^{-t(p^2+M_n^2)}. \quad (16)$$

The contribution from the lowest state of the tower is simply

$$A(t) \sim \frac{1}{t^{D/2}} e^{-tM_0^2} \quad (17)$$

and in particular if the lowest state is tachyonic $M_0^2 < 0$, the annulus amplitude diverges at $t \to \infty$. This is an IR divergence.

There is another way of understanding this divergence. The same amplitude can be written in terms of the variable $\tau = \frac{1}{t}$,

$$\int \frac{d\tau}{\tau} A(\tau), \quad (18)$$

with

$$A(\tau) = \tau^{D/2} e^{|M_0^2|/\tau}. \quad (19)$$

The interpretation of the amplitude, written in terms of the variable τ, is of a propagation of closed string between a stack of two branes. The behaviour at $\tau \to 0$ is a property of the UV regime. The exponential divergence at the UV (19) should be interpreted as an exponential density of bosonic (NS–NS or R–R) closed string states, exactly as in [12]. Thus we found the following relation between the IR of the open string theory and the UV of the closed string theory: an open string tachyon exists if and only if the asymptotic density of closed strings that propagate in the annulus diagram is exponential.

Let us consider one particular case. A system of a brane and an anti-brane in type II string theory. When the branes are brought close to each other a tachyonic mode develops, since the mass of the lightest open string modes is $\alpha' M^2 = -\frac{1}{2} + \frac{y^2}{4\pi^2\alpha'}$. Physically, this is due to the instability of the system: a lower energy state can be achieved by the annihilation of the branes. The explanation in terms of the closed string channel is also straightforward: the NS–NS and R–R sectors contribute equally and *with equal signs*. Thus one can multiply the NS–NS contribution by two. The amplitude exhibits an asymptotic exponential density of states. This is perhaps the simplest example of a bosonic string amplitude that exhibits our claimed IR/UV connection.

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References

[1] J.L.F. Barbon, E. Rabinovici, hep-th/0407236.
[2] E. Witten, Nucl. Phys. B 202 (1982) 253.
[3] G. Veneziano, S. Yankielowicz, Phys. Lett. B 113 (1982) 231.
[4] M.A. Shifman, A.I. Vainshtein, Nucl. Phys. B 296 (1988) 445, Sov. Phys. JETP 66 (1987) 1100.
[5] G.R. Dvali, M.A. Shifman, Phys. Lett. B 396 (1997) 64, hep-th/9612128; G.R. Dvali, M.A. Shifman, Phys. Lett. B 407 (1997) 452, Erratum.
[6] E. Witten, Nucl. Phys. B 507 (1997) 658, hep-th/9706109.
[7] M. Shifman, A. Yung, Phys. Rev. D 67 (2003) 125007, hep-th/0212293.
[8] B.S. Acharya, C. Vafa, hep-th/0103011.
[9] A. Armoni, M. Shifman, Nucl. Phys. B 670 (2003) 148, hep-th/0303109.
[10] A. Armoni, T.J. Hollowood, JHEP 0507 (2005) 043, hep-th/0505213.
[11] A. Armoni, T.J. Hollowood, JHEP 0602 (2006) 072, hep-th/0601150.
[12] D. Kutasov, N. Seiberg, Nucl. Phys. B 358 (1991) 600.
[13] V. Niarchos, JHEP 0106 (2001) 048, hep-th/0010154.
[14] D. Gaiotto, N. Itzhaki, L. Rastelli, Nucl. Phys. B 688 (2004) 70, hep-th/0304192.
[15] S. Sarangi, S.H.H. Tye, Phys. Lett. B 573 (2003) 181, hep-th/0307078.