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Full Length Article

A novel adaptive genetic algorithm for global optimization of mathematical test functions and real-world problems

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ABSTRACT

Genetic algorithm (GA) is a population-based stochastic optimization technique that has two major problems, i.e. low convergence speed and falling down in local optimum points. This paper introduces an adaptive genetic algorithm (AGA) consisting of new crossover and mutation operators to handle these drawbacks. The crossover operator is based on a combination of the traditional crossover mechanism and the particle swarm optimization (PSO) operator. The proposed mutation operator intelligently uses sliding mode control (SMC) to escape from local minimums and converges to the global optimum. The performance of the proposed genetic algorithm is challenged by using twenty well-known test functions. The comparison of the obtained numerical results with those of the other optimization algorithms reported in literature demonstrates the superiority of the proposed algorithm in finding the global optimum points. At the end, the proposed method is employed to estimate the oil demand in Iran based on socio-economic indicators and using linear and exponential forms as a real-world problem that shows the AGA's effectiveness.

1. Introduction

In the most general terms, the optimization theory is a body of mathematical results and numerical methods to find and identify the best candidate from a collection of alternatives without having to explicitly enumerate and evaluate all possible ones [1]. Nowadays, it is fully accepted that optimization is widely applied in different branches of science, industry and commerce [2–5]. Many real-world optimization problems in engineering are increasingly becoming complicated, so optimization algorithms with high performance are needed [6,7]. Optimization algorithms have developed and evolved rapidly in recent years leading to reduced computation times and the improved accuracy of desired results.

Moreover, evolutionary optimization algorithms specially the Genetic algorithm (GA) and particle swarm optimization (PSO) have attracted much attention of many researches. Usually, they have combined the optimization algorithms together to use their advantages simultaneously [8–10]. Shieh et al. combined PSO with Simulated Annealing (SA) by a proper procedure for parameters selection to improve the solution quality than SA and fast the searching ability than PSO [11]. Kiran et al. incorporated PSO with Ant Colony Optimization (ACO) and proposed the hybrid ant particle optimization algorithm to find the global minimum [12]. In their algorithm, PSO and ACO work separately in each iteration and the obtained best solutions are applied to select the new position of particles and ants at the next iteration. Moradi and Abedini suggested a new combined GA and PSO for optimal Distributed Generation (DG) location and sizing on distribution systems [13]. Mahmoodabadi et al. also proposed a novel fuzzy combination of PSO and GA for Pareto optimal design of a five-degree of freedom vehicle vibration model [14]. Akpinar et al. presented a novel hybrid of ACO and GA for a mixed-model assembly line balancing problem with some particular traits of real world problems [15]. In a different work, Örkcü offered a new hybrid algorithm based on adaptive genetic and simulated annealing algorithms for the variable selection problem of multiple linear regression models [16]. Valdez et al. proposed a novel hybrid procedure based on PSO and GA that uses the fuzzy logic to integrate the results of the two algorithms [17]. Kuo et al. also described a new hybrid approach in which global best and particle best solutions of PSO are combined with crossover and mutation operators of GA [18]. In these studies [11–18], the numerical results have been obtained for general and simple test functions, and compared with pure optimization algorithms, and not tested for complicated problems.

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Further, GA premature convergence to the local optimum is an undesirable phenomenon often reported in literature [19–21]. Many researchers have shown that by using an adaptive mutation one could overcome this issue. For instance, Tang and Tseng suggested a simple adaptive directed mutation for real-coded GA [22]. Their proposed operator uses the abilities of GAs in searching global optima as well as in speeding convergence by integrating results of local directional and adaptive random search strategies. Linda and Nair gave a new GA by employing an adaptive mutation strategy for optimization of a global multi-machine power system stabilizer [23]. Alfi presented a PSO-based optimization technique with two new aspects, namely an adaptive mutation mechanism and a dynamic inertia weight, in order to enhance the global search ability and to increase accuracy [24]. Wang et al. proposed a PSO variant with new adaptive mutation to escape particles from local optimal and solve multimodal optimization problems [25].

In the present research, an adaptive genetic algorithm (AGA) is proposed that its crossover operator named GB-crossover uses a new composition of the traditional crossover of GA and the global best position of PSO. In other words, two chromosomes are selected for the crossover operator, from the mating pool, one of them is chosen randomly and the other is the position of the best particle of the entire swarm. The mutation operator used for the AGA applies an intelligent algorithm originated from sliding mode control (SMC) concepts. This innovative mutation operator (named quasi sliding surface-mutation) gradually decreases the changing of the genes of the selected chromosome. It also quite prevents particles from converging towards local optimum points. The capability of the proposed approach is evaluated on some well-known benchmark functions and the results are compared with several recent optimization algorithms applied to the same benchmark functions. Finally, the AGA is applied to estimate the future oil demand values in Iran based on population, Gross Domestic Product (GDP) and import and export data. In the recent years, researchers have proposed different approaches for modeling the energy demand [26–35]. Furthermore, Yu and Zhu developed a PSO-GA optimal model to predict energy demand in China using Gross Domestic Product (GDP), population, economic structure,

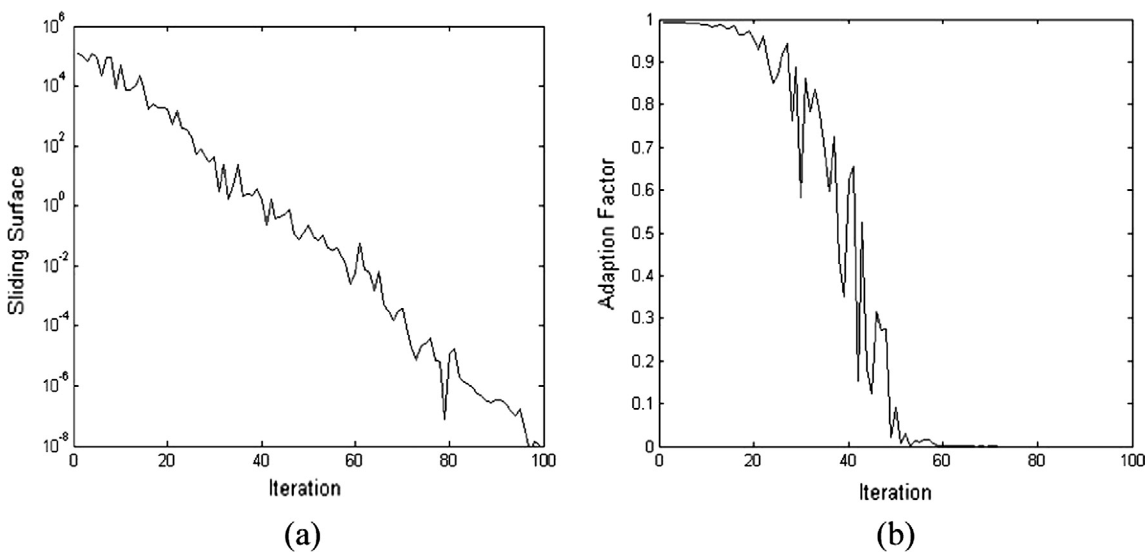


Fig. 1. The obtained trajectory of the sliding surface (a) and the adaptation factor (b) for the Sphere test function.

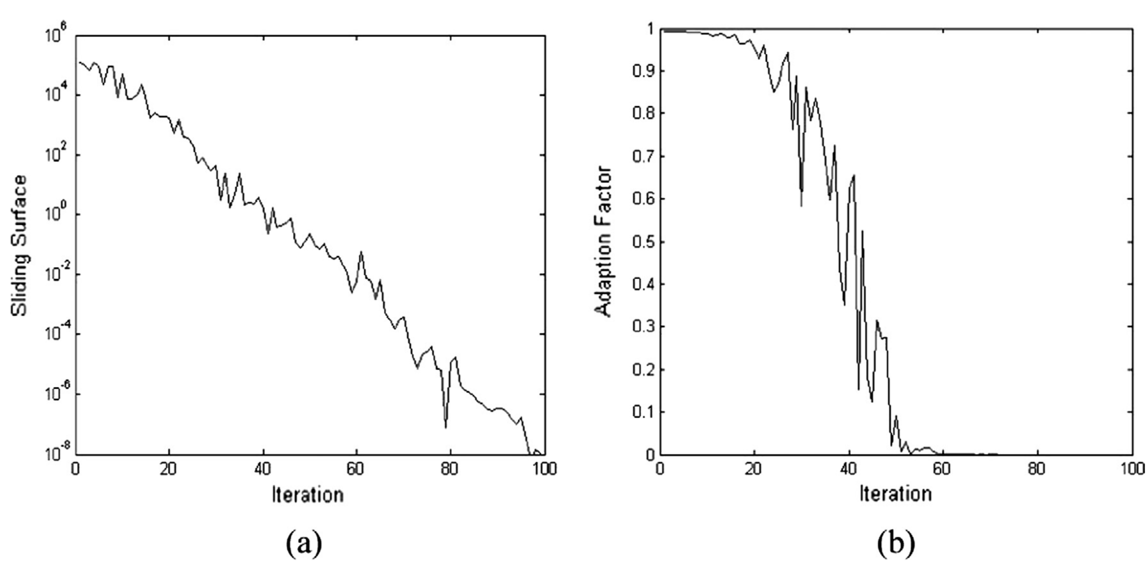


Fig. 2. The obtained trajectory of the sliding surface (a) and the adaptation factor (b) for the Griewank test function.

urbanization rate, and energy structure with linear, exponential and quadratic forms [36]. Kiran et al. presented a novel hybrid algorithm based on PSO and ACO for energy demand forecasting in Turkey [37]. They supposed that the main affecting factors of energy demand in Turkey include GDP, population, import and export. They also applied two new models in order to estimate electricity energy demand in Turkey by using Artificial Bee Colony (ABC) and PSO algorithms [38]. Further, Yu and Zhu proposed a hybrid technique (PSO-GA) to improve energy demand estimation in China by applying linear, exponential, and quadratic models and considering GDP, population, economic structure, urbanization rate, and energy consumption structure [39]. Piltan et al. used PSO and GA to attain the parameters of the energy demand forecasting model in Iranian metal industry [40]. The coefficients of two linear and three nonlinear functions are optimized by considering a function of different variables such as electricity tariff, manufacturing value added, prevailing fuel prices, the number of employees, the investment in equipment, and consumption. Ghanbari et al. presented a cooperative ACO-GA approach to construct a knowledge-based expert system for simulating fluctuations of energy demand [41]. They evaluate the ability of this algorithm by applying it on three case studies; annual electricity demand, natural gas demand and oil products demand in Iran. Rahmani et al. introduced a new hybrid of ACO and PSO to predict the

Define the objective function, design variables and algorithm parameters

Initialize the population randomly: $\vec{X}_{ij}(0);\ \ i = 1, 2, \ldots, N;\ \ j = 1, 2, \ldots, D$

Determine $\vec{X}_{gbest}$

t=1;

While t<maximum iteration

  Store the old population $\vec{X}_i(t-1)$ for calculating the sliding surface s

  Find fitness values of chromosomes and store $\vec{X}_{gbest}$

  If $rand < P_{GBc}$

    $i = random[1, N]$ as an integer number

    Select randomly one chromosome from population and obtain $\vec{X}_{i_{new}}(t)$ using the GB-crossover

    If $fitness(\vec{X}_{i_{new}}(t)) < fitness(\vec{X}_i(t))$

      $\vec{X}_i(t+1) = \vec{X}_{i_{new}}(t)$

    end

  end

  If $rand < P_{QSSm}$

    $i = random[1, N]$ as integer number

    Determine the sliding surface s

    Calculate the adaption factor μ

    Select randomly one chromosome from population and update it using the QSS-mutation

  End

t=t+1;

end.

Fig. 3. The pseudo code of the AGA.

Table 1
General mathematical test functions used to evaluate the algorithms.

	Name	Formulation	Search domain	Global $f_{\min}$
Unimodal	Sphere	$f_1(x) = \sum_{i=1}^{D} x_i^2$	$[-100, 100]^D$	0
	Schwefel's P2.22	$f_2(x) = \sum_{i=1}^{D} \lvert x_i \rvert + \prod_{i=1}^{D} \lvert x_i \rvert$	$[-10, 10]^D$	0
	Schwefel's P1.2	$f_3(x) = \sum_{i=1}^{D} (\sum_{j=1}^{i} x_j)^2$	$[-100, 100]^D$	0
	Schwefel's P2.21	$f_4(x) = \max \lvert x_i \rvert; i \in [1.D]$	$[-100, 100]^D$	0
	Rosenbrock	$f_5(x) = \sum_{i}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-10, 10]^D$	0
	Quaric	$f_6(x) = \sum_{i=1}^{D} i x_i^4 + \mathrm{random}[0, 1]$	$[-1.28, 1.28]^D$	0
	High Conditioned Eliptic	$f_7(x) = \sum_{i=1}^{D} (10^6)^{\frac{i-1}{D-1}} x_i^2$	$[-100, 100]^D$	0
Multimodal	Rastrigin	$f_8(x) = \sum_{i=1}^{D} [x_i^2 - 10\cos(2\pi x_i) + 10]$	$[-5.12, 5.12]^D$	0
	Ackley	$f_9(x) = -20\ \exp\ \left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) + 20 - \exp\ \left(\frac{1}{D}\sum_{i=1}^{D} cos(2\pi x_i)\right) + e$	$[-32, 32]^D$	0
	Griewank	$f_{10}(x) = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-600, 600]^D$	0
	Weierstrass	$f_{11}(x) = \sum_{i=1}^{D} (\sum_{k=0}^{K_{max}} \left[a^k \cos(2\pi b^k (x_i + 0.5))\right]) - D\sum_{k=0}^{K_{max}} \left[a^k \cos(\pi b^k)\right] a = 0.5, b = 3, K_{max} = 20$	$[-0.5, 0.5]^D$	0

Table 2
Shifted mathematical test functions used to evaluate the algorithms.

	Name	Formulation	f_{bias}	Search domain	Global $f_{\min}$
Unimodal	Shifted Sphere	$f_{12}(x)=\sum_{i=1}^{D} z_i^2+f_{bias}; z_i=x_i-0$	−450	$[-100, 100]^D$	−450
	Shifted Schwefel's P1.2	$f_{13}(x)=\sum_{i=1}^{D}\left(\sum_{j=1}^{i} z_j\right)^2+f_{bias}; z_i=x_i-0.5$	−450	$[-100, 100]^D$	−450
	Shifted Schwefel's P1.2 with Noise in Fitness	$f_{14}(x)=\left(\sum_{i=1}^{D}\left(\sum_{j=1}^{i} x_j\right)^2\right)(1+4(\text{random}[0,1]))+f_{bias}; z_i=x_i-0.5$	−450	$[-100, 100]^D$	−450
	Shifted High Conditioned Eliptic	$f_{15}(x)=\sum_{i=1}^{D}(10^6)^{\frac{i-1}{D-1}} z_i^2+f_{bias}; z_i=x_i-0$	−450	$[-100, 100]^D$	−450
	Shifted Rosenbrock	$f_{16}(x)=\sum_{i}^{D-1}\left[100(z_{i+1}-z_i^2)^2+(z_i-1)^2\right]+f_{bias}; z_i=x_i+1$	390	$[-100, 100]^D$	390
Multimodal	Shifted Rastrigin	$f_{17}(x)=\sum_{i=1}^{D}\left[z_i^2-10\cos(2\pi z_i)+10\right]+f_{bias}; z_i=x_i-0$	−330	$[-5, 5]^D$	−330
	Shifted Ackley with Global Optimum on Bounds	$f_{18}(x)=-20\ \exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} z_i^2}\right)+20-\exp\left(\frac{1}{D}\sum_{i=1}^{D} cos(2\pi z_i)\right)$ $+e+f_{bias}; z_i=x_i-0$	−140	$[-32, 32]^D$	−140
	Shifted Rotated Griewank without Bounds	$f_{19}(x)=\frac{1}{4000}\sum_{i=1}^{D} z_i^2-\prod_{i=1}^{D}\cos\left(\frac{z_i}{\sqrt{i}}\right)+1+f_{bias}; z_i=(x_i-0.5)(1+3(\text{random}[0,1]))$	−180	$[0, 600]^D$	−180
	Shifted Weierstrass	$f_{20}(x)=\sum_{i=1}^{D}\left(\sum_{k=0}^{K_{max}}\left[a^k\cos\left(2\pi b^k(z_i+0.5)\right)\right]\right)-D\sum_{k=0}^{K_{max}}\left[a^k\cos\left(\pi b^k\right)\right]$ $+f_{bias}; a=0.5, b=3, K_{max}=20, z_i=x_i-1$	90	$[-0.5, 0.5]^D$	90

energy output of a real wind farm located in Binaloud, Iran with the meteorological data consisting of the wind speed and the ambient temperature [42]. Askarzadeh compared standard PSO with six variants of PSO for estimation of the electricity demand in Iran [43].

The rest of this paper is organized as follows. Section 2 gives briefly an overview of GA, PSO and SMC. Section 3 describes the operators and structure of the AGA and the ability of this proposed algorithm for preventing of premature convergence to local optimum points and rapidly converges to the global optimum. Then, in Section 4, well-known general and shifted test functions and the used algorithms for comparison are illustrated. Furthermore, simulation results and comparisons on the solution accuracy and the convergence speed are shown in this section to verify the sufficiency of the AGA. Moreover, this algorithm is implemented to estimate Iran's oil demand in Section 5 to indicate the capability of the presented algorithm for solving the real-world and constrained problems. Lastly, Section 6 concludes the paper.

Table 3
Parameter settings for optimization algorithms.

Algorithm	Parameter
AGA	$P_{GBc}=0.9, P_{QSSm}=0.1$
GA-TC	$P_r=0.2, P_{tc}=0.4, P_m=0.1, s=0.05$, the tournament selection method
GA-MC	$P_r=0.2, P_{mc}=0.4, P_m=0.1, s=0.05$, the tournament selection method
S-PSO	$w=0.9, C_1=C_2=2.0$
F-GA&PSO	$w_1=0.9, w_2=0.4, C_{1i}=C_{2f}=2.5, C_{1f}=C_{2i}=0.5$, $\zeta_m=0.001, \xi_{tc}=\xi_{mc}=0.2$

2. Background

2.1. Genetic algorithm

John Holland presented GA in 1975 with the inspiration of Darwin's theoryabout the survival of fittest [44]. One of the capabilities of stochastic algorithms is to work over a set of solutions called population. Each member of the population is called a chromosome $\vec{X}_i=[x_1, x_2, \ldots, x_D]$ where D is the number of gens. The standard version of the GA is organized by three operators; reproduction, crossover, and mutation. After applying these operators, the new population would be created. This process is iterated until the stopping criterion is met, and the chromosome with the best

Table 4
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Sphere test function.

Dimension		10	20	30	40
GA-TC	mean	2.456×10^{-5}	3.001×10^{-3}	1.158	27.468
	maximum	3.386×10^{-3}	2.574×10^{-1}	10.935	46.941
	minimum	1.258×10^{-7}	1.858×10^{-4}	9.864×10^{-2}	2.562
	std. dev.	9.254×10^{-6}	3.021×10^{-3}	1.646	22.364
GA-MC	mean	$9.499\times10^{+3}$	$3.158\times10^{+4}$	$5.616\times10^{+4}$	$8.007\times10^{+4}$
	maximum	$1.198\times10^{+4}$	$7.651\times10^{+4}$	$9.362\times10^{+4}$	$1.543\times10^{+5}$
	minimum	$8.687\times10^{+2}$	$7.786\times10^{+3}$	$9.158\times10^{+3}$	$2.139\times10^{+4}$
	std. dev.	$2.847\times10^{+3}$	$5.576\times10^{+3}$	$7.788\times10^{+3}$	$1.094\times10^{+4}$
S-PSO	mean	$1.793\times10^{+3}$	$1.298\times10^{+4}$	$3.022\times10^{+4}$	$4.99\,3\times10^{+4}$
	maximum	$9.769\times10^{+3}$	$3.354\times10^{+4}$	$5.007\times10^{+4}$	$7.727\times10^{+4}$
	minimum	$5.834\times10^{+2}$	$7.075\times10^{+3}$	$9.899\times10^{+3}$	$1.006\times10^{+4}$
	std. dev.	434.875	$2.141\times10^{+3}$	$2.827\times10^{+3}$	$3.828\times10^{+3}$
F-GA&PSO	mean	1.629×10^{-19}	2.917×10^{-11}	4.671×10^{-7}	1.703×10^{-4}
	maximum	1.385×10^{-18}	3.645×10^{-10}	4.743×10^{-6}	8.930×10^{-4}
	minimum	2.476×10^{-23}	1.254×10^{-15}	9.379×10^{-12}	2.647×10^{-7}
	std. dev.	2.174×10^{-19}	3.200×10^{-11}	5.321×10^{-7}	2.780×10^{-4}
AGA	mean	$\mathbf{3.081\times10^{-99}}$	$\mathbf{3.878\times10^{-60}}$	$\mathbf{1.098\times10^{-43}}$	$\mathbf{4.282\times10^{-35}}$
	maximum	$\mathbf{3.177\times10^{-98}}$	$\mathbf{7.702\times10^{-59}}$	$\mathbf{1.387\times10^{-42}}$	$\mathbf{5.995\times10^{-34}}$
	minimum	$\mathbf{5.589\times10^{-105}}$	$\mathbf{6.217\times10^{-65}}$	$\mathbf{5.175\times10^{-49}}$	$\mathbf{4.542\times10^{-43}}$
	std. dev.	8.629×10^{-99}	1.426×10^{-59}	3.453×10^{-43}	1.411×10^{-34}

Table 5
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Schwefel's P2.22 test function.

Dimension		10	20	30	40
GA-TC	mean	7.700×10^{-3}	0.058	0.242	1.062
	maximum	5.364×10^{-2}	0.476	1.002	12.419
	minimum	1.021×10^{-5}	7.598×10^{-4}	9.436×10^{-3}	5.858×10^{-2}
	std. dev.	2.100×10^{-3}	0.013	0.068	0.510
GA-MC	mean	51.427	$5.850 \times 10^{+5}$	$5.451 \times 10^{+10}$	$5.309 \times 10^{+15}$
	maximum	$1.953 \times 10^{+2}$	$4.864 \times 10^{+6}$	$7.842 \times 10^{+11}$	$8.953 \times 10^{+16}$
	minimum	4.497	$2.370 \times 10^{+3}$	$5.923 \times 10^{+8}$	$5.964 \times 10^{+10}$
	std. dev.	48.268	$1.483 \times 10^{+6}$	$2.067 \times 10^{+11}$	$2.682 \times 10^{+16}$
S-PSO	mean	10.132	43.007	841.197	$6.742 \times 10^{+6}$
	maximum	21.075	72.096	$1.356 \times 10^{+4}$	$5.354 \times 10^{+7}$
	minimum	1.007	9.972	212.065	$9.892 \times 10^{+4}$
	std. dev.	1.600	5.631	$1.557 \times 10^{+3}$	$1.527 \times 10^{+7}$
F-GA&PSO	mean	3.346×10^{-11}	6.696×10^{-7}	1.784×10^{-4}	7.103×10^{-3}
	maximum	3.947×10^{-10}	6.724×10^{-6}	1.043×10^{-3}	5.643×10^{-2}
	minimum	4.421×10^{-14}	7.121×10^{-10}	9.361×10^{-7}	5.515×10^{-5}
	std. dev.	2.332×10^{-11}	4.294×10^{-7}	1.608×10^{-4}	6.596×10^{-3}
AGA	mean	$\mathbf{1.005 \times 10^{-57}}$	$\mathbf{3.222 \times 10^{-39}}$	$\mathbf{2.036 \times 10^{-31}}$	$\mathbf{1.522 \times 10^{-26}}$
	maximum	$\mathbf{1.639 \times 10^{-56}}$	$\mathbf{1.798 \times 10^{-38}}$	$\mathbf{1.343 \times 10^{-30}}$	$\mathbf{1.959 \times 10^{-25}}$
	minimum	$\mathbf{1.030 \times 10^{-60}}$	$\mathbf{2.295 \times 10^{-41}}$	$\mathbf{1.008 \times 10^{-32}}$	$\mathbf{4.185 \times 10^{-28}}$
	std. dev.	3.038×10^{-57}	4.997×10^{-39}	2.819×10^{-31}	3.615×10^{-26}

Table 6
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Schwefel's P1.2 test function.

Dimension		10	20	30	40
GA-TC	mean	2.500×10^{-3}	6.400×10^{-3}	0.019	0.042
	maximum	3.553×10^{-2}	7.774×10^{-2}	0.307	0.908
	minimum	5.546×10^{-5}	9.610×10^{-5}	7.746×10^{-4}	3.497×10^{-3}
	std. dev.	8.800×10^{-3}	0.017	0.079	0.102
GA-MC	mean	0.495	0.897	1.428	1.281
	maximum	1.598	2.037	3.135	3.213
	minimum	0.042	0.121	0.837	0.756
	std. dev.	1.037	1.585	2.253	2.532
S-PSO	mean	5.148×10^{-5}	1.437×10^{-4}	1.928×10^{-4}	2.683×10^{-4}
	maximum	$\mathbf{5.590 \times 10^{-4}}$	9.346×10^{-4}	$\mathbf{2.127 \times 10^{-4}}$	9.386×10^{-3}
	minimum	6.125×10^{-8}	1.021×10^{-7}	9.739×10^{-7}	1.205×10^{-6}
	std. dev.	1.016×10^{-4}	3.581×10^{-4}	4.488×10^{-4}	5.727×10^{-4}
F-GA&PSO	mean	$\mathbf{2.273 \times 10^{-6}}$	$\mathbf{7.491 \times 10^{-6}}$	$\mathbf{9.836 \times 10^{-6}}$	$\mathbf{1.939 \times 10^{-5}}$
	maximum	6.754×10^{-4}	9.409×10^{-4}	1.114×10^{-3}	3.463×10^{-3}
	minimum	$\mathbf{3.759 \times 10^{-10}}$	$\mathbf{8.087 \times 10^{-10}}$	$\mathbf{9.908 \times 10^{-10}}$	$\mathbf{1.212 \times 10^{-9}}$
	std. dev.	8.212×10^{-6}	1.898×10^{-5}	2.131×10^{-5}	6.194×10^{-5}
AGA	mean	6.843×10^{-5}	8.536×10^{-5}	1.217×10^{-4}	1.477×10^{-4}
	maximum	1.543×10^{-3}	$\mathbf{6.104 \times 10^{-4}}$	5.112×10^{-3}	$\mathbf{1.692 \times 10^{-3}}$
	minimum	1.632×10^{-9}	6.952×10^{-9}	4.721×10^{-9}	8.204×10^{-9}
	std. dev.	2.215×10^{-4}	1.492×10^{-4}	5.953×10^{-4}	2.921×10^{-4}

Table 7
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Schwefel's P2.21 test function.

Dimension		10	20	30	40
GA-TC	mean	0.027	7.978	14.790	19.571
	maximum	0.124	17.953	21.603	32.052
	minimum	0.010	0.693	2.493	7.987
	std. dev.	0.079	3.163	3.248	3.525
GA-MC	mean	54.347	75.910	82.907	86.07
	maximum	77.903	84.076	90.047	98.128
	minimum	27.975	50.092	72.860	79.075
	std. dev.	8.355	6.081	4.642	3.386
S-PSO	mean	24.545	52.349	64.470	72.483
	maximum	52.974	66.426	73.947	87.091
	minimum	2.351	21.856	40.032	64.632
	std. dev.	3.205	3.876	3.415	3.021
F-GA&PSO	mean	7.038×10^{-8}	0.022	**1.671**	**7.775**
	maximum	8.087×10^{-7}	0.947	**5.547**	**14.087**
	minimum	8.358×10^{-10}	2.574×10^{-6}	8.230×10^{-6}	4.359×10^{-3}
	std. dev.	5.983×10^{-8}	0.016	0.711	2.037
AGA	mean	$\mathbf{2.742 \times 10^{-31}}$	$\mathbf{1.022 \times 10^{-5}}$	17.102	67.269
	maximum	$\mathbf{5.335 \times 10^{-30}}$	$\mathbf{3.044 \times 10^{-4}}$	57.377	92.522
	minimum	$\mathbf{4.354 \times 10^{-37}}$	$\mathbf{1.119 \times 10^{-15}}$	$\mathbf{3.243 \times 10^{-6}}$	$\mathbf{1.443 \times 10^{-3}}$
	std. dev.	9.998×10^{-31}	5.557×10^{-5}	31.169	32.036

Table 8
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Rosenbrock test function.

Dimension		10	20	30	40
GA-TC	mean	6.776	35.700	$1.128 \times 10^{+2}$	$2.184 \times 10^{+2}$
	maximum	9.470	$1.147 \times 10^{+2}$	$2.244 \times 10^{+2}$	$3.738 \times 10^{+2}$
	minimum	6.446	17.348	29.877	$1.110 \times 10^{+2}$
	std. dev.	1.473	28.578	54.056	61.833
GA-MC	mean	$1.380 \times 10^{+5}$	$1.155 \times 10^{+6}$	$2.244 \times 10^{+6}$	$3.427 \times 10^{+6}$
	maximum	$9.768 \times 10^{+5}$	$8.268 \times 10^{+6}$	$1.460 \times 10^{+7}$	$5.903 \times 10^{+7}$
	minimum	$1.246 \times 10^{+3}$	$9.775 \times 10^{+3}$	$1.555 \times 10^{+4}$	$3.431 \times 10^{+4}$
	std. dev.	$8.684 \times 10^{+4}$	$3.476 \times 10^{+5}$	$5.601 \times 10^{+5}$	$9.315 \times 10^{+5}$
S-PSO	mean	$5.392 \times 10^{+3}$	$2.060 \times 10^{+5}$	$7.525 \times 10^{+5}$	$1.554 \times 10^{+6}$
	maximum	$1.059 \times 10^{+4}$	$1.358 \times 10^{+6}$	$1.460 \times 10^{+7}$	$5.903 \times 10^{+7}$
	minimum	$1.205 \times 10^{+3}$	$7.879 \times 10^{+3}$	$2.476 \times 10^{+4}$	$2.876 \times 10^{+4}$
	std. dev.	$2.617 \times 10^{+3}$	$5.649 \times 10^{+4}$	$1.528 \times 10^{+5}$	$2.240 \times 10^{+5}$
F-GA&PSO	mean	**1.216**	27.191	44.751	72.775
	maximum	**5.615**	77.000	$1.36 \times 10^{+2}$	$2.191 \times 10^{+2}$
	minimum	**0.072**	**0.050**	**16.747**	**33.675**
	std. dev.	1.403	25.861	31.557	39.207
AGA	mean	6.317	**16.843**	**27.034**	**37.194**
	maximum	6.874	**17.352**	**27.648**	**37.534**
	minimum	5.822	15.921	26.483	36.448
	std. dev.	0.253	0.283	0.293	0.274

Table 9
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Quaric test function.

Dimension		10	20	30	40
GA-TC	mean	3.213×10^{-3}	9.307×10^{-3}	0.020	0.038
	maximum	3.465×10^{-2}	9.233×10^{-2}	0.123	0.404
	minimum	3.450×10^{-4}	9.980×10^{-4}	3.764×10^{-3}	8.359×10^{-3}
	std. dev.	1.313×10^{-3}	3.121×10^{-3}	5.727×10^{-3}	0.011
GA-MC	mean	5.709	64.711	183.880	393.701
	maximum	14.976	72.987	197.770	402.394
	minimum	1.201	17.086	121.898	330.359
	std. dev.	3.430	19.510	49.780	80.690
S-PSO	mean	0.971	12.416	65.551	176.087
	maximum	3.987	21.212	77.970	212.900
	minimum	0.006	0.120	11.910	144.801
	std. dev.	0.367	3.357	11.417	24.575
F-GA&PSO	mean	1.941×10^{-3}	7.149×10^{-3}	0.019	0.038
	maximum	2.121×10^{-2}	7.771×10^{-2}	0.212	0.444
	minimum	1.110×10^{-4}	6.899×10^{-4}	4.110×10^{-3}	9.992×10^{-3}
	std. dev.	9.772×10^{-4}	2.668×10^{-3}	6.771×10^{-3}	0.011
AGA	mean	$\mathbf{2.797 \times 10^{-4}}$	$\mathbf{7.067 \times 10^{-4}}$	$\mathbf{2.041 \times 10^{-3}}$	$\mathbf{2.266 \times 10^{-3}}$
	maximum	$\mathbf{1.452 \times 10^{-3}}$	$\mathbf{1.903 \times 10^{-3}}$	$\mathbf{8.143 \times 10^{-3}}$	$\mathbf{5.753 \times 10^{-3}}$
	minimum	$\mathbf{1.506 \times 10^{-5}}$	$\mathbf{6.421 \times 10^{-5}}$	$\mathbf{3.783 \times 10^{-4}}$	$\mathbf{2.930 \times 10^{-4}}$
	std. dev.	2.905×10^{-4}	4.436×10^{-4}	1.719×10^{-3}	1.503×10^{-3}

Table 10
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for High Conditioned Eliptic test function.

Dimension		10	20	30	40
GA-TC	mean	0.560	$3.932 \times 10^{+2}$	$1.413 \times 10^{+4}$	$1.142 \times 10^{+5}$
	maximum	2.009	$3.233 \times 10^{+3}$	$5.631 \times 10^{+4}$	$2.790 \times 10^{+5}$
	minimum	0.090	32.961	$1.712 \times 10^{+3}$	$2.976 \times 10^{+4}$
	std. dev.	0.430	$5.821 \times 10^{+2}$	$1.229 \times 10^{+4}$	$6.578 \times 10^{+4}$
GA-MC	mean	$7.150 \times 10^{+7}$	$6.118 \times 10^{+8}$	$1.527 \times 10^{+9}$	$2.538 \times 10^{+9}$
	maximum	$1.462 \times 10^{+8}$	$1.018 \times 10^{+9}$	$2.617 \times 10^{+9}$	$3.744 \times 10^{+9}$
	minimum	$1.183 \times 10^{+7}$	$3.299 \times 10^{+8}$	$8.175 \times 10^{+8}$	$1.151 \times 10^{+9}$
	std. dev.	$3.532 \times 10^{+7}$	$1.868 \times 10^{+8}$	$4.409 \times 10^{+8}$	$6.891 \times 10^{+8}$
S-PSO	mean	$3.130 \times 10^{+6}$	$6.069 \times 10^{+7}$	$2.340 \times 10^{+8}$	$5.308 \times 10^{+8}$
	maximum	$7.416 \times 10^{+6}$	$8.726 \times 10^{+7}$	$3.646 \times 10^{+8}$	$7.547 \times 10^{+8}$
	minimum	$1.211 \times 10^{+6}$	$2.847 \times 10^{+7}$	$1.541 \times 10^{+8}$	$3.196 \times 10^{+8}$
	std. dev.	$1.555 \times 10^{+6}$	$1.429 \times 10^{+7}$	$4.897 \times 10^{+7}$	$1.020 \times 10^{+8}$
F-GA&PSO	mean	1.707×10^{-15}	3.298×10^{-7}	1.047×10^{-2}	6.031
	maximum	8.436×10^{-15}	1.442×10^{-6}	4.127×10^{-2}	19.665
	minimum	4.729×10^{-17}	1.585×10^{-8}	1.281×10^{-3}	0.581
	std. dev.	2.259×10^{-19}	3.451×10^{-7}	8.417×10^{-3}	4.868
AGA	mean	$\mathbf{3.164 \times 10^{-111}}$	$\mathbf{3.457 \times 10^{-69}}$	$\mathbf{9.849 \times 10^{-52}}$	$\mathbf{3.505 \times 10^{-41}}$
	maximum	$\mathbf{9.281 \times 10^{-110}}$	$\mathbf{7.851 \times 10^{-68}}$	$\mathbf{2.748 \times 10^{-50}}$	$\mathbf{9.758 \times 10^{-40}}$
	minimum	$\mathbf{5.408 \times 10^{-119}}$	$\mathbf{1.937 \times 10^{-76}}$	$\mathbf{1.049 \times 10^{-58}}$	$\mathbf{5.657 \times 10^{-48}}$
	std. dev.	1.693×10^{-110}	1.434×10^{-68}	5.010×10^{-51}	1.781×10^{-40}

Table 11
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Rastrigin test function.

Dimension		10	20	30	40
GA-TC	mean	6.300	14.698	25.394	36.837
	maximum	17.953	25.876	34.098	44.871
	minimum	0.874	7.890	18.987	21.112
	std. dev.	3.136	4.920	7.610	9.225
GA-MC	mean	97.526	251.950	410.879	569.759
	maximum	121.001	297.009	444.245	601.007
	minimum	21.998	40.980	99.071	111.923
	std. dev.	10.615	21.889	24.508	29.148
S-PSO	mean	52.982	173.939	309.696	448.241
	maximum	110.005	212.330	313.086	491.060
	minimum	2.758	14.986	35.091	72.946
	std. dev.	6.727	10.973	12.281	18.188
F-GA&PSO	mean	3.704	14.359	32.959	59.409
	maximum	17.990	21.005	39.343	73.222
	minimum	0.017	0.123	0.990	2.983
	std. dev.	1.738	4.867	8.725	14.198
AGA	mean	**0**	**0**	$\mathbf{3.978 \times 10^{-9}}$	$\mathbf{3.506 \times 10^{-1}}$
	maximum	**0**	**0**	$\mathbf{1.193 \times 10^{-7}}$	**10.520**
	minimum	**0**	**0**	**0**	**0**
	std. dev.	0	0	2.178×10^{-8}	1.920

Table 12
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Ackley test function.

Dimension		10	20	30	40
GA-TC	mean	0.067	1.550	2.783	4.045
	maximum	0.761	12.998	17.998	19.001
	minimum	5.513×10^{-5}	1.865×10^{-3}	8.991×10^{-3}	1.911×10^{-2}
	std. dev.	(0.277)	(0.956)	(0.959)	(1.058)
GA-MC	mean	18.854	20.053	20.387	20.478
	maximum	21.004	21.954	22.002	22.861
	minimum	17.087	19.154	19.900	20.020
	std. dev.	0.761	0.374	0.290	0.294
S-PSO	mean	12.834	17.769	19.020	19.553
	maximum	14.001	19.007	21.213	21.650
	minimum	1.970	12.870	14.768	15.000
	std. dev.	1.186	0.586	0.299	0.237
F-GA&PSO	mean	1.613×10^{-10}	0.115	0.590	1.209
	maximum	8.560×10^{-10}	0.359	0.791	3.130
	minimum	9.986×10^{-11}	0.097	0.127	0.576
	std. dev.	1.355×10^{-10}	0.115	0.745	0.833
AGA	mean	$\mathbf{4.440 \times 10^{-15}}$	$\mathbf{6.217 \times 10^{-15}}$	$\mathbf{7.638 \times 10^{-15}}$	$\mathbf{9.414 \times 10^{-15}}$
	maximum	$\mathbf{7.993 \times 10^{-15}}$	$\mathbf{7.993 \times 10^{-15}}$	$\mathbf{1.509 \times 10^{-14}}$	$\mathbf{1.509 \times 10^{-14}}$
	minimum	$\mathbf{1.881 \times 10^{-15}}$	$\mathbf{4.440 \times 10^{-15}}$	$\mathbf{4.440 \times 10^{-15}}$	$\mathbf{4.440 \times 10^{-15}}$
	std. dev.	1.319×10^{-15}	1.806×10^{-15}	1.945×10^{-15}	3.311×10^{-15}

Table 13
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Griewank test function.

Dimension		10	20	30	40
GA-TC	mean	0.063	0.032	0.674	1.457
	maximum	0.344	0.211	1.003	3.013
	minimum	4.232×10^{-7}	7.812×10^{-4}	7.241×10^{-3}	0.072
	std. dev.	0.067	0.052	0.236	0.543
GA-MC	mean	86.440	285.253	506.544	721.645
	maximum	100.654	310.657	536.096	976.871
	minimum	12.346	47.987	97.818	138.054
	std. dev.	25.765	50.200	70.092	98.450
S-PSO	mean	15.584	122.104	273.025	445.451
	maximum	21.875	146.086	303.936	490.043
	minimum	3.870	21.087	86.900	110.072
	std. dev.	4.192	16.454	26.019	44.027
F-GA&PSO	mean	0.108	9.772×10^{-3}	2.917×10^{-3}	8.541×10^{-5}
	maximum	1.870	0.881	0.034	8.251×10^{-4}
	minimum	3.974×10^{-3}	9.341×10^{-5}	1.943×10^{-5}	7.961×10^{-7}
	std. dev.	0.056	0.033	0.021	7.100×10^{-5}
AGA	mean	$\mathbf{4.121 \times 10^{-2}}$	$\mathbf{3.621 \times 10^{-3}}$	**0**	**0**
	maximum	$\mathbf{3.345 \times 10^{-1}}$	$\mathbf{1.086 \times 10^{-1}}$	**0**	**0**
	minimum	**0**	**0**	**0**	**0**
	std. dev.	2.489×10^{-2}	1.983×10^{-2}	0	0

Table 14
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for Weierstrass test function.

Dimension		10	20	30	40
GA-TC	mean	0. 752	2.395	5.391	10.786
	maximum	0.833	2.553	5.971	11.301
	minimum	0.701	2.312	4.645	10.353
	std. dev.	0.070	0.136	0.678	0.479
GA-MC	mean	9.193	24.247	39.784	60.062
	maximum	9.324	25.983	41.107	60.896
	minimum	9.088	23.248	37.885	59.018
	std. dev.	0.120	1.509	1.686	0.956
S-PSO	mean	6.690	22.625	37.551	54.815
	maximum	7.813	23.620	38.183	55.488
	minimum	5.330	21.878	37.223	54.253
	std. dev.	1.258	0.896	0.547	0.624
F-GA&PSO	mean	1.371×10^{-3}	2.425×10^{-2}	0.701	2.216
	maximum	4.114×10^{-3}	6.159×10^{-2}	1.024	3.166
	minimum	1.561×10^{-8}	1.708×10^{-3}	0.238	1.214
	std. dev.	2.375×10^{-3}	3.257×10^{-2}	0.411	0.977
AGA	mean	**0**	**0**	**0**	**0**
	maximum	**0**	**0**	**0**	**0**
	minimum	**0**	**0**	**0**	**0**
	std. dev.	0	0	0	0

Table 15
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for shifted Sphere test function.

Dimension		10	20	30	40
GA-TC	mean	−449.9999837745328	−449.9972976720476	−448.4499163144820	−420.4920911387130
	maximum	−449.9999588066938	−449.9931744325014	−442.8819511247692	−388.1314662592574
	minimum	−449.9999966502348	−449.9990235218959	−449.8730327580137	−442.1867204606277
	std. dev.	1.026×10^{-5}	1.546×10^{-3}	1.518	13.093
GA-MC	mean	830.8271206286860	3094.207702924056	5641.599439114948	8274.321337189026
	maximum	1514.509529036338	4110.281953857641	7442.622749401216	1009.078179111214
	minimum	247.8552583488984	1918.299086586165	3626.586856353388	5714.100661851109
	std. dev.	$3.118 \times 10^{+3}$	$5.502 \times 10^{+3}$	$9.493 \times 10^{+3}$	$9.734 \times 10^{+3}$
S-PSO	mean	1182.494874960175	1316.767566124047	2994.365504144866	4.812075560223211
	maximum	2218.564717591757	1652.536751461939	3490.325122363107	5.451251571649575
	minimum	301.9667875124316	874.8863270600046	2249.261854790727	3.554409650249442
	std. dev.	$4.782 \times 10^{+2}$	$2.084 \times 10^{+3}$	$3.029 \times 10^{+3}$	$4.635 \times 10^{+3}$
F-GA&PSO	mean	**−450**	−449.9999999999440	−449.9999995251106	−449.9997860481680
	maximum	**−450**	−449.9999999994244	−449.9999979520674	−449.9991817488010
	minimum	**−450**	−449.9999999999972	−449.9999999318787	−449.9999844329565
	std. dev.	0	1.188×10^{-10}	4.539×10^{-7}	2.105×10^{-4}
AGA	mean	**−450**	**−450**	**−450**	**−450**
	maximum	**−450**	**−450**	**−450**	**−449.9999999999999**
	minimum	**−450**	**−450**	**−450**	**−450**
	std. dev.	0	0	0	3.949×10^{-14}

Table 16
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for shifted Schwefel's P1.2 test function.

Dimension		10	20	30	40
GA-TC	mean	−449.9993829874049	−449.9980536071201	−449.9925408112686	−449.9077783918694
	maximum	−449.9982201126205	−449.9974226101959	−449.9776724346482	−449.7873248307561
	minimum	−449.9999993741910	−449.9990732585442	−449.9999999634825	−449.9960312315511
	std. dev.	1.007×10^{-3}	8.913×10^{-4}	1.287×10^{-2}	0.108
GA-MC	mean	−449.6486380943293	−448.8685346225282	−449.4291628332339	−449.2532590634946
	maximum	−449.0614807525150	−446.6620793665169	−448.4741922241739	−449.0007699389189
	minimum	−449.9568529485293	−449.9964717689761	−449.9201180985775	−449.5160109335621
	std. dev.	0.508	1.911	0.827	0.257
S-PSO	mean	−449.9999910054258	−449.9999659474472	−449.9998652270189	−449.9998414362930
	maximum	−449.9999793767399	−449.9999402643471	−449.9996435574347	−449.9996813116705
	minimum	−449.9999987666490	−449.9999942560277	−449.9999886529012	−449.9999698307262
	std. dev.	1.025×10^{-5}	2.709×10^{-5}	1.923×10^{-4}	1.468×10^{-4}
F-GA&PSO	mean	**−449.9999978871052**	**−449.9999966405426**	−449.9999955258148	−449.9999824102650
	maximum	**−449.9999939847133**	**−449.9999908661935**	**−449.9999875822466**	−449.9999337437559
	minimum	**−450**	**−450**	**−449.9999999990837**	**−450**
	std. dev.	2.344×10^{-6}	4.549×10^{-6}	5.089×10^{-6}	2.794×10^{-5}
AGA	mean	−449.9999959360885	−449.9999921236908	**−449.9999960810008**	**−449.9999880772277**
	maximum	−449.9999707487508	−449.9999474812837	−449.9999863454146	**−449.9999431427385**
	minimum	−449.9999999602784	−449.9999999919742	−449.9999999937223	−449.9999999975880
	std. dev.	9.039×10^{-6}	1.626×10^{-5}	5.807×10^{-6}	2.512×10^{-5}

fitness would be introduced as the optimal solution. The details of the reproduction, crossover, and mutation operators are described in the following.

- *Reproduction:* In a simple way, two members of the population are selected randomly then the member with less fitness is removed from the population and the one with more fitness is put in place. This operator is done for $(P_r \times N)$ members of the population. P_r and N are the probability of the reproduction and the size of the population, respectively.
- *Crossover:* A crossover operator selects two members of the population randomly. Then, it creates two new chromosomes and puts them at the place of the old chromosomes. The crossover operator is usually applied to a number of pairs determined as $(P_{tc} \times N)/2$, where P_{tc} and N are the probability of the crossover and the population size, respectively. Let $\vec{X}_i(t)$ and $\vec{X}_j(t)$ be two randomly selected chromosomes and $\vec{X}_i(t)$ has the smaller fitness value than $\vec{X}_j(t)$, then the crossover relations are as follows.

$$\begin{aligned}\vec{X}_i(t+1) &= \vec{X}_i(t) + \vec{\gamma}_1(\vec{X}_i(t) - \vec{X}_j(t)) \\ \vec{X}_j(t+1) &= \vec{X}_j(t) + \vec{\gamma}_2(\vec{X}_i(t) - \vec{X}_j(t))\end{aligned} \tag{1}$$

where $\vec{\gamma}_1$ and $\vec{\gamma}_2 \in [0,1]^D$ are random vectors.

- *Mutation:* Mutation operator causes variations on the values of a number of chromosomes in the population (determined as $P_m \times N$, where P_m and N are the probability of the mutation and the population size, respectively). Let $\vec{X}_i(t)$ a randomly selected chromosome, and then the mutation formulation is defined as:

$$\vec{X}_i(t+1) = \vec{X}_i(t) + (\vec{b} \times \eta) \tag{2}$$

where $\vec{b} \in [0,1]^D$ is a random vector and η is a constant value.

2.2. Particle swarm optimization

In the PSO algorithm [45], the position vector $\vec{U}_i(\mathrm{t})$ is changed by adding a velocity vector $\vec{v}_i(\mathrm{t})$ to it as follows

Table 17
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for shifted Schwefel's P1.2 test function with noise in fitness.

Dimension		10	20	30	40
GA-TC	mean	−449.9985646936117	−449.9995366580214	−449.9768482611706	−449.8689327193002
	maximum	−449.9973857109426	−449.9989148703982	−449.9320490211749	−449.6109926351037
	minimum	−449.9988366510013	−449.9999325471117	−449.9994119744920	−449.9846700225463
	std. dev.	1.407×10^{-3}	5.627×10^{-4}	3.894×10^{-2}	0.223
GA-MC	mean	−449.9186438791738	−446.4141073209536	−449.9544126604372	−449.7128661887552
	maximum	−449.8147846442799	−441.2545801124971	−449.9298434911697	−449.4688667139914
	minimum	−449.9900004053890	−449.8147561449861	−449.9996427847039	−449.9745888937195
	std. dev.	0.070	3.846	0.039	0.253
S-PSO	mean	−449.9999284658217	−449.9999885284668	−449.9998138476615	−449.9999437261948
	maximum	−449.9998117567730	−449.9999780811913	−449.9994285900362	−449.9997224890036
	minimum	−449.9999995133398	−449.9999960390725	−449.9999420414245	−449.999963152574
	std. dev.	1.054×10^{-4}	9.332×10^{-6}	3.508×10^{-4}	1.626×10^{-4}
F-GA&PSO	mean	−449.9999950023186	**−449.9999990451206**	−449.9999968631385	−449.9999923693189
	maximum	−449.9999840194949	−449.9999952698601	−449.9999905894156	−449.9999863195532
	minimum	**−449.9999999999784**	**−450**	**−450**	−449.9999994119204
	std. dev.	7.092×10^{-6}	2.110×10^{-6}	5.433×10^{-6}	6.602×10^{-6}
AGA	mean	−449.9999996303929	−449.9999989551646	**−449.9999999262753**	**−449.9999996911625**
	maximum	**−449.9999985214742**	**−449.9999956029188**	**−449.9999997946533**	**−449.9999991521398**
	minimum	**−449.9999999951697**	−449.9999999998866	−449.9999999995843	**−449.9999999972478**
	std. dev.	6.279×10^{-7}	1.892×10^{-6}	8.275×10^{-8}	4.682×10^{-7}

Table 18
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for shifted High Conditioned Eliptic test function with noise in fitness.

Dimension		10	20	30	40
GA-TC	mean	−89.800120311823065	−44.785659135786744	6175.906660381945	9678.863028581396
	maximum	0	0	30879.53330190972	48394.31514290698
	minimum	−449.0006015591154	−223.9282956789337	0	0
	std. dev.	$2.007 \times 10^{+2}$	$1.001 \times 10^{+2}$	$1.3809 \times 10^{+4}$	$2.164 \times 10^{+4}$
GA-MC	mean	95038613.62111142	628139467.1060340	1183655090.315825	2668993846.784902
	maximum	166548699.3689040	842993720.4283164	1781170337.972066	3447049174.110232
	minimum	19262053.58890160	378705600.2122736	718784314.9118657	1798963434.666303
	std. dev.	$6.016 \times 10^{+7}$	$2.102 \times 10^{+8}$	$4.447 \times 10^{+8}$	$6.801 \times 10^{+8}$
S-PSO	mean	4024464.947335149	47643637.75159591	194539391.7604014	530961373.5759113
	maximum	6619122.049961107	63723089.21787029	262225505.6660125	691997922.2474580
	minimum	2866912.949787115	24933497.00746602	141578937.2758800	393195819.7200962
	std. dev.	$1.559 \times 10^{+6}$	$2.022 \times 10^{+7}$	$6.165 \times 10^{+7}$	$1.507 \times 10^{+8}$
F-GA&PSO	mean	**−450**	−449.9999987344112	−449.9879738051980	−444.5714916245727
	maximum	**−450**	−449.9999925130472	−449.9635396521812	−431.1295022914000
	minimum	**−450**	−449.9999999183896	−449.9980208697401	−448.8713572043769
	std. dev.	0	2.229×10^{-6}	0.011	5.441
AGA	mean	**−450**	**−450**	**−450**	**−450**
	maximum	**−450**	**−449.9999999999999**	**−449.9999999999999**	**−449.9999999999999**
	minimum	**−450**	**−450**	**−450**	**−450**
	std. dev.	0	1.055×10^{-14}	2.111×10^{-14}	4.352×10^{-14}

$$\vec{U}_i(t+1) = \vec{U}_i(t) + \vec{v}_i(t+1) \quad (3)$$

$$\vec{v}_i(t+1) = w\omega\vec{v}_i(t) + C_1\vec{r}_1(\vec{U}_{pbest_i} - \vec{U}_i(t)) + C_2\vec{r}_2(\vec{U}_{gbest} - \vec{U}_i(t)) \quad (4)$$

where C_1, C_2 and w are the cognitive learning factor, social learning factor and inertia weight, respectively. $\vec{U}_{pbest_i}$ is the personal best position of the particle i. $\vec{U}_{gbest}$ is the position of the best particle within the swarm, and $\vec{r}_1, \vec{r}_2 \in [0,1]^d$ are random vectors where d is dimension.

2.3. Sliding mode control

The sliding mode controller is a powerful control strategy applied to robust handle of nonlinear systems [46–55]. In this section, the general concepts of SMC for a second-order dynamic system defined by the following state-space equation are described.

$$\dot{x} = f(h, u, t) \quad (5)$$

where $h \in R^n$ is the state vector, $u \in R^m$ is the input vector, and t is the time. The sliding surface $s(e,t)$ is defined as:

$$s(e,t) = \left(\frac{d}{dt} + \lambda\right)e = 0 \quad (6)$$

$$s = \dot{e} + \lambda e; e = h - h_d \quad (7)$$

where λ is a strictly positive constant and h_d is the desired state vector. In fact, s is a sum-weighted of the position and velocity errors. In this control method, by changing the control law according to certain predefined rules which depend on the position of state errors of the system with respect to sliding surfaces, the states are switched between stable and unstable trajectories until they reach the sliding surface [55].

Table 19
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for shifted Rosenbrock test function.

Dimension		10	20	30	40
GA-TC	mean	396.7169325292743	669.8500426088053	22201.25716313337	292680.0310020061
	maximum	399.5639733800460	1063.760936413901	73595.40483050168e	966506.8051038038
	minimum	392.5458839400190	459.4194942689739	3078.850362025212e	59766.56716796412
	std. dev.	2.391	$2.378 \times 10^{+2}$	$2.256 \times 10^{+4}$	$2.643 \times 10^{+5}$
GA-MC	mean	1312989928.450298	12698736804.81050	20158404514.52971	29239627456.19702
	maximum	3641413484.367582	18994422155.75515	31187077254.86490	46485373401.66466
	minimum	237833353.0822434	8042887099.252314	9869203003.529253	18724947626.21279
	std. dev.	$1.048 \times 10^{+9}$	$3.207 \times 10^{+9}$	$5.701 \times 10^{+9}$	$8.605 \times 10^{+9}$
S-PSO	mean	47948530.92204220	1928556741.888466	$7095243433.6 \times 10^{+7}97580$	15345055869.48889
	maximum	75216003.65289119	2658482090.983409	9865562889.146740	19289997678.74270
	minimum	14707664.63443860	1330601848.694514	4723389491.999683	11864908313.17943
	std. dev.	1.835	$4.053 \times 10^{+8}$	$1.580 \times 10^{+9}$	$2.664 \times 10^{+9}$
F-GA&PSO	mean	400.7631178052712	423.2722138655833	454.7913176607509	531.0464844829378e
	maximum	449.1466581757315	564.7035492755612	531.2735612358079	855.6660029402847
	minimum	390.6951831994961	394.6441395009354	415.3489590734728	428.8583860117879
	std. dev.	18.230	54.130	38.011	$1.236 \times 10^{+2}$
AGA	mean	**390.0103693163959**	**390.0000000000293**	**390.0000055661383**	**390.0000000000462**
	maximum	**390.0812110353763**	**390.0000000002930**	**390.0000555189113**	**390.0000000002925**
	minimum	**390**	**390**	**390**	**390.0000000000001**
	std. dev.	2.560×10^{-2}	9.265×10^{-11}	1.755×10^{-5}	9.585×10^{-11}

Table 20
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for shifted Rastrigin test function.

Dimension		10	20	30	40
GA-TC	mean	−323.9613956917980	−313.9139387095056	−305.3315578704945	−295.1801946529780
	maximum	−319.0522968263761	−298.0720932223101	−285.9633972995544	−268.0650434297603
	minimum	−329.0033923519364	−323.9469696783326	−318.5657942558461	−310.7506211122298
	std. dev.	2.624	7.027	6.340	9.577
GA-MC	mean	−231.9698052931943	−83.941368063095069	70.760317929220861	236.1408259510231
	maximum	−212.4165576083890	−53.885011698788048	111.8040067233131	276.5894158248347
	minimum	−262.5106046824117	−123.9854940309564	26.133970258747922	150.8674398598100
	std. dev.	12.938	18.654	20.148	30.619
S-PSO	mean	−277.1768516616635	−153.0470626955032	−32.783281161176426	101.7092016372488
	maximum	−268.1007442226782	−139.6359702847255	−16.531704151738040	133.8456720051392
	minimum	−288.2437691151679	−162.1774641862251	−58.313023008207892	20.859000129232982
	std. dev.	5.430	7.050	13.656	36.315
F-GA&PSO	mean	−326.3145780230623	−315.7467701788274	−295.6464454608587	−268.8474911347565
	maximum	−322.0403204297655	−306.1209888679108	−272.4151217728556	−236.0237198109013
	minimum	−329.8819537334009	−326.0193383619661	−311.9872240480715	−299.6417972654198
	std. dev.	1.707	4.674	9.346	14.257
AGA	mean	**−330**	**−329.9999999999987**	**−329.4288611416667**	**−327.5174367040779**
	maximum	**−330**	**−329.9999999999690**	**−312.8658342597547**	**−283.9127388666287**
	minimum	**−330**	**−330**	**−330**	**−330**
	std. dev.	0	5.793×10^{-12}	3.128	9.729

Table 21
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for shifted Ackley test function with global optimum on bounds.

Dimension		10	20	30	40
GA-TC	mean	−139.9954751841448	−138.5374635701375	−137.3833104527502	−135.6958985337245
	maximum	−139.9929757117656	−136.0417450878459	−136.2048541231768	−133.7063907814467
	minimum	−139.9976555619460	−139.9800837892175	−138.3306921219629	−137.3554190074501
	std. dev.	1.212×10^{-3}	1.013	0.564	0.980
GA-MC	mean	−121.1409083448861	−119.8299232061201	−119.5856638490304	−119.4649582920145
	maximum	−120.2192789849038	−119.4096083447712	−119.2552438861343	−119.2193540349563
	minimum	−123.0959999329135	−120.7758534028307	−120.2132778756837	−119.9239611136939
	std. dev.	0.739	0.334	0.231	0.203
S-PSO	mean	−119.6054526561096	−119.1529554692678	−118.9645897540434	−118.8646535066549
	maximum	−119.4424835750159	−119.0626933933543	−118.8757692849515	−118.8032615023607
	minimum	−119.7493443842990	−119.2989478861391	−119.0609793345992	−118.9783372946030
	std. dev.	0.073	0.059	0.056	0.044
F-GA&PSO	mean	−139.9999999998468	−139.9386637254656	−139.6363650743944	−138.8401343023154
	maximum	−139.9999999994627	−138.1599814760067	−137.7790022180207	−137.6150003151231
	minimum	−139.9999999999735	−139.9999992741500	−139.9999200049385	−139.9982777771635
	std. dev.	1.120×10^{-10}	0.335	0.646	0.779
AGA	mean	**−140**	**−139.9999999999993**	**−139.9999999999664**	**−139.9999999996101**
	maximum	**−140**	**−139.9999999999899**	**−139.9999999998142**	**−139.9999999974686**
	minimum	**−140**	**−139.9999999999999**	**−139.9999999999995**	**−139.9999999999809**
	std. dev.	2.531×10^{-14}	1.812×10^{-12}	4.609×10^{-11}	5.101×10^{-10}

Table 22
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for shifted rotated Griewank test function without bounds.

Dimension		10	20	30	40
GA-TC	mean	−179.9825801365417	−179.9216607422553	−177.6983433953596	−168.2485964490810
	maximum	−179.9477438416087	−179.8623381121829	−176.2525137511280	−157.1037671577080
	minimum	−179.9999993441533	−179.9656428020683	−178.5260630896768	−175.4867898152660
	std. dev.	0.030	0.053	1.256	9.794
GA-MC	mean	−111.4686797775816	134.0160670343220	420.9536469767337	681.8078396562808
	maximum	−56.158183775051839	204.4546030053080	446.5056746454128	733.3147163165897
	minimum	−154.2261958169383	69.201969519069763	396.5079368826565	591.6814377382542
	std. dev.	38.412	54.806	18.670	78.317
S-PSO	mean	−154.1584115072451	8.764564008438990	877.6754105683400	904.7778144006653
	maximum	−144.9857313017676	50.405065221202250	100.9304347704108	123.7426472312736
	minimum	−165.9091851052426	−34.815108795819953	786.4061470590939	585.0415780198359
	std. dev.	10.697	42.643	$1.168 \times 10^{+2}$	$3.263 \times 10^{+2}$
F-GA&PSO	mean	−179.8943923251610	**−179.9999999973435**	**−179.9948025577891**	−179.8576922572178
	maximum	−179.7920755427302	**−179.9999999580797**	−179.8511962884156	−179.1465172991599
	minimum	**−180**	**−179.9999999998250**	**−179.9999859612236**	**−179.9832091828384**
	std. dev.	0.045	7.497×10^{-9}	0.027	0.169
AGA	mean	−179.9809385148956	−179.9808945605819	−179.9687126675721	−179.9545297792528
	maximum	−179.8155751363782	−179.8705780086872	**−179.9589673530622**	**−179.9419697746883**
	minimum	−179.9984210554754	−179.9905315632542	−179.9768199123363	**−179.9655346905215**
	std. dev.	0.041	0.021	4.469×10^{-3}	4.939×10^{-3}

Table 23
Mean, maximum, minimum and standard deviation of the best fitness values for 30 runs found by the AGA, GA-TC, GA-MC, S-PSO and F-GA&PSO for shifted Weierstrass test function.

Dimension		10	20	30	40
GA-TC	mean	90.68243160562338	92.62769869767138	95.62835293704381	99.81133668986833
	maximum	90.78207132556143	93.05874425386949	95.76754799404334	101.45077463412084
	minimum	90.54257205148032	91.99132366029672	95.37637069053305	98.23230431295631
	std. dev.	0.124	0.562	0.218	1.610
GA-MC	mean	100.17278603329092	116.04672600049534	131.97931686585904	148.83583247497340
	maximum	100.32941099753873	116.24504459299220	133.12709849005102	150.62029191477802
	minimum	100.00413481797922	115.71385676948675	131.15002913544846	146.38712996556176
	std. dev.	0.162	0.290	1.026	2.193
S-PSO	mean	96.99352358606079	112.10121312994039	127.61205531207980	145.34705410945317
	maximum	97.32595439980692	113.13443418136816	128.18282685503970	145.72366308886145
	minimum	96.33181487399530	110.47705550735766	127.22273409253024	145.06015884251654
	std. dev.	0.573	1.423	0.505	0.340
F-GA&PSO	mean	90.000000007731160	90.106262709541355	90.577498056046537	92.639861430215703
	maximum	90.000000018520836	90.136831048824774	90.828838630791068	93.070974028222338
	minimum	90.000000002096073	90.068452113976250	90.247195173366663	91.876795310525438
	std. dev.	$9.3471.657 \times 10^{-9}$	0.034	0.298	0.662
AGA	mean	**90**	**90.000000000000981**	**90.000000000085905**	**90.305509925165367**
	maximum	**90**	**90.000000000002899**	**90.000000000128864**	**90.916527562355597**
	minimum	**90**	**90.000000000000028**	**90.000000000007105**	**90.000000000225270**
	std. dev.	0	1.657×10^{-12}	6.833×10^{-11}	0.529

3. Adaptive genetic algorithm

In this section, a novel adaptive genetic algorithm is introduced and the details of the new crossover and mutation operators are described.

3.1. Global best-crossover

The global best-crossover (GB-crossover) operator chooses two chromosomes as parents. One of them is selected from the mating pool randomly and the other one is the best chromosome of the population that is the same $\vec{U}_{gbest}$ of Eq. (4) for the PSO approach [45]. Then, the selected parents create an offspring which is replaced with the selected chromosome. Consider P_{GBC} as the probability of GB-crossover and N as the population size, then $P_{GBC} \times N$ is the number of chromosomes selected to promote by the GB-crossover. Let $\vec{X}_i(t)$ and $\vec{X}_{gbest}$ represent the selected chromosome and the global best chromosome respectively. Then, the offspring is calculated by the following equation.

$$\vec{X}_i(t+1) = \vec{X}_{gbest} + \vec{r}_1\vec{X}_{gbest} - \vec{r}_2\vec{X}_i(\mathrm{t}) \tag{8}$$

where $\vec{r}_1$ and $\vec{r}_2 \in [0,1]^D$ are random vectors. After calculation of Eq. (8), the superior between $\vec{X}_i(t)$ and $\vec{X}_i(t+1)$ should be selected. In fact, the position of the produced chromosome by the traditional crossover operator, Eq. (1), is between the positions of its parents. But, in GB-crossover, the selection of the global best chromosome as one of the parents causes that the position of the obtained offspring tends toward the best chromosome and it increases the convergence speed.

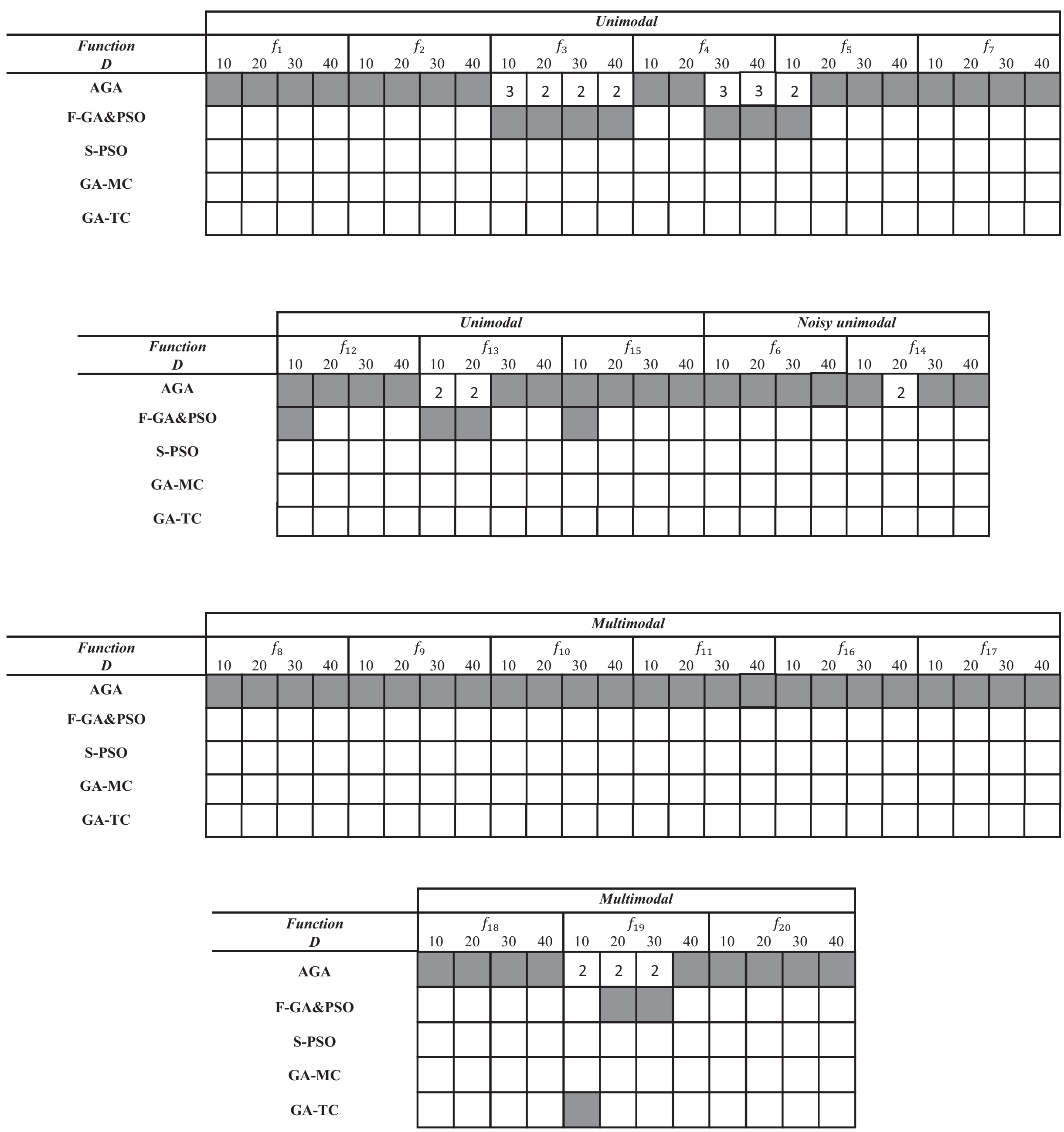


Fig. 4. Indicators of the best test algorithm in the experiments: The cell with grey color represents that the corresponding algorithm outperforms the others for a particular test function at a particular dimension.

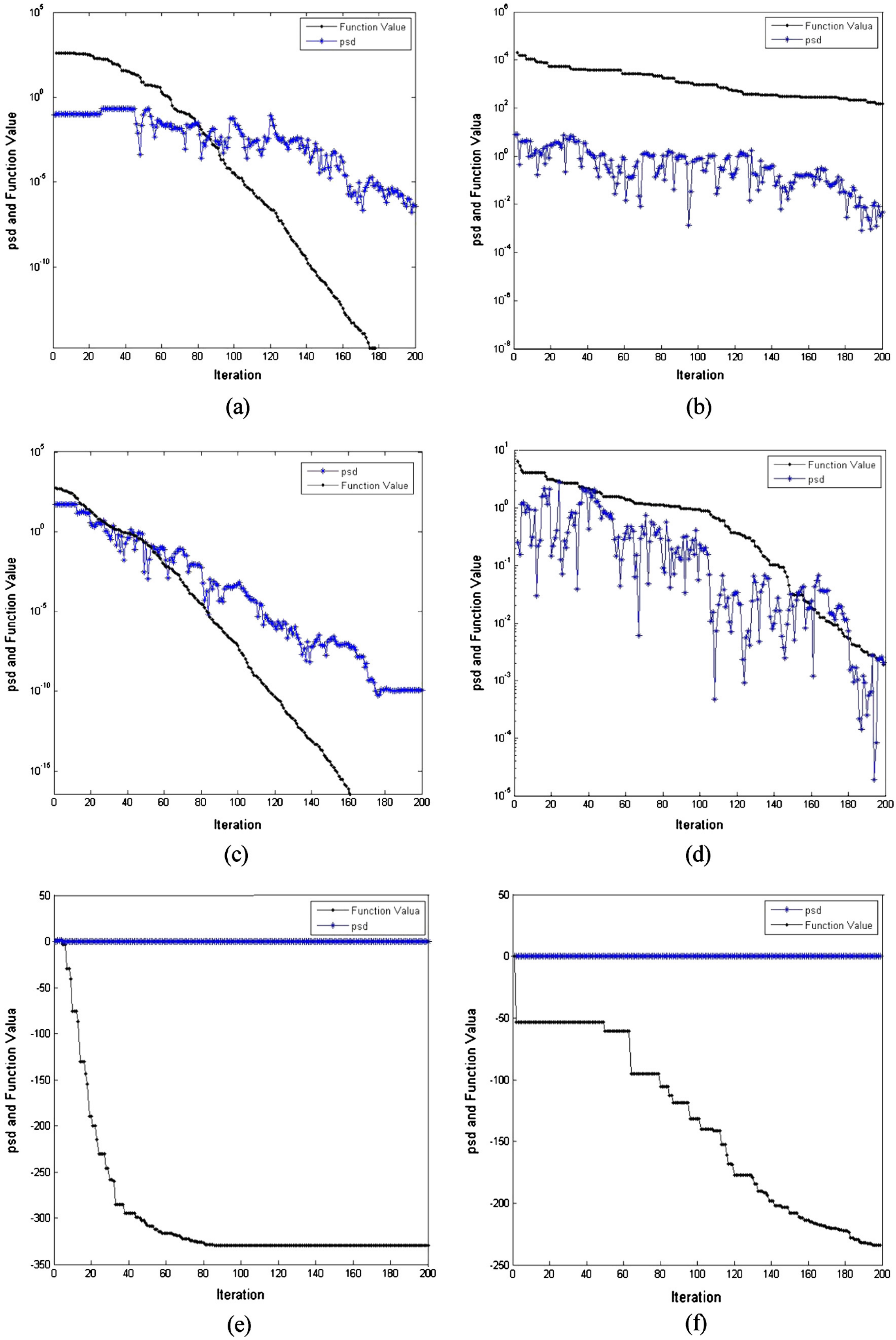


Fig. 5. The mean *psd* and the mean function value with dimension 30 during 200 iterations; (a, b) f_8, (c, d) f_{10}, (e, f) f_{17}, and (g, h) f_{19}; (a, c, e, g) AGA, and (b, d, f, h) F-GA&PSO.

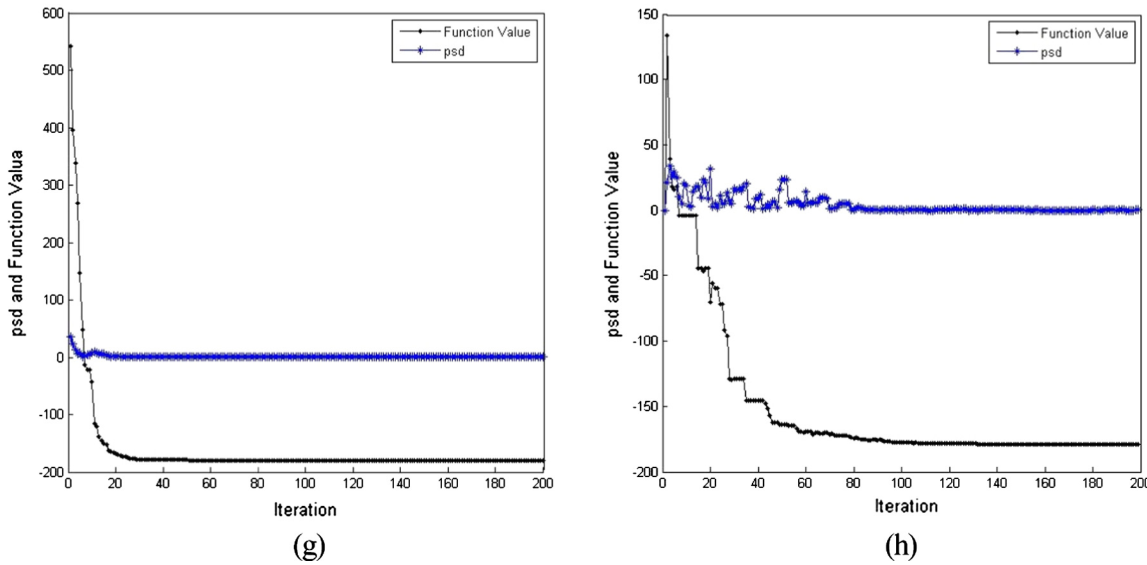


Fig. 5 (*continued*)

3.2. Quasi sliding surface-mutation

The quasi sliding surface-mutation (QSS-mutation) operator intelligently changes the value of a number of chromosomes in the population based on the SMC concepts. This number is determined by $P_{QSSm} \times N$, where P_{QSSm} and N are the probability of QSS-mutation and population size respectively. Let $\vec{X}_i(t)$ is a randomly selected chromosome, then the QSS-mutation is defined as bellow.

$$\vec{X}_i(t+1) = \vec{X}_i(t) + (\vec{a} \times \mu) \tag{9}$$

where $\vec{X}_i(0)$ is the initial position and equal to $\vec{X}_i(1)$, $\vec{a} \in [0,1]^D$ is a random vector, and the adaptation factor μ will be calculated by the following formulation.

$$\mu = 10^{\left(\frac{-1}{\sqrt{|s|}}\right)} \tag{10}$$

Which

$$s = \dot{e} + e; e = f(\vec{X}_i(t)); \dot{e} = \frac{f(\vec{X}_i(t)) - f(\vec{X}_i(t-1))}{t-(t-1)}$$
$$= f(\vec{X}_i(t)) - f(\vec{X}_i(t-1)) \tag{11}$$

where $f(\vec{X}_i(t))$ is the fitness value of $\vec{X}_i(t)$ and t is the iteration number.

It is observable from Eqs. (10) and (11) that if $f(\vec{X}_i(t))$ or $f(\vec{X}_i(t)) - f(\vec{X}_i(t-1))$ are large then the adaptation factor would be also large. However, if those are small then the adaptation factor would be small. Hence, QSS-mutation changes the chromosomes with respect to its fitness value. Figs. 1 and 2 illustrate this subject for both unimodal and multimodal test functions; i. e. Sphere and Griewank (Table 1).

3.3. The configuration of the proposed algorithm

The details of the adaptive genetic algorithm are described in the following. First, the initial population is randomly generated. After evaluation of the fitness values of all members, $\vec{X}_{gbest}$ would be determined. Then, according to the values of probabilities (P_{GBc} and P_{QSSm}), some chromosomes would be randomly selected to be changed by either GB-crossover or QSS-mutation operators. This cycle must be repeated until the user-defined stopping criterion is satisfied. The pseudo code of this algorithm is shown in Fig. 3.

4. Experimental results on the mathematical test functions

In this section, benchmark functions with experimental tests are carried out to validate the proposed AGA in comparison with three other optimization algorithms.

4.1. Mathematical test functions

Twenty well-known general and shifted benchmark problems with some their essential information are summarized in Tables 1 and 2 [56,57] including eleven unimodal test functions and nine multimodal designed with a considerable amount of local minima that should be minimized. These general and shifted test functions are used to evaluate the performance of the AGA, because they are dimension-wise scalable.

4.2. Algorithms for comparison

In order to validate the performance of the proposed genetic algorithm, we compare the optimal fitness values found by the AGA with four other evolutionary algorithms. These algorithms include GA with Traditional Crossover (GA-TC) [44], GA with Multiple Crossover (GA-MC) [58], Standard PSO (S-PSO) [45], and Fuzzy GA and PSO (F-GA&PSO) [14].

4.3. Settings for comparison

The population size is set at 100 for the AGA, GA-TC, GA-MC, and F-GA&PSO. For standard PSO, the swarm size is considered as 100. To provide a fair comparison among the test algorithms, the maximum number of iterations for all algorithms and functions is fixed at 400. All test functions are tested with dimensions 10, 20, 30, and 40. The performances of the algorithms on each test function are evaluated based on statistics data obtained from 30 independent runs. A list of other necessary parameters to run the algorithms for comparison is stated in Table 3.

4.4. Simulation results

At first, Tables 4–23 show the performance in terms of accuracy (quality of the averaged optimal fitness) of the AGA on general and shifted f_2 test functions in comparison with GA-TC, GA-MC, S-PSO, PSOMS, and F-GA&PSO. They list the average, worst, best and the standard deviation of the optimal fitness for 30 trials. The bold values indicate that the corresponding algorithm is the best algorithm among others on a particular test function at a particular dimension for mean, maximum and minimum values. It is observable that the AGA achieves the highest accuracy on functions f_1, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}, f_{15}, f_{16}, f_{17}, f_{18}, f_{20} at all dimensions. Furthermore,

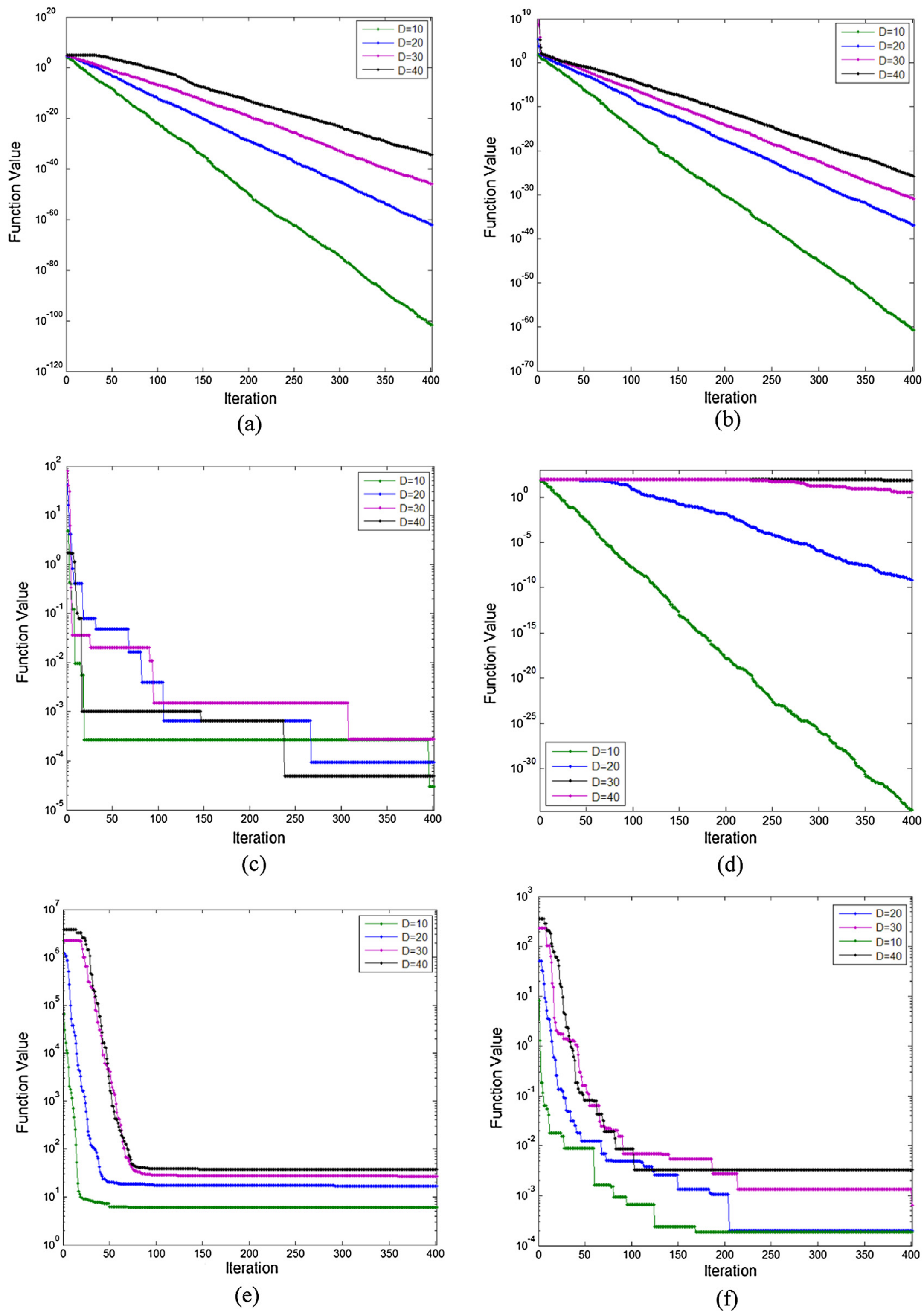
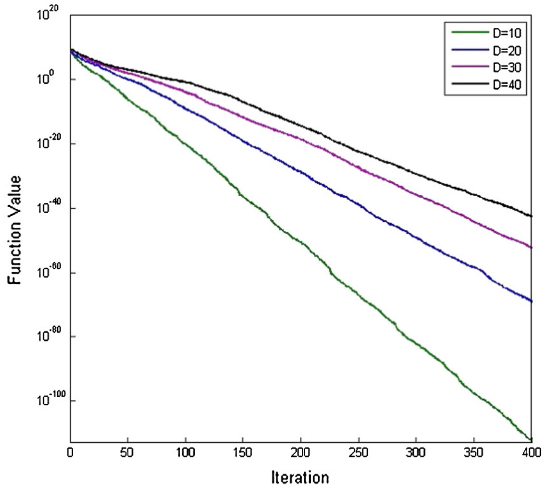
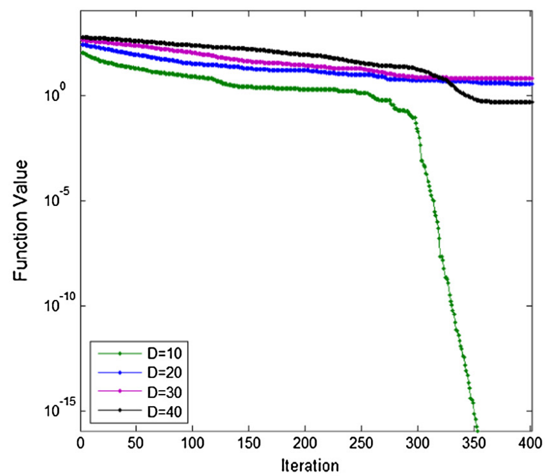


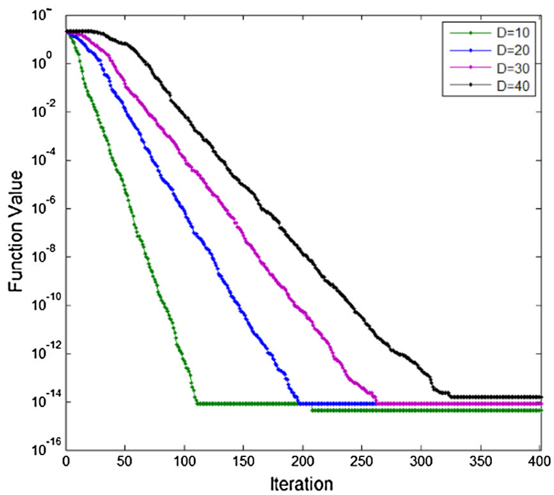
Fig. 6. The evolutionary traces of the AGA on the test functions; (a) f_1, (b) f_2, (c) f_3, (d) f_4, (e) f_5, (f) f_6, (g) f_7, (h) f_8, (i) f_9, (j) f_{10}, and (k) f_{11} for different dimensions; D = 10, 20, 30, 40.



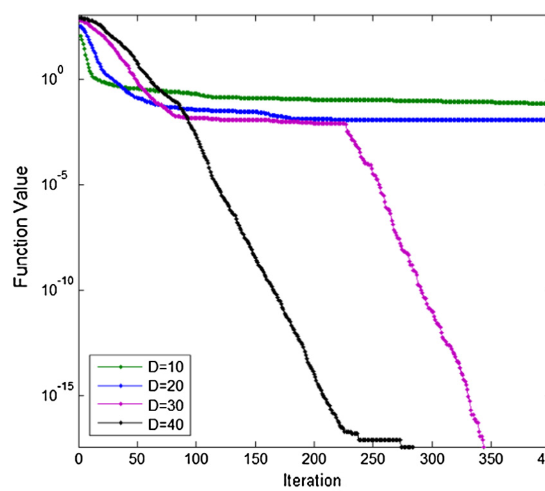
(g)



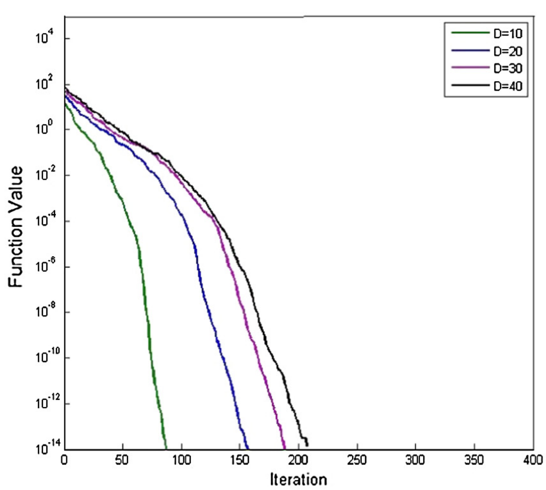
(h)



(i)



(j)



(k)

Fig. 6 (*continued*)

the AGA surpasses all other algorithms in solving functions f_4 for low dimensions, f_5, f_{13}, f_{14} and f_{19} for high dimensions. Fig. 4 shows a summary of the mean values presented in the previous Tables. The shaded cells in this figure indicate that the corresponding algorithm is the best on a particular test function at a particular dimension. When the AGA is not the best algorithm, the numbers inside the cells indicate its ranking on the relevant test function at the corresponding dimension. Simply, it can be realized that the AGA

Table 24
Oil demand, GDP, population, import and export data of Iran between 1981 and 2005 [60].

Year	Oil consumption(Mboe)[a]	Population (Thousand persons)	GDP (10^9 Iranian rials)	Import (Mboe)[a]	Export (Mboe)[a]
1981	176.2	40,825.6	170,281.2	21.4	339.8
1982	191.9	42,420	191,666.8	31.2	787.7
1983	234.4	44,076.6	212,876.5	61.7	764.3
1984	257.6	45,720.7	208,515.9	39.6	610.6
1985	264.6	47,541.4	212,686.3	65.3	652.3
1986	241	49,445	193,235.4	62	566.5
1987	262.8	50,650	191,312.4	72.9	635
1988	264.5	51,890	180,822.5	69.6	682.5
1989	280	53,167	191,502.6	50	765.5
1990	284.5	54,483	218,538.7	46.8	919.5
1991	306.1	55,837	245,036.4	48.4	964.8
1992	330.9	56,963	254,822.5	64.6	1023.3
1993	349.4	58,114	258,601.4	57.9	1058.6
1994	366.8	59,290	259,876.3	42.6	991
1995	344.8	59,151	267,534.2	28.5	1002.8
1996	370.9	60,055.5	283,806.6	29.1	880.4
1997	383.5	60,936.5	291,768.7	28.7	855.1
1998	402.8	61,830	300,139.6	24.6	854.6
1999	379.8	62,736	304,941.2	26.9	810.6
2000	382.7	63,663.9	320,068.9	39.6	955.9
2001	392.4	64,528.2	330,565	51.3	901.1
2002	406	65,540.2	355,554	67.3	928.4
2003	414.1	66,991.6	379,838	99.6×10^6	1109.6
2004	427.1	67,477.5	398,234.6	121.6	1184.9
2005	457.4	68,467.4	419,705	116.7	1182.3

[a] Mboe: Million barrel of oil equivalents. 1 barrels of oil equivalent (boe) = 6119 J (J).

Table 25
Maximum and minimum values of the effective parameters used for normalization.

	X_{min}	X_{max}
Population (Thousand persons)	40,825.6	62,736
GDP (billion Iranian rials)	170,281.2	304,941.2
Import (Mboe)	21.4	72.9
Export (Mboe)	339.8	1058.6
Oil consumption (Mboe)	176.2	402.8

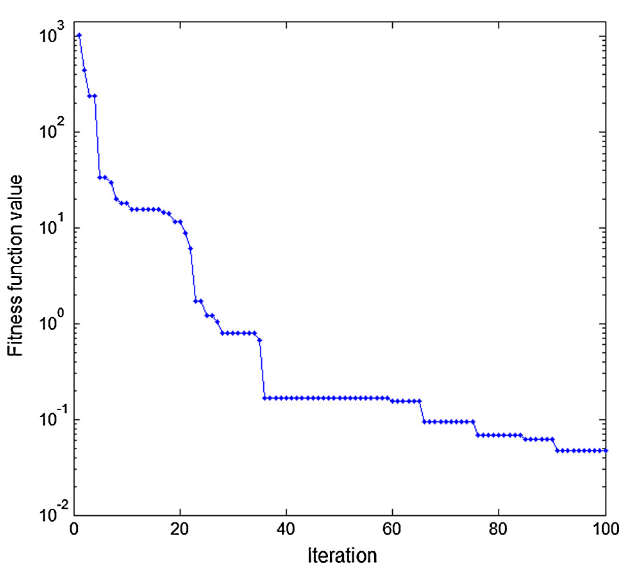


Fig. 7. The evolutionary trace of the oil demand estimation obtained by the AGA.

outperforms the other algorithms, because the ranking of the AGA is the first in 67 cases and is the second with very little differences in 10 other cases out of a total of 80 cases.

High convergence speed and escaping from local optimum point for converging to global optimum points are two important characteristics for an optimization algorithm. In this work, the GB-crossover operator increases the convergence speed and QSS-mutation prevents sticking to local optima. The comparisons in Tables 4–23 and Fig. 4 show that for both unimodal and multi-modal problems, the AGA offers the highest accuracy and the best performance on most test functions. Furthermore, the diversity of the population shows the ability of the algorithm to escape from local optimum points. Hence, in this paper, the presented criteria in Ref. [59] named popular standard deviation (*psd*) as the following equation is implemented to evaluate the diversity of the population.

$$psd = \sqrt{\left[\sum_{i=1}^{N}\sum_{j=1}^{D}\left(x_i^j - \bar{x}^j\right)^2\right]/(N-1)} \quad (12)$$

where N, D and $\bar{x}$ are the population size, dimension and the average position of all chromosomes, respectively. Fig. 5 shows the mean *psd* and function value of the AGA and F-GA&PSO on four multi-modal test functions for 30 runs. The larger *psd* rather than function value reflects the high diversity of the population, while small *psd* show that the population is converging to local optimum. This figure indicate that the mean *psd* of the AGA is larger for test functions f_8, f_{10}, and f_{17} in comparison with the mean *psd* of F-GA&PSO. Thus, the AGA escapes successfully from local optimum points and converges to global optimum point with considering Table 11, 13 and 20. Whereas, Fig. 5 and Table 22 show that F-GA&PSO is better than the AGA in test function f_{19} with dimension 30.

Fig. 6 describes the evolutionary traces of the general test functions (Table 1) and illustrates their mean values at every iteration for the AGA which are obtained at dimensions (D) 10, 20, 30, and 40 for 30 runs. This figure graphically presents the convergence characteristics of the evolutionary processes for solving the eleven different problems and proves that the AGA can successfully jump out from the local optimum on the test functions. It can be observed from results comparison that the used techniques and formulations for the AGA can significantly improve its performance for finding the global optimal point of an optimization problem.

Table 26
Optimum value of the weight coefficients for Eq. (14) by proposed model to predict the oil demand.

Weight coefficient	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}	w_{13}
GA-TC	+0.3473	+0.2678	+0.7194	−0.4092	+0.2459	+0.4576	+0.2910	−0.7735	−0.3282	−0.1299	−1.8898	−0.7538	−0.3527
GA-MC	−0.4165	+0.6202	−0.2116	+0.0082	−0.0421	+0.1347	−0.6074	+0.9782	+0.1154	−0.2422	−1.6632	−0.2890	−1.1382
S-PSO	−0.4083	+0.2024	+0.5414	−0.1990	+0.4762	−0.0282	+0.1645	+0.6819	−0.6102	−0.4041	−3.5071	−0.8345	−7.9933
F-GA&PSO	+0.2066	+0.6983	+0.2749	−0.1130	−0.1027	−0.4269	−0.2957	−0.3087	−0.0303	−0.3166	−0.7480	−0.8492	−4.3880
AGA	−0.2447	+0.5290	+0.2200	−0.0283	+0.0394	−0.0675	−0.0048	−0.2823	+0.7029	−1.3309	−0.0537	−0.0568	−0.1134

Table 27
Comparison of the AGA with actual data, estimated value with other algorithms by proposed model, and predicted values in Ref. [27].

Year	2000	2001	2002	2003	2004	2005	Average
Actual data	382.700	392.400	406.000	414.100	427.100	457.400	–
AGA with the proposed model	**383.519**	**392.254**	**406.071**	**419.565**	**427.069**	440.943	–
Relative error of AGA (%)	**0.214**	**−0.037**	**0.018**	**1.320**	**−0.007**	−3.598	**0.87**
F-GA&PSO with the proposed model	372.152	385.370	405.419	421.558	426.974	444.752	–
Relative error of F-GA&PSO (%)	−2.756	−1.791	−0.142	1.801	−0.029	−2.765	1.547
S-PSO with the proposed model	388.585	392.188	407.130	427.982	433.355	**453.694**	–
Relative error of S-PSO (%)	1.537	−0.053	0.278	3.352	1.464	**−0.810**	1.250
PSO-DEM$_{exponential}$	384.045	388.317	401.876	421.502	431.958	443.353	–
Relative error of PSO-DEM$_{exponential}$ (%)	0.351	−1.041	−1.016	1.787	1.137	−3.071	1.40
PSO-DEM$_{linear}$	386.773	389.436	404.594	425.571	436.778	452.985	–
Relative error of PSO-DEM$_{linear}$ (%)	1.064	−0.755	−0.346	2.770	2.266	−0.965	1.36
GA-MC with the proposed model	380.515	392.729	405.185	423.983	428.778	431.508	–
Relative error of GA-MC (%)	−0.570	0.084	−0.200	2.386	0.393	−5.660	1.549
GA-TC with the proposed model	391.925	388.236	4.09622	422.831	423.410	467.558	–
Relative error of GA-TC (%)	2.410	−1.060	0.892	2.108	−0.863	2.220	1.592
GA- DEM$_{exponential}$	392.103	394.697	413.502	433.202	448.191	469.126	–
Relative error of GA-DEM$_{exponential}$ (%)	2.457	0.585	1.848	4.613	4.938	2.564	2.83
GA-DEM$_{linear}$	393.349	390.998	403.335	426.912	437.095	452.484	–
Relative error of GA-DEM$_{linear}$ (%)	2.783	−0.357	−0.656	3.094	2.340	−1.075	1.72

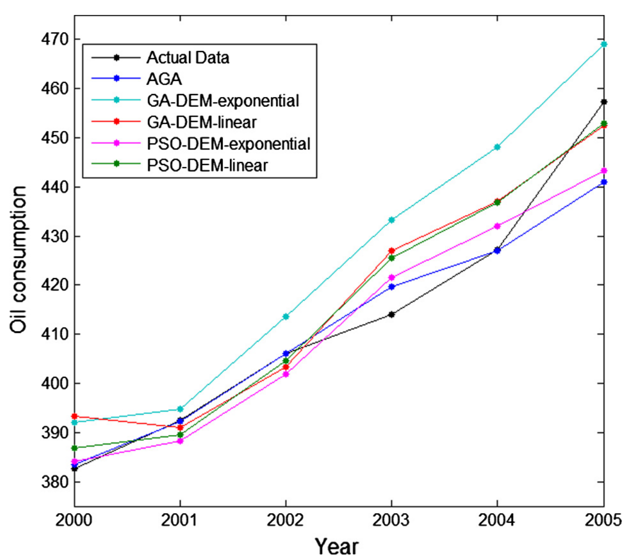


Fig. 8. Comparison between the actual and predicted values by AGA and Assareh et al. [27] for the oil consumption among years 2000 and 2005.

5. Application of the AGA on demand estimation of oil in Iran

5.1. Problem definition

In this section, in order to challenge the performance of the AGA for a real-world problem, it has been developed to estimate the future oil demand values based on population, Gross Domestic Product (GDP) and import and export data. For this purpose, the fitness function is considered as follows that should be minimized.

$$F(x) = \sum_{j=1}^{m} (E_{actual} - E_{predicted})^2 \tag{13}$$

where E_{actual} and $E_{predicted}$ are the actual and predicted oil demand, respectively, and m is the number of observation. The data related to the effective parameters (Iran's population, GDP, import, export and oil consumption) is taken from the energy balance annual report of the energy ministry of Iran in 2005 [60] and shown in Table 12. Furthermore, the prediction of the oil demand, based on the socio-economic indicators, is modeled by using an equation with a new form as follows.

$$\begin{aligned} E_{predicted} &= w_1 + w_2X_1 + w_3X_2 + w_4X_3 + w_5X_4 + w_6 \\ &\quad \times \exp\,(w_{10}X_1) + w_7 \exp\,(w_{11}X_2) + w_8 \\ &\quad \times \exp\,(w_{12}X_3) + w_9 \exp\,(w_{13}X_4) \end{aligned} \tag{14}$$

where X_1, X_2, X_3, and X_4 are the normalized value of the population, GDP, import and export of oil respectively, and w_i $(i = 1, 2, 3, \ldots 13)$ are the weight coefficient that should be found by the optimization algorithm. Moreover, the variables of this optimization problem are constrained as

$$\begin{aligned} &-1 \leqslant w_p \leqslant 1; p = 1, 2, \ldots, 9 \\ &w_q \leqslant 0; q = 10, 11, 12, 13 \end{aligned} \tag{15}$$

In the other words, the AGA is applied to find optimal values of weight parameters based on actual data for estimation of the oil consumption. In order to normalize the population, GDP, import, export and oil consumption, Eq. (16) is applied.

$$X_N = (X_R - X_{min})/(X_{max} - X_{min}) \tag{16}$$

where X_N, X_R, X_{min}, and X_{max} are the normalized value (N = 1, 2, 3, 4), the real value (Table 24), the minimum and maximum value (Table 25), respectively.

5.2. Estimation of oil demand by using the AGA

The following configurations are considered to perform the optimization process for the prediction of the oil demand. Population size: 20, Probability of GB-crossover: 0.9, Probability of SS-mutation: 0.1, Maximum number of iterations: 100.

Fig. 7 shows the evolutionary trace of the estimation of oil demand obtained via the AGA, and Table 26 gives the achieved weight coefficients in proposed model (Eq. (14)) at the end of this process for all algorithms. Table 27 and Fig. 8 illustrate the values of oil demand predicted by using the AGA, from 2000 to 2005. These values are compared with the actual data, estimated value with other algorithms by proposed model, and predicted values by Assareh et al. [27] (The bold values indicate the best prediction for each year). In fact, Assareh et al. utilized two algorithms (genetic algorithm and particle swarm optimization) and two models (linear and exponential) for the prediction of the oil demand. Table 27 depicts that the average relative error of the proposed model optimized by PSO and GA is smaller than linear and exponential models and the introduced model by this work is a proper model for the estimation of the oil demand. Furthermore, this table indicates that the predicted data by AGA based on the proposed model is in a very good agreement with the actual values in comparison with the obtained values by other models and algorithms.

6. Conclusion

In this work, a new adaptive genetic algorithm using two novel operators called GB-crossover and QSS-mutation is presented. In GB-crossover, the best chromosome from the entire population (Global Best Chromosome) is utilized as one of the parents which originated from particle swarm optimization**.** This operator creates one offspring from two parents in the mating pool. QSS-mutation changes the value of the selected chromosomes intelligently based on the sliding mode concept organized from sliding mode control. To consider the performance of the AGA, it is applied for eleven unimodal and nine multimodal test functions. The depicted results are compared with the obtained results from several well-known and recent optimization algorithms. The comparisons indicate that the AGA offers the highest accuracy and the best performance on most unimodal and multimodal test functions. Also, the ability of the AGA for avoiding being trapped into local optimum points for multimodal functions is shown. Moreover, the AGA is successfully used to estimate the oil demand of Iran based on the socio-economic conditions. Validations of the proposed model show that it is in a good agreement with regard the observed results and is a satisfactory tool for successful oil demand forecasting. So, the obtained results prove that the AGA is verily an effective and successful algorithm for optimization of both constrained and unconstrained mathematical test functions and real-world problems.

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