Leveraged fault identification method for receiver autonomous integrity monitoring

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Abstract Receiver autonomous integrity monitoring (RAIM) provides integrity monitoring of global positioning system (GPS) for safety-of-life applications. In the process of RAIM, fault identification (FI) enables navigation to continue in the presence of fault measurement. Affected by satellite geometry, the leverage of each measurement in position solution may differ greatly. However, the conventional RAIM FI methods are generally based on maximum likelihood of ranging error for different measurements, thereby causing a major decrease in the probability of correct identification for the fault measurement with high leverage. In this paper, the impact of leverage on the fault identification is analyzed. The leveraged RAIM fault identification (L-RAIM FI) method is proposed with consideration of the difference in leverage for each satellite in view. Furthermore, the theoretical probability of correct identification is derived to evaluate the performance of L-RAIM FI method. The experiments in various typical scenarios demonstrate the effectiveness of L-RAIM FI method over conventional FI methods in the probability of correct identification for the fault with high leverage.

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1. Introduction

Global positioning system (GPS) has become the core element of modern air traffic system by greatly enhancing the operational efficiency. To ensure the safety of flight, excessive ranging errors on any navigation signals broadcasted by GPS satellites that would cause unaccepted positioning error must be detected, identified and excluded. To achieve this goal, one effective method is called receiver autonomous integrity monitoring (RAIM), an augmentation to GPS which uses self-consistency check among measurements of navigation satellite signals to detect and identify potential excessive ranging errors arising from satellite hardware, signal propagation, and receiver, i.e. faults. RAIM is essential for safety-of-life applications and is a mandatory function embedded in aviation navigation receiver to support the air navigation for enroute, terminal, and non-precision approach (NPA) phases of flight.1–3

The key function of RAIM to identify faults is called fault identification (FI). Various RAIM FI methods were studied over the past decades and could be classified into three categories: maximum likelihood estimation fault identification

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algorithm (MLE FI), characteristic bias line fault identification algorithm (CBL FI) and subset measurement fault identification algorithm (SM FI). These three kinds of methods were proved to have equivalent FI performance but with different calculation costs. Novel RAIM FI methods that identify faults among measurements from different navigation satellite constellations, e.g. GPS, Beidou and Galileo, have become hot spots in recent years. In the event of simultaneous multiple faults, the identification process is repeated until no more faults are identified.

In spite of different implementations, current RAIM FI methods are generally based on the same basic idea, which is to determine the satellite measurement that maximizes the likelihood of ranging error. However, the impact of ranging error on positioning error has not been considered in previous RAIM FI method. Actually, the ranging measurements from different satellites have different impacts on the positioning solutions. This effect is defined as “leverage” in regression theory. The measurement with higher leverage has larger impact on position estimation than that with lower leverage. Therefore, faults on high leverage measurement tend to cause larger positioning errors. Whereas, the probability of correct identification using traditional RAIM FI methods may decrease in the presence of faults on high leverage measurements.

In this paper, a leveraged RAIM fault identification (L-RAIM FI) method considering the difference in measurement leverage is proposed. The theoretical probability of correct identification is derived. Based on this, the performance of L-RAIM FI method and traditional FI method in the probability of correct identification is compared and discussed. Experimental results with simulated and real data show that the L-RAIM FI method outperforms the traditional method in the probability of correct identification.

The remainder of this paper is organized as follows. In Section 2, the traditional RAIM FI method is described. In Section 3, the L-RAIM FI method is proposed which takes account into different leverage of the measurement. In Section 4, the probability of correct identification is derived to compare the performance of L-RAIM FI method with the traditional method. The experiments are conducted in Section 5 to demonstrate the performance of our approach. Finally the conclusions are shown in Section 6.

2. Traditional RAIM FI method

Because of the equivalence of traditional RAIM FI methods, only MLE FI method is described in this section as a baseline for further discussion.

The MLE FI method employs the maximum likelihood criterion to estimate fault bias. After that, the likelihood probability under the estimated bias is exploited to identify the fault measurements.

The basic linearized GPS measurement equation is described by an over-determined system.

\[
z = Hx + e + f
\]

where \(e \in \mathbb{R}^n\) is a vector of pseudorange measurement residuals, in which \(n\) is the number of satellites in view. \(H \in \mathbb{R}^{m \times n}\) is the observation matrix consisting of line-of-sight vectors. \(x \in \mathbb{R}^4\) is a vector of estimated position and clock bias correction.

\(e \in \mathbb{R}^n\) is Gaussian measurement errors with the covariance of \(\sigma^2\).

The existing methods model the fault as measurement bias added to the measurement noise. Then the measurement equation with fault can be expressed as follows:

\[
z = Hx + e + f
\]

where \(f\) denotes the fault bias vector.

Currently the satellite navigation system with RAIM can only be applied to the phases from en-route to non-precise approaches. For more stringent precise approach, specific standard for RAIM has not been developed yet. In this paper, only single fault is considered corresponding to the requirement for non-precise approaches. That means only one element in fault bias vector \(f\) is non-zero. The vector \(f\) is determined by multiplying the fault bias magnitude \(b\) and fault mode \(\mu_i\), i.e.

\[
f = b \mu_i, \quad i = 1, 2, \ldots, n
\]

where \(\mu_i\) is \(n \times 1\) fault mode matrix. Corresponding to the fault on the \(i\)th satellite, the \(i\)th element of \(\mu_i\) is one and the other elements of \(\mu_i\) are zeros.

As the components of pseudorange measurement residual vector are not completely independent of each other, the state space is transformed to parity space to eliminate the correlation between the components. Using QR decomposition, matrix \(H\) can be decomposed as follows,

\[
H = UT = \begin{bmatrix} U_1, U_2 \end{bmatrix} \begin{bmatrix} T_1 \; T_2 \end{bmatrix}
\]

where \(U_1 \in \mathbb{R}^{m \times 1}\) and \(U_2 \in \mathbb{R}^{m \times (n-4)}\) constitute the unitary matrix \(U \in \mathbb{R}^{m \times n}\). \(T_1 \in \mathbb{R}^{4 \times 4}\) is the first four rows of matrix \(T \in \mathbb{R}^{m \times n}\).

Then the parity vector \(p \in \mathbb{R}^{n-4}\) is defined as

\[
p = U_1^T z
\]

The elements of parity vector are uncorrelated following joint Gaussian distribution with the expected value \(b U_2^T \mu_i\) and the covariance \(\sigma^2 I_{n-4}\). The probability density function of parity vector conditioned on bias magnitude and fault mode is

\[
p(p | b, \mu_i) = (2\pi \sigma^2)^{-(n-4)/2} \exp[-J(b, \mu_i)/2]
\]

where \(J(b, \mu_i) = (p - b U_2^T \mu_i)^T (p - b U_2^T \mu_i)\). To describe the probability density distribution \(p(p | b, \mu_i)\) with the changing of bias magnitude \(b\) corresponding to different fault mode \(\mu_i\), the expansion of \(J(b, \mu_i)\) in Eq. (6) is given by

\[
J(b, \mu_i) = S_i b^2 - 2 b S_i^T z + z^T S_i z
\]

where \(S_i\) and \(S_{ii}\) are the \(i\)th column vector and the \(i\)th diagonal element of matrix \(S = I_n - (H^T H)^{-1} H^T\) respectively.

In the process of the MLE FI method, the estimated bias magnitude \(\hat{b}\) is calculated using MLE principle to maximize \(p(p | b, \mu_i)^2 i.e.

\[
\hat{b_i} = \frac{S_i^T z}{S_{ii}}
\]

If \(p(b | \mu_i)\) follows the uniform distribution, \(\hat{b_i}\) estimated by MLE and maximum a posteriori probability (MAP) is equivalent. Then the ranging source \(I\) is identified as fault if

\[
I = \arg \max_i p(\hat{b_i} | \mu_i)
\]
Substitute Eqs. (6)-(8) into Eq. (9), we have

$$I = \arg \max_{\mu} \left( \left( S_i^T \mathbf{z} \right)^2 / S_{ii} \right)$$

(10)

After fault identification, the isolation procedure is conducted by removing fault measurement $I$ from the position solution.

However, the traditional RAIM FI method only considers the fault magnitude $b_i$ estimated by the parameter estimation method. The difference in the shape of $p(p|b, \mu_i)$ resulting from different leverage over each $\mu_i$ is ignored. It is probably to cause wrong identification especially when the fault occurs on the measurement with high leverage, as shown in the following sections.

3. L-RAIM FI method

In this section, the impact of leverage on the FI method is analyzed and the L-RAIM FI method is proposed. Then, the implementation process of the L-RAIM FI method is provided at the end of the section.

The fault identification aims at finding the failing ranging sources after fault detection alarm, which is pattern recognition in nature. Bayesian decision theory is a basic method with minimum error rate in pattern recognition. According to Bayesian decision theory, the most probable fault satellite is the one which maximizes the posterior probability $P(\mu|p)$ over observed parity vector $\mathbf{z}$, i.e.

$$I = \arg \max_{\mu} P(\mu|p)$$

(11)

The posterior probability $P(\mu|p)$ of the fault mode induced by the measurement data can be computed using Bayesian formula as follows:

$$P(\mu|p) = \frac{P(\mu)p(p|\mu)}{\sum_{i=1}^{n} P(\mu)p(p|\mu)}$$

(12)

where $\sum_{i=1}^{n} P(\mu)p(p|\mu)$ keeps constant for each $\mu$, $p(p|\mu)$ is class-conditional probability density distribution and $P(\mu)$ is prior probability of fault mode $\mu$.

According to the satellite fault model, the characteristic of each satellite fault in one constellation is independent of each other following the same statistical distribution. For $i = 1, 2, \ldots, n$, the probability $P(\mu_i)$ in Eq. (12) remains the same. Thus, the process to find the $\mu_i$ with maximum $P(\mu|p)$ is therefore transformed to find the $\mu_i$ with maximum $p(p|\mu_i)$, i.e.

$$\arg \max_{\mu} P(\mu|p) = \arg \max_{\mu} p(p|\mu)$$

(13)

$p(p|\mu)$ is the probability of observed parity vector under the fault mode $\mu_i$ with unknown fault bias $b$. According to Eq. (6), many different fault biases under $\mu_i$ have the potential to produce the observed $p$. The probability density distribution $p(p|b, \mu_i)$ under different fault mode $\mu_i$ is different resulting from different value of $S_{ii}$. For different satellite, $S_{ii}$ is related to the importance of the corresponding measurement in position solution. In regression theory, leverage is defined to measure the contribution of the measurement to the regression result. The smaller $S_{ii}$ is, higher leverage the measurement has. The high leverage measurement with smaller $S_{ii}$ has larger impact on the estimated position while the low leverage one has little impact.

In the light of the leverage difference, the L-RAIM FI method identifies the fault by comparing the class-conditional probability density $p(p|\mu_i)$ which involves all possible bias magnitude $b$. As the gradient of $p(p|\mu_i)$ is not big enough, many values of $b$ have probabilities to fit the observed parity vector especially for the fault with high leverage. The $p(p|\mu_i)$ takes the different measurement leverage into accounts and thereby is an optimal classifier for fault identification.

By integrating the likelihood probability density of parity vector over all possible biases, $p(p|\mu_i)$ is determined as follows:

$$p(p|\mu_i) = \int_{\mathbb{R}} db p(b|\mu_i) p(p|b, \mu_i)$$

(14)

The prior model for bias magnitude $b$ is presented as uniformly and identically distributed for each $\mu_i$, given by $p(b|\mu_i) \sim \text{lim}_{M \to \infty} U[-M/2, M/2]$. Substituting the prior model of $b$ to Eq. (14) yields

$$p(p|\mu_i) = \lim_{M \to \infty} \int_{-M/2}^{M/2} \exp\left[-J(b, \mu_i)/2\right] db$$

(15)

Substitute Eq. (7) to Eq. (15), then the class-conditional probability $p(p|\mu_i)$ can be simplified as

$$p(p|\mu_i) = \lim_{M \to \infty} \frac{1}{M} \left(2\pi\sigma^2\right)^{-\frac{n-4}{2}} \int_{-M/2}^{M/2} \exp \left[-(b^2 S_{ii} - 2 b \mathbf{z}^T S_{ii}^T \mathbf{z} + \mathbf{z}^T S_{ii} \mathbf{z})/2\right] db$$

(16)

For each satellite, $\mathbf{z}^T S_{ii} \mathbf{z}$ remains to be constant. Defining constant coefficient as $K = 1/M \left(2\pi\sigma^2\right)^{-\frac{n-4}{2}} \exp(-\mathbf{z}^T S_{ii} \mathbf{z}/2)$ yields

$$p(p|\mu_i) = K S_{ii}^{1/2} \exp \left(\mathbf{z}^T S_{ii}^T \mathbf{z}/2 S_{ii}\right)$$

(17)

According to the property of normal distribution, $(2\pi)^{-1/2} \int_{-\infty}^{\infty} \exp(-x^2/2) dx = 1$, we can get

$$p(p|\mu_i) = (2\pi)^{1/2} K S_{ii}^{-1/2} \exp \left(\mathbf{z}^T S_{ii}^T \mathbf{z}/2 S_{ii}\right)$$

(18)

In practice, logarithmic form is adopted for the convenience of computing, as

$$\ln p(p|\mu_i) = \frac{1}{2} \left(\mathbf{z}^T S_{ii} \mathbf{z}/S_{ii} - \ln S_{ii}\right) + \ln(\sqrt{2\pi}K)$$

(19)

According to Eqs. (11), (13) and (19), the decision function to identify the fault is simplified by removing constant part, i.e.

$$I = \arg \max \left(\frac{\mathbf{z}^T S_{ii} \mathbf{z}/S_{ii} - \ln S_{ii}}{\ln(\sqrt{2\pi}K)}\right)$$

(20)

As shown in Eq. (20), the decision function decreases with the increase of $S_{ii}$, i.e. the smaller $S_{ii}$ is, the larger value of decision function is. Thus, the proposed method in this paper compensates the difference of leverage effect above the framework of MLE FI method to effectively identify the fault measurement with different leverage.

Based on Bayesian minimum error theory, our approach can obtain the identification results with least risk as,

$$\text{PRisk} = 1 - \frac{\max_{i} \exp \left(\mathbf{z}^T S_{ii} \mathbf{z}/2 S_{ii}\right)/\sqrt{S_{ii}}}{\sum_{i=1}^{n} \exp \left(\mathbf{z}^T S_{ii} \mathbf{z}/2 S_{ii}\right)/\sqrt{S_{ii}}}$$

(21)
Based on the analysis above, the process of L-RAIM FI can be summarized as follows: (1) Calculate the decision function \((S_i^T z)^2/S_{ii} - \ln S_{ii}\) for \(i = 1, 2, \ldots, n\) respectively. (2) The \(i\)th satellite which maximizes the decision function is determined as fault one.

4. Performance analysis

To evaluate the probability of correct identification \(P_{CI}\), Wang et al.\(^\text{16}\) and Pervan et al.\(^\text{19}\) proposed to use the correlation coefficient and Bayesian posterior probability as assessment criteria respectively. However, the relations between criteria and \(P_{CI}\) have not been established precisely. In this section, the theoretical \(P_{CI}\) of FI method is derived and the \(P_{CI}\) of MLE FI method and L-RAIM FI method are compared.

4.1. Theoretical \(P_{CI}\)

For a given satellite geometry, \(P_{CI}\) is the probability to correctly identify the \(i\)th measurement under fault mode \(\mu_i\) and the bias magnitude \(b\). By the total probability formula, \(P_{CI}\) can be expressed as

\[
P(I = i|\mu_i, b) = \int_\mathcal{P} dp P(I = i|p)p(p|b, \mu_i)
\]

(22)

As RAIM fault identification is performed only after fault detection alarm, \(\mathcal{P}\) denotes the region in which the parity vector exceeds the detection threshold. Then we have \(p^T P > T_D\), \(\forall p \in \mathcal{P}\), where \(T_D\) is the threshold of RAIM fault detection.\(^\text{3}\)

According to Eq. (6), the parity vector under fault mode \(\mu_i\) and bias magnitude \(b\) follows the joint Gaussian distribution, noted as \(p(p|b, \mu_i) \sim N(bU_i^T \mu_i, \sigma^2 I_{n-1}).\)

With the observed \(p\), the probability of identifying the \(i\)th measurement \(P(I = i|p)\) can be expressed as an indicative function

\[
P(I = i|p) = \begin{cases} 1 & p \in \Omega_i \\ 0 & p \notin \Omega_i \end{cases}
\]

(23)

\(\Omega_i\) is the distribution region of \(p\) where the \(i\)th measurement will be identified as fault by the FI method. Left multiply Eq. (5) by \(U_2\) and we can get

\[
U_2 p = U_2 U_2^T z
\]

(24)

As \(S = U_2 U_2^T\), the \(j\)th element of the vector in Eq. (24) can be expressed as

\[
S_{jj} z = p^T U_2^T z
\]

(25)

where \(S_{jj}\) is the \(j\)th column of \(S\) and \(U_{2j}\) the \(j\)th row of \(U_2\).

Substitute Eq. (25) into Eq. (10) and Eq. (20), the \(\Omega_i\) for MLE FI method and the L-RAIM method in the parity space are obtained:

**MLE:**

\[
\Omega_i = \left\{ p \in \mathcal{P} \mid i = \arg \max \left( \frac{(p^T U_2^T)^2}{S_{ii}} \right) \right\}
\]

(26)

**L-RAIM FI:**

\[
\Omega_i = \left\{ p \in \mathcal{P} \mid i = \arg \max \left( \frac{(p^T U_2^T)^2}{S_{ii} - \ln(S_{ii})} \right) \right\}
\]

(27)

For a given satellite geometry and a given fault measurement, the theoretical probability of correct identification can be obtained in Eq. (22). In the following section, the \(P_{CI}\) of MLE FI method and L-RAIM FI method are compared and discussed.

4.2. Comparison between L-RAIM FI method and MLE FI method

To compare the L-RAIM FI method with MLE FI method in theoretical \(P_{CI}\), a scenario with 6 visible satellites is presented as an example, which can be easily extended to scenarios with more number of visible satellites.

With 6 satellites in view, the parity vector is a 2-dimensional vector.\(^\text{18}\) The columns of parity matrix \(U_2\) can be depicted in the parity space as characteristic bias lines (CBL) corresponding to each measurement. Denote the angle between the \(i\)th CBL and the parity vector \(p\) as \(\theta_i\), then, we have

\[
\cos \theta_i = \frac{p^T U_2^T}{\|p\| \cdot \|U_2^T\|}
\]

(28)

Since \(S = U_2 U_2^T\), we have \(\|U_2^T\| = \sqrt{S_{ii}}\). With Eq. (25), we can obtain

\[
\cos \theta_i = \frac{p^T U_2^T}{\|p\| \cdot \sqrt{S_{ii}}} = \frac{(S_{ii} z) / \sqrt{S_{ii}}}{\|p\|} = \frac{\ln(S_{ii}/S_{ij})}{\|p\| \sin(\theta_i + \theta_j)}
\]

(29)

According to Eqs. (10) and (29), \(\cos^2 \theta_i\) is in proportion to the decision function of MLE FI. Thus, the MLE FI method is equivalent to finding the satellite, of which CBL is the closest to (with minimum \(\theta_i\)) the observed parity vector. For the fault on the \(i\)th measurement, the boundary of \(\Omega_i\) is two angular bisectors of the \(i\)th CBL and the two closest characteristic bias lines on the two sides. As shown in Fig. 1, when the parity vector lies between the \(i\)th and the \(j\)th CBL, the boundary of \(\Omega_i\) is the bisector of the \(i\)th CBL and the \(j\)th CBL, i.e.

**MLE FI:** \(\theta_i = \theta_j\)

(30)

For L-RAIM FI method, the boundary of \(\Omega_i\) can be deduced as (proof in Appendix)

**L-RAIM FI:**

\[
\sin(\theta_i - \theta_j) = \frac{\ln(S_{ij}/S_{ij})}{\|p\| \sin(\theta_i + \theta_j)}
\]

(31)

![Fig. 1](image.png)

Diagram of theoretical correct identification calculation.
where \( \sin(\theta_i + \theta_j) \) is a positive constant related to the satellite geometry, \( |\mathbf{p}| \) the length of the observed parity vector.

Without loss of generality, we assume the leverage for the \( i \)th measurement is higher than that of the \( j \)th measurement \((S_{ij} > S_{ji})\) as shown in Fig. 1. Then we have \( \ln(S_{ij}/S_{ji}) > 0 \), yields \( \theta_i > \theta_j \). That means the \( \Omega_i \) of L-RAIM FI method for the fault with higher leverage is larger than that of the fault with lower leverage.

For the fault on the \( i \)th measurement, the mean value of \( \mathbf{p} \) projected to the \( i \)th CBL has the projection length

\[
E(|\mathbf{p}| \cos \theta_i) = E\left(\frac{(S_i^T b_i)}{\sqrt{S_i}}\right) = \left(\frac{S_i^T b_i}{\sqrt{S_i}}\right)
\]

(32)

With the same bias \( b \), the projection length is smaller for the fault with higher leverage than for the fault with lower leverage.

The elements of parity vector \( \mathbf{p} \) with different satellite faults remain the same variance as the measurement noise \( \sigma^2 \). Therefore the probability density distribution of \( \mathbf{p} \) projected to the 2-dimensional parity space is a circle. For a given probability, the ends of vector \( \mathbf{p} \) resulting from different satellite failures are distributed on the circle with the same radius. The distance from the center of the circle to the origin of coordinate is equal to \( \sqrt{S_i}b \). For clarity, the circles to describe the distribution of parity vectors are shown in Fig. 1. It can be noted that the distribution area of \( \mathbf{p} \) resulting from fault with higher leverage has larger angle range than the fault with lower leverage.

However, the MLE FI method uses the angular bisector as the boundary to distinct the source of failure, which underestimates the distribution range of parity vector caused by fault measurement with high leverage. Consequently, the \( P_{CI} \) of MLE FI method for the fault with high leverage will decrease. Instead of using angular bisector as the boundary of \( \Omega_i \), the L-RAIM method divides the parity space according to different leverage, which will improve the \( P_{CI} \) for the fault with high leverage.

5. Experiment and discussion

In this section, experiments are designed with both simulated data and real data to evaluate the performance of L-RAIM FI method. The first simulation experiment is to compare the \( P_{CI} \) of L-RAIM FI method and MLE FI method in the presence of fault with high and low leverage separately. The second simulation experiment is to evaluate the minimum performance that L-RAIM method and MLE FI method can achieve for the fault on any satellite measurement. The final experiment uses real data to evaluate the performance of the two FI methods in practice.

To compare the minimum performance of the two FI methods, a performance indicator named minimal identifiable bias (MIB) is defined in this paper. MIB is derived from \( P_{CI} \) which indicates that biases larger than MIB can be identified to achieve the required \( P_{CI} \) for any measurement fault. For a given geometry, smaller value of MIB reflects that higher \( P_{CI} \) can be achieved.

The experiment scenarios and results with simulated and real data are presented in detail as the following two sections.

5.1. Experiment with simulated data

The probability of correct identification for fault bias is the statistical average for measurements noise with a given geometry. As the satellite geometries vary continuously over time, the simulated experiments are designed to evaluate the \( P_{CI} \) and MIB of the FI methods.

The world-wide (longitude \(-180^\circ \) to \(+170^\circ \) and latitude \(-65^\circ \) to \(+65^\circ \)) grid of locations is simulated at one epoch (GPS seconds = 432,000 s) with an interval of \( 10^\circ \) and masking angle of 7.5°. In the simulations, the nominal GPS constellation with 24 satellites is used.\(^{21}\) The measurement noise with selective availability (SA) OFF is assumed to follow the normal distribution with \( \sigma = 12.5 \) m, which is specified in TSO-196.\(^{22}\) As the fault identification method is executed after the fault detection (FD) alarm, the chi-square method is applied as the baseline FD algorithm according to DO-208,\(^{23}\) where the missed detection \( P_{md} \) = 0.001 and the false alarm \( P_{fa} = 6.7 \times 10^{-5} \) correspond to the requirement for non-precision approach.

5.1.1. Evaluation of \( P_{CI} \)

To evaluate the \( P_{CI} \) of L-RAIM FI method in the presence of fault measurement with different leverage, the bias is added to the measurement with the highest and lowest leverage.

![Average of \( P_{CI} \) for all the simulated geometries.](image)

(a) Fault with high leverage

(b) Fault with low leverage

Fig. 2
separately for all the simulated geometries. By generating the Gaussian noise, the $P_{CI}$ with the increase of bias magnitude for each geometry can be obtained. Fig. 2(a) is the average of $P_{CI}$ for all the simulated geometries in the presence of fault with high leverage. Fig. 2(b) is the average of $P_{CI}$ for all the simulated geometries in the presence of fault with low leverage.

As shown in Fig. 2(a), for the fault on the high leverage measurement, the average of $P_{CI}$ for all the simulated geometries with L-RAIM FI method outperforms the MLE FI method by about 20% when the bias is less than 50 m. If the bias increases to 100 m, the average of $P_{CI}$ with L-RAIM FI method outperforms the MLE FI method by about 10%. When the bias magnitude is more than 300 m, the results of these two methods are almost the same. To sum up, when compared with MLE FI method, the L-RAIM FI method can obtain higher probability of correct identification for the fault measurement with high leverage such that smaller fault bias can be identified to achieve the required probability of correct identification. As shown in Fig. 2(b), the L-RAIM FI method and MLE FI method tend to approximate performance when the fault occurs on the low leverage measurement. For example, when the bias magnitude increases to 100 m, the average of $P_{CI}$ with both methods can reach 95%.

The experimental results show that our approach can improve the probability of correct identification in the case of high leverage measurement fault; while for the fault on low leverage measurement, the performance of our approach is approximate to the existing method. As the measurement with higher leverage has larger impact in the positioning solution, the fault on the measurement with higher leverage will cause larger positioning errors. The improvement by the L-RAIM FI method for the fault with high leverage is meaningful to ensure the safety of flight.

5.1.2. Evaluation of MIB

MIB is a performance indicator to describe the minimum performance for FI method. As any measurement may be contaminated with fault, the simulation adds the fault bias on each satellite measurement separately. MIB is the minimal bias for which the performance of each measurement fault can achieve the required $P_{CI}$.

Fig. 3 shows the distribution diagrams of MIB for which the $P_{CI}$ of any measurement fault can reach 80%. As shown in Fig. 3(a) and (b), 70% of the simulated geometries is able to identify the fault bias less than 100 m to achieve 80% $P_{CI}$ by using L-RAIM FI method, while only 55% of the geometries can reach $P_{CI}$ of 80% when applying MLE FI method.

Fig. 4 shows the distribution diagrams of MIB with $P_{CI}$ of 90% for each satellite to be identified. When the probability of correct identification arrives at 90%, the MIB with L-RAIM
Fig. 5 Identification flag for L-RAIM FI and MLE FI.

Fig. 6 Sum of the identification flag with L-RAIM FI and MLE FI.
FI method is less than 120 m for 60% of the geometries, while only 50% of all the simulated geometries can achieve $P_{CI}$ of 90% with the bias less than 120 m by using MLE FI method.

The L-RAIM FI method pays attention to the difference in parity vector distribution caused by the leverage of measurement. For a given probability of correct identification, lower bias magnitude can be identified by L-RAIM FI method compared with MLE FI method.

5.2. Experiment with real data

The real data is downloaded from the website of continuously operation reference station (CORS) at http://www.ngs.noaa.gov/CORS/. The observation data in the format of RINEX are collected from ICT1 ($X = -643821.392, Y = -501964.1155, Z = 386950.5366$) station on 23th November, 2013. The interval of sampling is 1 s from 0:00 to 1:00 and 3600 samples are collected. The fault biases of 10 m, 20 m and 30 m are separately added to the measurement at each epoch and the results of L-RAIM FI and MLE FI are compared, as shown in Figs. 5 and 6. For clarity, we define identification flag to express the result of fault identification, where 0 indicates that the L-RAIM FI method has correctly identified the fault, and $-1$ indicates that the MLE FI method has correctly identified the fault.

The identification flag for each epoch using L-RAIM FI method and MLE FI method is shown in Fig. 5. With the increase of the fault bias, both L-RAIM FI method and MLE FI method can improve the performance of identification. To make the comparison between L-RAIM FI and MLE FI more clearly, the identification flag of MLE FI is added to the flag of L-RAIM FI to demonstrate the differences between the two methods as shown in Fig. 6.

With the fault bias of 10 m, there are 86 epochs when the L-RAIM FI outperforms the MLE-FI method. At most of the rest epochs with no difference between the two methods, neither L-RAIM FI nor MLE-FI can identify the fault since the fault bias is not big enough for these geometries. As the fault bias is increased to 20 m, 135 epochs which cannot be identified by the two methods with 10 m bias can be identified by L-RAIM FI method but still fail to be identified by MLE FI method. As the fault bias is increased to 30 m, although most of the epochs can achieve correct identification by both methods, there are still 77 epochs when the L-RAIM FI method outperforms the MLE FI method.

6. Conclusions

The fault identification is the key function of the receiver autonomous integrity monitoring to ensure the safety of aviation. This paper aims to propose an RAIM FI method with the consideration of the difference in measurement leverage.

(1) By analyzing how the leverage impacts the fault identification, the L-RAIM FI method is proposed.

(2) The theoretical probability of correct identification is deduced to evaluate the performance of L-RAIM FI method. Moreover, L-RAIM FI method and traditional FI method are compared from the probability of correct identification.

(3) The simulations demonstrate that the L-RAIM FI method outperforms the MLE FI method in the probability of correct identification for the fault measurement with high leverage. In the world-wide simulations, 70% of the simulated geometries can achieve 80% $P_{CI}$ with fault bias less than 100 m by using L-RAIM FI method, while only 55% of the geometries can reach 80% $P_{CI}$ by using MLE FI method.

(4) The experiment with real data shows the practice utility of L-RAIM FI method over MLE FI method. With the bias of 10 m, 20 m and 30 m, the L-RAIM FI outperforms the MLE FI by 86, 135 and 77 epochs, respectively.

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Appendix A

Define the decision function in Eq. (27) as
\[
\Gamma_i = \left( \frac{p^T U_i^2}{S_{ii}} \right) / S_{ii} - \ln S_{ii} \tag{A1}
\]
For L-RAIM FI method, the boundary of $\Omega_i$ between the $i$th CBL and the $j$th CBL (as shown in Fig. 1) is expressed as
\[
\Gamma_i - \Gamma_j = 0 \tag{A2}
\]
According to Eq. (A1) and Eq. (29), we have
\[
\begin{align*}
\Gamma_i - \Gamma_j &= \left( \frac{p^T U_i^2}{S_{ii}} \right) / S_{ii} - \ln S_{ii} \\
&- \left( \frac{p^T U_j^2}{S_{jj}} \right) / S_{jj} - \ln S_{jj} \\
&= [\cos^2 \theta_i ||p||^2 - \ln S_{ii}] \\
&- [\cos^2 \theta_j ||p||^2 - \ln S_{jj}] \\
&= [\cos^2 \theta_i - \cos^2 \theta_j] ||p||^2 - \ln(S_{ii}/S_{jj})
\end{align*} \tag{A3}
\]
Substituting triangle formula $\cos^2 \theta_i - \cos^2 \theta_j = -\sin(\theta_i + \theta_j)\sin(\theta_i - \theta_j)$ into Eq. (A3) yields,
\[
\begin{align*}
\Gamma_i - \Gamma_j &= -\sin(\theta_i + \theta_j) \sin(\theta_i - \theta_j) ||p||^2 \\
&- \ln(S_{ii}/S_{jj}) \tag{A4}
\end{align*}
\]
With Eq. (A2), we can obtain
\[
\begin{align*}
||p||^2 \sin(\theta_i + \theta_j) \sin(\theta_i - \theta_j) &= -\ln(S_{ii}/S_{jj}) \\
&= \ln(S_{jj}/S_{ii}) \tag{A5}
\end{align*}
\]
Then, we have
\[
\sin(\theta_i - \theta_j) = \frac{\ln(S_{jj}/S_{ii})}{||p||^2 \sin(\theta_i + \theta_j)} \tag{A6}
\]
References


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