



# Electron and muon $g - 2$ contributions from the $T'$ Higgs sector

Chiu Man Ho\*, Thomas W. Kephart

Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235, USA

## ARTICLE INFO

### Article history:

Received 20 February 2010

Accepted 15 March 2010

Available online 17 March 2010

Editor: T. Yanagida

### Keywords:

Discrete flavor symmetry

Higgs

Anomalous magnetic moment

## ABSTRACT

We study the experimental constraints from electron and muon  $g - 2$  factors on the Higgs masses and Yukawa couplings in the  $T'$  model, and thereby show that the discrepancy between the standard model prediction and experimental value of muon  $g - 2$  factor can be easily accommodated.

© 2010 Elsevier B.V. Open access under CC BY license.

## 1. Introduction

The electron anomalous magnetic moment has been measured to an extremely high precision and agrees with the theoretical prediction calculated from the standard model (SM) [1], with the result

$$\Delta a_e = |a_e^{\text{SM}} - a_e^{\text{Expt}}| < 1 \times 10^{-10}. \quad (1.1)$$

On the other hand, the most recent theoretical calculation of the muon anomalous magnetic moment gives [2]:

$$a_\mu^{\text{SM}} = (11659183.4 \pm 4.9) \times 10^{-10}, \quad (1.2)$$

where the errors are dominated by the hadronic contribution. The corresponding most updated experimental value is [3]:

$$a_\mu^{\text{Expt}} = (11659208.0 \pm 5.4 \pm 3.3) \times 10^{-10}. \quad (1.3)$$

This implies that  $a_\mu^{\text{SM}}$  differs from  $a_\mu^{\text{Expt}}$  by  $3.1\sigma$ , and suggests that a contribution beyond standard model may be required. As we will show, this discrepancy between the theoretical and experimental values can be easily accommodated in the  $T'$  model [4–6] due to the existence of a new and unique Higgs coupling to the muon. While many authors have developed models that resolve this discrepancy [7], only a few have invoked a discrete flavor symmetry.

## 2. Higgs contributions to $g - 2$ factors in the $T'$ model

The  $T'$  model [4–6] relates quarks and electrons through a discrete flavor symmetry, the binary tetrahedral group  $T'$ , whose irreducible representations are three singlets, three doublets and a triplet. The renormalizable  $T'$  model has led to successful predictions of the tribimaximal neutrino mixing matrix as well as the Cabibbo angle [5,6]. More details about the  $T'$  model, its variants and other related models can be found in the literature [8].

In the  $T'$  model, electrons and muons couple to the different components of the triplet Higgs  $H'_3$  through the interaction terms  $Y_e \bar{e} H'_{3,e} e$  and  $Y_\mu \bar{\mu} H'_{3,\mu} \mu$ . To compute the contribution of a virtual Higgs to the electron and muon  $g - 2$  factors, we need to study its contribution to the electron/muon–photon vertex. For  $f = e, \mu$ , the vertex function is given by

$$\begin{aligned} & -ie\bar{u}(p')\Lambda_f^\nu(p', p)u(p) \\ &= (-ie)(-iY_f)^2 \int \frac{d^4k}{(2\pi)^4} \bar{u}(p') \frac{i}{k^2 - M_{H_f}^2 + i\epsilon} \\ & \quad \times \frac{i(\not{p}' - \not{k} + m)}{(p' - k)^2 - m^2 + i\epsilon} \gamma^\nu \frac{i(\not{p} - \not{k} + m_f)}{(p - k)^2 - m_f^2 + i\epsilon} u(p), \quad (2.1) \end{aligned}$$

where  $\bar{u}(p')$  and  $u(p)$  are the spinors obeying the equation of motions  $\bar{u}(p')(\not{p}' - m_f) = (\not{p} - m_f)u(p) = 0$ , and  $M_{H_f}$  is the mass of the Higgs which couples to the electron or muon whose mass is denoted by  $m_f$ .

After some calculations, we obtain

$$\bar{u}(p')\Lambda_f^\nu(p, p')u(p) = F_f(q^2)\bar{u}(p')\frac{i\sigma^{\nu\alpha}q_\alpha}{2m_f}u(p) + \dots, \quad (2.2)$$

\* Corresponding author.

E-mail addresses: chiu.man.ho@vanderbilt.edu (C.M. Ho), tom.kephart@gmail.com (T.W. Kephart).

where  $F_f(q^2)$  is the form factor associated with the electron or muon, and  $\sigma^{\nu\alpha} = \frac{i}{2}[\gamma^\nu, \gamma^\alpha]$ . The contributions from the  $T'$  Higgs sector to electron or muon anomalous magnetic moment is given by

$$\Delta a_f = \Delta \left( \frac{g_f - 2}{2} \right) = F_f(q^2 = 0), \quad (2.3)$$

$$= \frac{Y_f^2}{8\pi^2} \frac{m_f^2}{M_{H_f}^2} \int_0^1 dx \frac{(1-x^2)(1-x)}{x + (1-x)^2 \frac{m_f^2}{M_{H_f}^2}}. \quad (2.4)$$

For  $m_f \ll M_{H_f}$ , which is likely to be the case, there is a logarithmic divergence in the above integral as  $x \rightarrow 0$ . This divergence can be extracted by setting  $1-x \rightarrow 1$  and  $1-x^2 \rightarrow 1$  in the integrand. As a result, we obtain

$$\Delta a_f \approx \frac{Y_f^2}{4\pi^2} \left( \frac{m_f}{M_{H_f}} \right)^2 \ln \left( \frac{M_{H_f}}{m_f} \right). \quad (2.5)$$

Note that for a given value of  $Y_f$ ,  $\Delta a_f$  is strictly decreasing when the ratio  $M_{H_f}/m_f$  increases.

The condition (1.1) implies that any combinations of  $Y_e$  and  $M_{H_e}$  must be such that

$$|\Delta a_e| < 1 \times 10^{-10}, \quad (2.6)$$

which imposes the following constraint

$$Y_e \lesssim 21.4 \lambda_e \frac{M_{H_e}/m_e}{\sqrt{\ln(M_{H_e}/m_e)}}, \quad (2.7)$$

where  $\lambda_e \sim 3 \times 10^{-6}$  is the corresponding electron Yukawa coupling in SM. We required the ratio  $M_{H_e}/m_e \gg 1$  when we were deriving (2.5), but otherwise a free parameter. To have an assessment on the allowed range of  $Y_e$ , we need to have some experimental bounds on  $M_{H_e}$ . Apparently, we would have hoped that the LEP [9] bound on Higgs mass may help – due to the non-observation of the “Higgs-strahlung” process  $e^+e^- \rightarrow HZ$  at LEP, a lower bound has been given to the SM Higgs, namely  $M_{H_{SM}} \geq 114.5$  GeV. However, in the  $T'$  model, all the Higgs singlets and triplets couple to  $Z$ . Thus, the LEP bound does not apply directly to any of the masses of the Higgs singlets and triplets. If we simply assume that  $M_{H_e} \gtrsim 100$  GeV, then we require  $Y_e \lesssim 3.5$  in order to satisfy the condition (1.1). In this case, the upper bound on the Yukawa coupling  $Y_e$  is very loose and any value of  $Y_e$  that is perturbatively small would be allowed.

For the muon anomalous magnetic moment, the discrepancy between the theoretical and experimental values can be accounted for easily in the  $T'$  model if

$$\Delta a_\mu \sim |a_\mu^{\text{SM}} - a_\mu^{\text{Expt}}| = (24.6 \pm 8.0) \times 10^{-10}, \quad (2.8)$$

leading to the constraint

$$Y_\mu \sim 0.52 \lambda_\mu \frac{M_{H_\mu}/m_\mu}{\sqrt{\ln(M_{H_\mu}/m_\mu)}}, \quad (2.9)$$

where  $\lambda_\mu \sim 0.0006$  is the corresponding muon Yukawa coupling in SM. It is obvious that  $Y_\mu \gg \lambda_\mu$ , for any choice of  $M_{H_\mu}/m_\mu \gg 1$ . For instance, if we assume that  $M_{H_\mu} \gtrsim 100$  GeV, then in order to satisfy (2.9), we require  $Y_\mu \gtrsim 0.13$ .

### 3. Conclusions

In this Letter, we have computed the contributions to electron and muon  $g-2$  factors from the Higgs sector in the  $T'$  model. We then used the experimental data to constrain the  $T'$  model Higgs masses and Yukawa couplings.

If we assume that  $M_{H_e} \gtrsim 100$  GeV, then the upper bound on the electron Yukawa coupling  $Y_e$  would be very loose and any value of  $Y_e$  consistent with the perturbation theory would be allowed.

Our main result is the demonstration that the discrepancy between the standard model and experimental values of muon anomalous  $g-2$  factor can be accounted for easily in the  $T'$  model. Assuming  $M_{H_\mu} \gtrsim 100$  GeV, we found that the Yukawa coupling  $Y_\mu$  should be much larger than the corresponding SM value in order to explain the discrepancy.

### Acknowledgements

We thank Shinya Matsuzaki for useful comments. This work was supported by US DOE grant DE-FG05-85ER40226.

### References

- [1] D. Hanneke, S. Fogwell, G. Gabrielse, Phys. Rev. Lett. 100 (2008) 120801.
- [2] M. Davier, A. Hoecker, B. Malaescu, C.Z. Yuan, Z. Zhang, arXiv:0908.4300 [hep-ph].
- [3] G.W. Bennett, et al., Muon  $g-2$  Collaboration, Phys. Rev. D 73 (2006) 072003.
- [4] P.H. Frampton, T.W. Kephart, Int. J. Mod. Phys. A 10 (1995) 4689, arXiv: hep-ph/9409330.
- [5] P.H. Frampton, T.W. Kephart, JHEP 0709 (2007) 110, arXiv:0706.1186 [hep-ph].
- [6] P.H. Frampton, T.W. Kephart, S. Matsuzaki, Phys. Rev. D 78 (2008) 073004, arXiv:0807.4713 [hep-ph].
- [7] E. Ma, M. Raidal, Phys. Rev. Lett. 87 (2001) 011802; E. Ma, M. Raidal, Phys. Rev. Lett. 87 (2001) 159901, arXiv:hep-ph/0102255, Erratum; T.W. Kephart, H. Pas, Phys. Rev. D 65 (2002) 093014, arXiv:hep-ph/0102243; Z.H. Xiong, J.M. Yang, Phys. Lett. B 508 (2001) 295, arXiv:hep-ph/0102259; Z.Z. Xing, Phys. Rev. D 64 (2001) 017304, arXiv:hep-ph/0102304; T. Ibrahim, U. Chattopadhyay, P. Nath, Phys. Rev. D 64 (2001) 016010, arXiv:hep-ph/0102324; J.R. Ellis, D.V. Nanopoulos, K.A. Olive, Phys. Lett. B 508 (2001) 65, arXiv: hep-ph/0102331; X. Calmet, H. Fritzsch, D. Holtmannspotter, Phys. Rev. D 64 (2001) 037701, arXiv:hep-ph/0103012; K. Choi, K. Hwang, S.K. Kang, K.Y. Lee, W.Y. Song, Phys. Rev. D 64 (2001) 055001, arXiv:hep-ph/0103048; S. Rajpoot, arXiv:hep-ph/0103069; C.A. de S. Pires, P.S. Rodrigues da Silva, Phys. Rev. D 64 (2001) 117701, arXiv:hep-ph/0103083; E.O. Iltan, H. Sundu, Acta Phys. Slov. 53 (2003) 17, arXiv:hep-ph/0103105; S. Baek, P. Ko, H.S. Lee, Phys. Rev. D 65 (2002) 035004, arXiv:hep-ph/0103218; M. Raidal, Phys. Lett. B 508 (2001) 51, arXiv:hep-ph/0103224; A. Dedes, H.E. Haber, arXiv:hep-ph/0105014; E.O. Iltan, JHEP 0305 (2003) 065, arXiv:hep-ph/0304097; H. Chavez, C.N. Ferreira, J.A. Helayel-Neto, Phys. Rev. D 74 (2006) 033006, arXiv:hep-ph/0410373; A. Mondragon, M. Mondragon, E. Peinado, J. Phys. A 41 (2008) 304035, arXiv:0712.1799 [hep-ph].
- [8] M. Schmaltz, Phys. Rev. D 52 (1995) 1643, arXiv:hep-ph/9411383; L.J. Hall, H. Murayama, Phys. Rev. Lett. 75 (1995) 3985, arXiv:hep-ph/9508296; C.D. Carone, R.F. Lebed, Phys. Rev. D 60 (1999) 096002, arXiv:hep-ph/9905275; P.H. Frampton, A. Rasin, Phys. Lett. B 478 (2000) 424, arXiv:hep-ph/9910522; R. Dermisek, S. Raby, Phys. Rev. D 62 (2000) 015007, arXiv:hep-ph/9911275; G. Altarelli, F. Feruglio, New J. Phys. 6 (2004) 106, arXiv:hep-ph/0405048; K.S. Babu, J. Kubo, Phys. Rev. D 71 (2005) 056006, arXiv:hep-ph/0411226; N. Haba, K. Yoshioka, Nucl. Phys. B 739 (2006) 254, arXiv:hep-ph/0511108; Y. Kajiyama, E. Itou, J. Kubo, Nucl. Phys. B 743 (2006) 74, arXiv: hep-ph/0511268; C. Hagedorn, M. Lindner, R.N. Mohapatra, JHEP 0606 (2006) 042, arXiv: hep-ph/0602244; T. Kobayashi, H.P. Nilles, F. Ploger, S. Raby, M. Ratz, Nucl. Phys. B 768 (2007) 135, arXiv:hep-ph/0611020;

- M.C. Chen, K.T. Mahanthappa, Phys. Lett. B 652 (2007) 34, arXiv:0705.0714 [hep-ph];  
M. Frigerio, E. Ma, Phys. Rev. D 76 (2007) 096007, arXiv:0708.0166 [hep-ph];  
S. Sen, Phys. Rev. D 76 (2007) 115020, arXiv:0710.2734 [hep-ph];  
G. Altarelli, arXiv:0711.0161 [hep-ph];  
N. Kifune, J. Kubo, A. Lenz, Phys. Rev. D 77 (2008) 076010, arXiv:0712.0503 [hep-ph];  
M. Honda, M. Tanimoto, Prog. Theor. Phys. 119 (2008) 583, arXiv:0801.0181 [hep-ph];  
G. Altarelli, F. Feruglio, C. Hagedorn, JHEP 0803 (2008) 052, arXiv:0802.0090 [hep-ph];  
F. Plentinger, G. Seidl, W. Winter, JHEP 0804 (2008) 077, arXiv:0802.1718 [hep-ph];  
C. Luhn, Phys. Lett. B 670 (2009) 390, arXiv:0807.1749 [hep-ph];  
P.H. Frampton, S. Matsuzaki, Mod. Phys. Lett. A 24 (2009) 2081, arXiv:0810.1029 [hep-ph];  
D.A. Eby, P.H. Frampton, S. Matsuzaki, Phys. Lett. B 671 (2009) 386, arXiv:0810.4899 [hep-ph];  
F. Bazzocchi, S. Morisi, Phys. Rev. D 80 (2009) 096005, arXiv:0811.0345 [hep-ph];  
P.H. Frampton, S. Matsuzaki, Phys. Lett. B 679 (2009) 347, arXiv:0902.1140 [hep-ph];  
D.A. Eby, P.H. Frampton, S. Matsuzaki, Phys. Rev. D 80 (2009) 053007, arXiv:0907.3425 [hep-ph];  
M.C. Chen, K.T. Mahanthappa, F. Yu, arXiv:0907.3963 [hep-ph];  
M.C. Chen, K.T. Mahanthappa, Phys. Lett. B 681 (2009) 444, arXiv:0904.1721 [hep-ph];  
F. Feruglio, C. Hagedorn, Y. Lin, L. Merlo, Nucl. Phys. B 809 (2009) 218, arXiv:0807.3160 [hep-ph];  
F. Feruglio, C. Hagedorn, Y. Lin, L. Merlo, Nucl. Phys. B 775 (2007) 120, arXiv:hep-ph/0702194.  
[9] R. Barate, et al., LEP Collaborations, Phys. Lett. B 565 (2003) 61.