Ain Shams Engineering Journal (2014) 5, 1233–1239

Ain Shams University

Ain Shams Engineering Journal

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MECHANICAL ENGINEERING

MHD stagnation point flow of Carreau fluid toward a permeable shrinking sheet: Dual solutions

N.S. Akbar [a], S. Nadeem [b], Rizwan Ul Haq [b,d,*], Shiwei Ye [c]

[a] *DBS&H, CEME, National University of Sciences and Technology, Islamabad, Pakistan*
[b] *Department of Mathematics, Quaid-i-Azam University, Islamabad 44000, Pakistan*
[c] *School of Mathematical Sciences, Peking University, Beijing 100871, PR China*
[d] *Mechanical and Materials Engineering, Spencer Engineering Building, Room 3055, University of Western Ontario, London, Ontario, Canada*

Received 24 November 2013; revised 22 April 2014; accepted 25 May 2014
Available online 26 June 2014

KEYWORDS

Stagnation flow;
Shrinking sheet;
Carreau fluid;
MHD;
Dual solutions;
Shooting method

Abstract Present analysis is carried out to study the two-dimensional stagnation-point flow of an in-compressible Carreau fluid toward a shrinking surface. The formulation of the Carreau fluid model has been developed first time for boundary layer problem of shrinking sheet and the governing partial differential equations are rehabilitated into ordinary differential equations using similarity transformations. The simplified nonlinear boundary value problem is solved by Runge-Kutta method after converting into the system of initial value problem using shooting method. Dual solutions are obtained graphically and results are shown for various parameters involved in the flow equations. Numerical values of skin friction coefficients are also computed.

1. Introduction

A stagnation point occurs whenever a flow impinges on a solid object. The pioneer work for a two dimensional stagnation point flow was done by Hiemenz [1]. He discussed the 2-dimensional flow of a fluid near a stagnation point. He exposed that the Navier–Stokes equations governing the flow can be reduced to an ordinary differential equation of third order using similarity transformation. The study of boundary layer flow over a stretching sheet is a topic of great attention due to a variety of applications in designing cooling system which includes liquid metals, MHD generators, accelerators, pumps and flow meters. At very start Sakiadis [2] examined the laminar boundary-layer behavior on a moving continuous flat surface and he used similarity transformations to simplified boundary-layer equations and then solved numerically. Crane [3] extended the work of Sakiadis [2] for linear and exponential stretching. The steady two-dimensional stagnation point flow of an incompressible micropolar fluid over a stretching sheet when the sheet is stretched in its own plane with a velocity proportional to the distance from the stagnation point has been

* Corresponding author at: Department of Mathematics, Quaid-i-Azam University, Islamabad 44000, Pakistan. Tel.: +92 51 90642182.
E-mail address: ideal_riz@hotmail.com (R.U. Haq).

Peer review under responsibility of Ain Shams University.

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http://dx.doi.org/10.1016/j.asej.2014.05.006

studied by Nazar et al. [4]. They solved the resulting non-linear ordinary coupled equations numerically using the Keller-box method. Two-dimensional stagnation-point flow of viscoelastic fluids is studied theoretically by Sadeghy et al. [5]. They assume that the fluid obeys the upper-convected Maxwell (UCM) model. Boundary-layer theory is used to simplify the equations of motion which are further reduced to a single non-linear third-order ODE using the concept of stream function coupled with the technique of the similarity solution. The obtained governing equation was solved using Chebyshev pseudo-spectral collocation-point method. The steady MHD mixed convection flow of a viscoelastic fluid in the vicinity of two-dimensional stagnation point with magnetic field has been investigated by Kumari and Nath [6]. They used upper-convected Maxwell (UCM) fluid as the proposed model. Boundary layer theory is used to simplify the equations of motion, induced magnetic field and energy which results in three coupled non-linear ordinary differential. These equations have been solved by using finite difference method. Ishak et al. [7] analyzed heat transfer over a stretching surface with uniform or variable heat flux in micropolar fluid. In this contest Nadeem and Hussain [8] discussed HAM solutions for boundary layer flow in the region of the stagnation point toward a stretching sheet. Nadeem et al. [9,10] developed the two and three dimensional boundary layer flow over stretching sheet for both Newtonian and non-Newtonian fluid. Analysis for MHD flow of a Maxwell fluid past a vertical stretching sheet in the presence of thermophoresis and chemical reaction was examined by Noor [11]. Recently Akbar et al [12] present the investigation on Magnetohydrodynamic boundary layer flow of tangent hyperbolic fluid toward a stretching sheet. In another article Nadeem and Haq [13] coated the effect of thermal radiation for MHD boundary layer flow of a nanofluid over a stretching sheet with convective boundary conditions. Stability of dual solutions in stagnation-point flow and heat transfer over a porous shrinking sheet with thermal radiation is given by Mahapatra and Nandy [14]. Later on many problems have been discussed by few authors [15–20].

In the present article model of Carreau fluid flow on a stretching sheet has been constructed along with the magnetic effects. To the best of author's knowledge no investigation has been done before in which Carreau fluid is model for shrinking/stretching sheet problems. The main objective of the article is to discuss the dual solution for MHD flow of Carreau fluid analysis on a stretching sheet. The formulation of the paper is organized as follows. The problem formulation is given in section two. The numerical solutions graphically with physical interpretation are presented in section three. Section four contains the conclusions of the current development.

2. Mathematical formulation

We discussed a two dimensional stagnation point flow of an incompressible Carreau fluid over a wall coinciding with plane $y = 0$, the flow is being confined to $y > 0$. The flow is generated due to the linear stretching. Extra stress tensor for Carreau fluid is [15],

$$\bar{\tau}_{ij} = \eta_o\left[1 + \frac{(n-1)}{2}(\Gamma\bar{\dot{\gamma}})^2\right]\bar{\dot{\gamma}}_{ij} \tag{1}$$

in which $\bar{\tau}_{ij}$ is the extra stress tensor, η_o is the zero shear rate viscosity, Γ is the time constant, n is the power law index and $\bar{\dot{\gamma}}$ is defined as

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i\sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\Pi}. \tag{2}$$

Here Π is the second invariant strain tensor. Flow equations for Carreau fluid model after applying the boundary layer approximations can be defined as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \nu\frac{3(n-1)\Gamma^2}{2}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho}(u_e - u) + u_e\frac{\partial u_e}{\partial x} \tag{4}$$

Here u and v are velocity components along x and y direction, respectively. Where ν is kinematic viscosity, σ is the electrical conductivity, ρ is the density. It is noticed that for power law index ($n = 1$) our problem reduced to the case of Newtonian fluid while for $n > 1$ phenomena remains for non-Newtonian fluid. The corresponding boundary conditions are

$$\begin{aligned} &u = u_w(x) = ax, \quad v = v_w(x), \quad at \quad y = 0, \\ &u \to u_e(x) = bx, \quad as \qquad\qquad\quad y \to \infty, \end{aligned} \tag{5}$$

in which $b > 0$ is constant, we assume that $u_w(x) = ax$ and $u_e(x) = bx$ are the velocities near and away from the wall respectively. Introducing the following similarity transformations

$$\eta = \sqrt{\frac{b}{\nu}}y, \ \psi = \sqrt{b\nu}xf(\eta), \tag{6}$$

where η is the similarity variable and ψ is the Stream function defined in the usual notation as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfy the equation of continuity define in Eq. (3). By using above similarity transformation defined in Eq. (5) on Eqs. (2)–(4), we get:

$$f''' - (f')^2 + ff'' + 1 + \frac{3(n-1)We^2}{2}f'''(f'')^2 + M^2(1 - f') = 0 \tag{7}$$

$$f = s, f' = \lambda, at \quad \eta = 0 \tag{8}$$

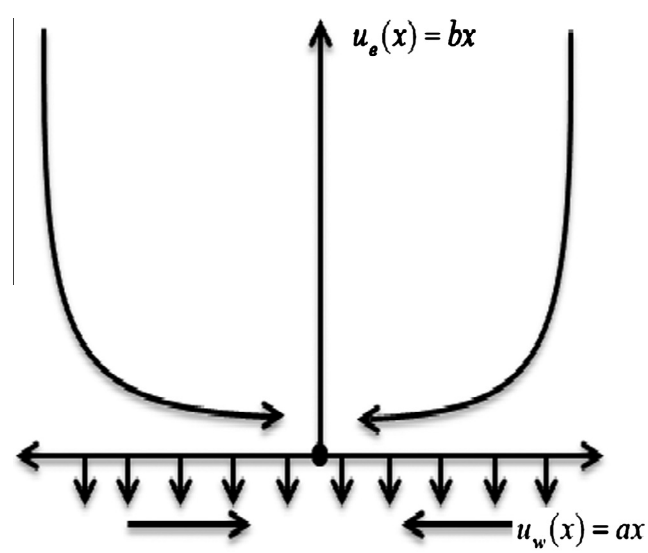

Figure 1 Geometry of the problem.

$$f' \to 1, at \quad \eta \to \infty, \tag{9}$$

where $s = -v_w/(av)^{1/2}$, $s > 0$ (i.e $v_w < 0$) corresponding to suction and $s < 0$ (i.e $v_w > 0$) corresponding to blowing case. Here $\lambda = a/b$ is the stretching/shrinking parameter, $We^2 = \frac{b^3x^2\Gamma^2}{v}$ is the Weissenberg number, $M^2 = \frac{\sigma B_0^2}{\rho b}$ is the magnetic parameter. After using boundary layer approximations wall shear stress τ_w is given by

$$\tau_w = \frac{\partial u}{\partial x} + \frac{(n-1)\Gamma^2}{2}\left(\frac{\partial u}{\partial y}\right)^3 \tag{10}$$

The coefficient of skin friction is defined as

$$c_f = \frac{\tau_w}{\rho u_w^2} \tag{11}$$

In dimensionless form skin friction is defined as

$$\sqrt{\mathrm{Re}}c_f = \left[f''(\eta) + \frac{(n-1)We^2}{2}(f''(\eta))^3\right]_{\eta=0} \tag{12}$$

3. Numerical method for solution

The nonlinear differential Eq. (7) along with the boundary conditions (8) and (9) is solved numerically with the help of shooting method after converting them into initial value problem. To handle the condition at infinity we consider the suitable value of $\eta \to \infty$, say η_∞. We set the following first-order system:

$$f' = p \tag{13}$$

$$f'' = p' = q \tag{14}$$

$$f''' = q'' = \frac{(p^2 - fq - M^2(1-p) - 1)}{1 + \left(\frac{3(n-1)We^2q^2}{2}\right)} \tag{15}$$

along with the boundary conditions are

$$f(0) = s, f'(0) = p(0) = \lambda, \tag{16}$$

To solve (15) with (16) as an IVP, the values for $q(0) = f''(0)$ are needed but no such values are given prior to the computation. The initial guess values of $f''(0)$ are chosen and fourth order Runge-Kutta method is applied to obtain a solution. We compared the calculated values of $f''(\eta)$ at the far field boundary condition $\eta_\infty(=20)$ with the given boundary condition $f''(\eta) \to \infty$ and the values of $f''(0)$ are adjusted using Secant method for better approximation. The step-size is taken as $\Delta\eta = 0.01$ and accuracy to the fifth decimal place as the criterion of convergence. It is important to note that the dual solutions are obtained by setting two different initial guesses for the values of $f''(0)$, where both profiles (first and second solutions) satisfy the far field boundary conditions (8 and 9) asymptotically but with different shape. (see Fig. 1)

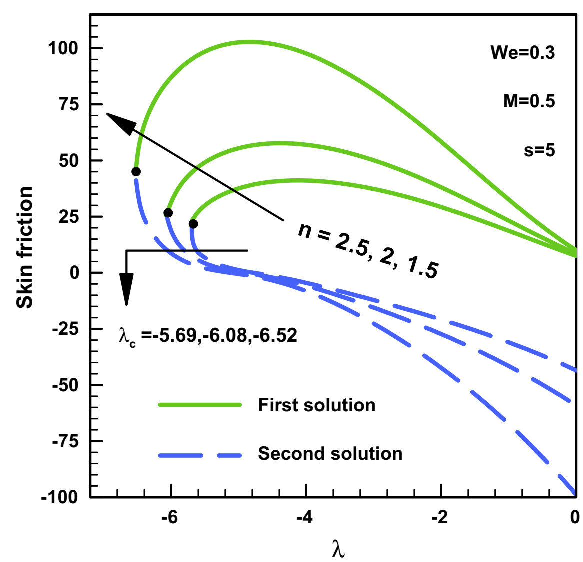

Figure 3 Variation of skin friction coefficient with λ and n.

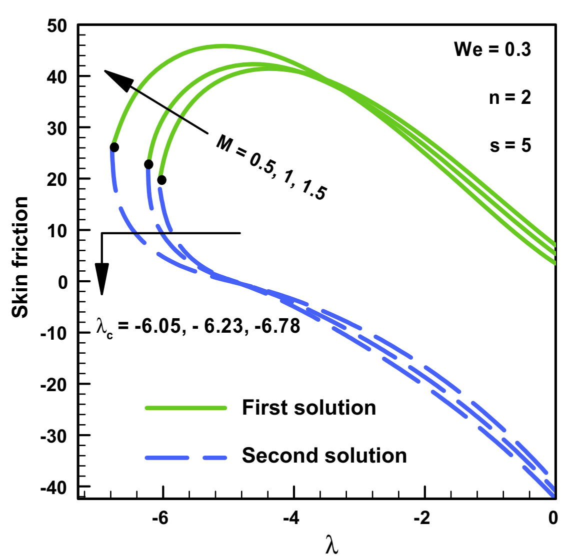

Figure 2 Variation of skin friction coefficient with λ and M.

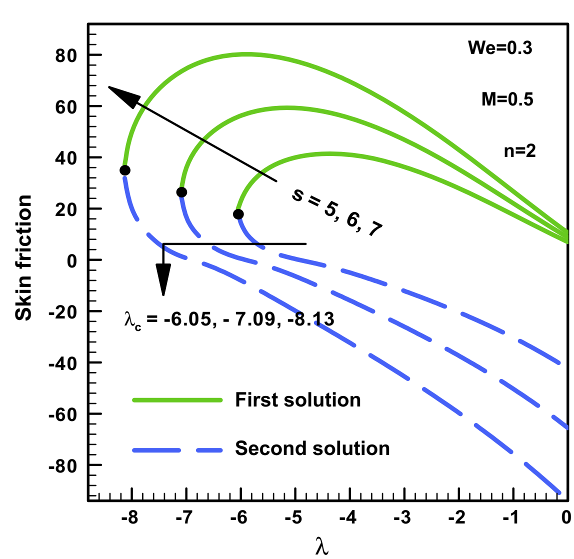

Figure 4 Variation of skin friction coefficient with λ and s.

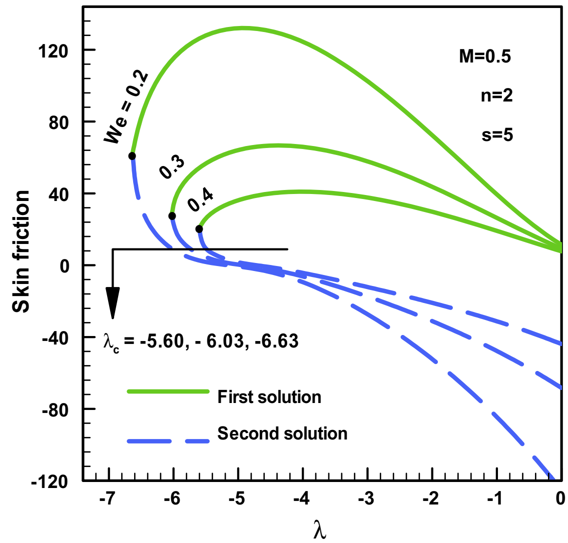

Figure 5 Variation of skin friction coefficient with λ and We.

4. Results and discussions

In the present section we will discuss the behavior of skin friction coefficient and velocity profile for various values of emerging parameter such as power law index n Weissenberg number We, Hartmann number M and suction/blowing parameter s. Behavior of skin friction coefficient against different values of physical parameter is presented in Figs. 2–5. In fact, when we consider the stagnation point flow in the case of shrinking sheet then ultimately two solution must be appear name as dual solution for the present region where the sheet being shrunk. In Fig. 2, two solutions being produced for skin friction coefficient against higher values of M. Moreover, critical values of λ, when solution being divided into two branches (lower and upper branch solutions), are $\lambda_c \approx -6.05, -7.09, -8.13$. It is also found from Fig. 2, for increasing values of M magnitude of skin friction coefficient also increases for both branches while rest of the parameters are kept fixed. Fig. 3 shows the variation of skin friction coefficient with λ for power law index n when $We = 0.3$, $M = 0.5$, $s = 5$. It is seen in Fig. 3 that there are

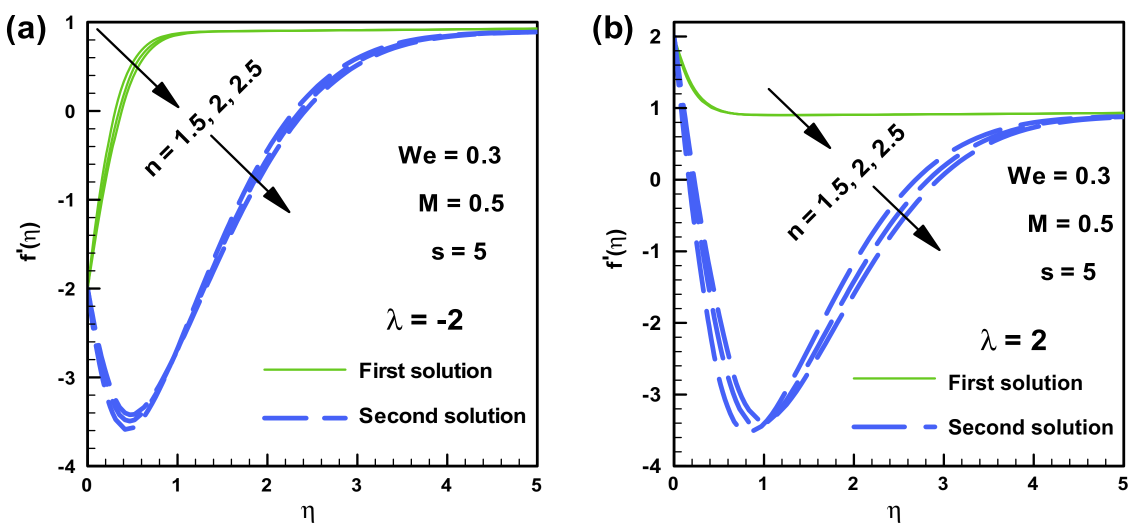

Figure 6 Effects of power law index n on velocity profile.

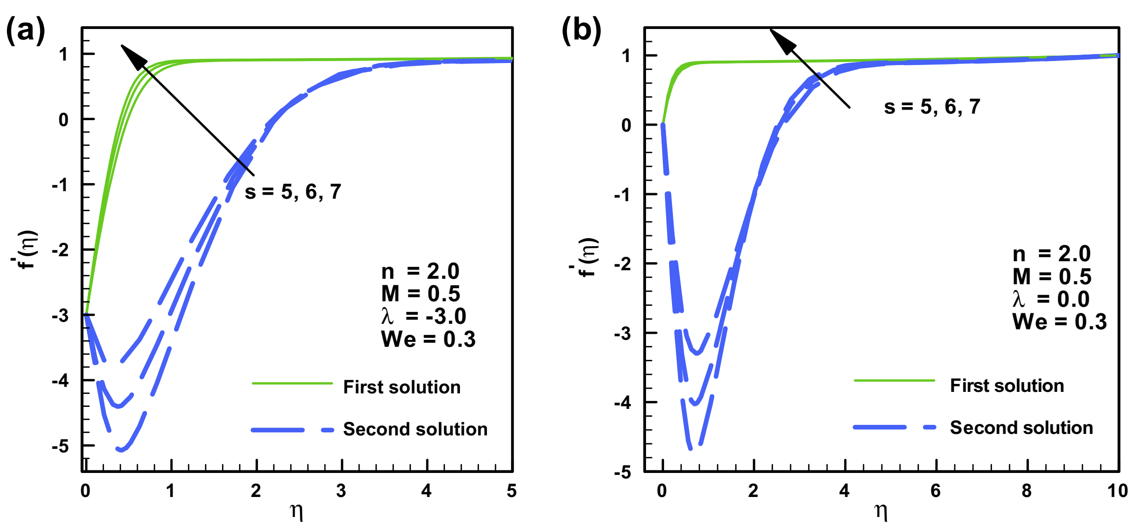

Figure 7 Effects of shrinking parameter s on velocity profile.

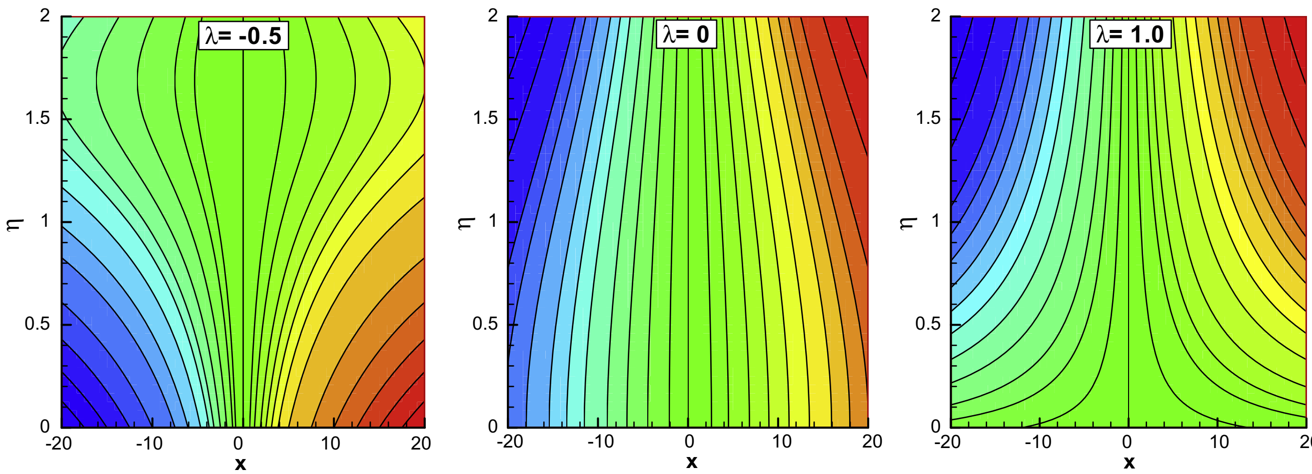

Figure 8 Variation of stream lines for various values of λ.

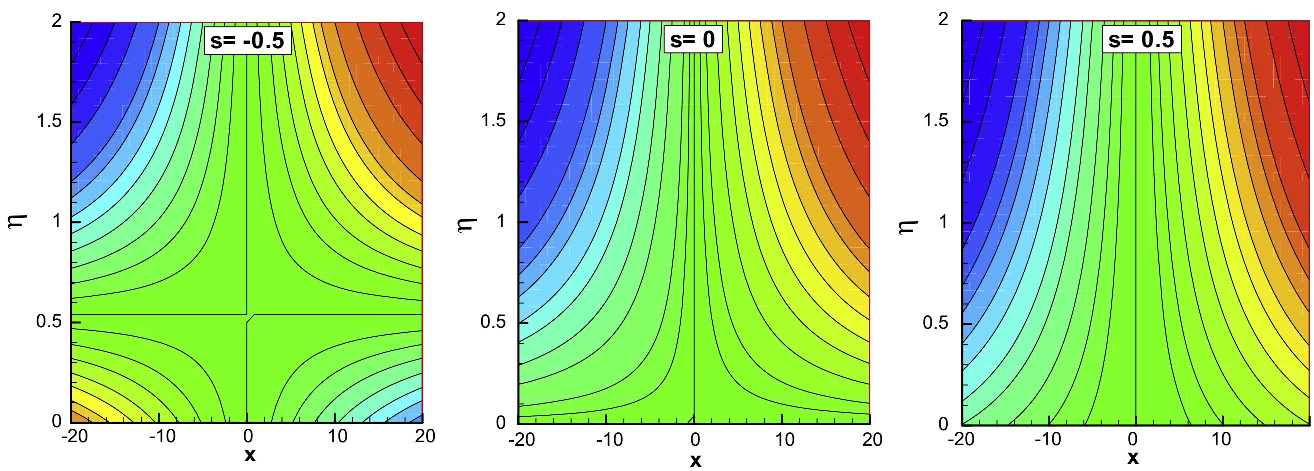

Figure 9 Variation of stream lines for various values of s.

Table 1 Comparison of the values of coefficient of skin friction (with $M = 0$) for shrinking sheet with different values of λ.

$\lambda \downarrow$	Present results $n = 1$, $We = 0$	Mahapatra and Nandy [14]	Wang [17]	Lok et al. [16]
0.0	1.2326	1.2326	1.2326	–
0.1	1.1466	1.1466	1.1466	–
0.2	1.0511	1.0511	1.0511	–
0.5	0.7133	0.7133	0.7133	0.7133
1.0	0	0	0	–
2.0	−1.8873	−1.8873	−1.8873	−1.8873
5.0	−10.2647	−10.2647	−10.2647	−10.2648

dual solutions for $\lambda_c < \lambda < 0$, where the critical values of λ are $\lambda_c \approx -5.69, -6.08, -6.5$. From Fig. 3, both the branches present increasing behavior for increasing values of power law index n. Similar pattern is observed when we compare Fig. 4 with Fig. 3 for suction parameter$s$. On the other hand, it is seen from Fig. 5 for increasing values of Weissenberg number it decreases the skin friction coefficient. Critical values of λ in Fig. 5 are $\lambda_c \approx -5.69, -6.08, -6.63$ when $n = 2, M = 0.5, s = 5$.

The variations of the velocity profile $f'(\eta)$ for emerging parameters are presented in Fig. 6 and Fig. 7. In Fig. 6(a), it depicts that when we consider the value of $\lambda = -2$ (shrinking case), velocity profile for first and second solutions present same sort of decreasing behavior with an increase of power law index n. On the other hand, Fig. 6(b), variation of velocity profile for higher values of n, illustrates the decreasing behavior when we consider the value of $\lambda = 2$ (stretching case) for both first and second solution. Fig. 6, also illustrates that in each case boundary layer thickness decreases with an increase in power law index n. In Fig. 7, it is observed that for increasing values of suction/blowing parameter s, for both cases when $\lambda = -3$ (shrinking case) and in the absence of shrinking parameter

Table 2 Comparison of the values of coefficient of skin friction (with $M = 0$) for shrinking sheet with different values of λ.

$\lambda\downarrow$	Present results for ($n = 1$, $We = 0$)		Mahapatra and Nandy [14]	
	First solution	Second solution	First solution	Second solution
−0.25	1.4022	–	1.4022	–
−0.50	1.4956	–	1.14957	–
−0.75	1.4893	–	1.4893	–
−1.0	1.3288	0	1.3288	0
−1.10	1.1867	0.0492	1.1867	0.0492
−1.15	1.0822	0.1167	1.0822	0.1167
−1.20	0.9325	0.2336	0.9324	0.2336
−1.246	0.5543	0.5542	0.5844	0.5542

($\lambda = 0$), velocity profile $f'(\eta)$ for first and second solution shows increasing behavior. Moreover, we have observed that boundary layer thickness decreases in both first and second solution in Fig. 7. Variations of stream lines behavior are plotted in Fig. 8 and 9, for various values of λ and s, respectively. It can be observed from Fig 8(a) and (b), that near the wall fluid flow behavior for various values of stretching/shrinking parameter λ, behavior of stream lines switches accordingly to the shrinking case ($\lambda < 0$) and stretching case ($\lambda > 0$). Moreover, we can observe that in case of simple flat plate ($\lambda = 0$), there is no variation in the behavior of stream lines that is obvious that flow remain normal to the wall. Fig. 9, represent the behavior of stream lines for each case when $s > 0$ (suction case), $s < 0$ (blowing case) and for $s = 0$ (absence of suction/blowing), while rest of parameters are kept fixed. We can see from Fig. 9(a) after blowing fluid flows along with the stretching wall and behavior of stream lines remains symmetric along both x and y-axis, while in case of ($s \geqslant 0$) behavior of the stream lines remain symmetric only along y-axis.

Table 1, Present the excellent correlation between previous literature [14,16,17] and the present study for skin friction coefficient in the absence of both non-Newtonian fluid parameter and MHD while for the value of $n = 1$. It can be seen from Table 1, with an increasing values of λ variation of skin friction coefficient decreases. Table 2, shows the comparison of dual solution with [14]. It is found that in the absence of $n = 1$ and ($\lambda < -1$) present the dual solution for skin friction coefficient.

5. Conclusion

The boundary layer stagnation-point flow of Carreau fluid model toward a Shrinking sheet with MHD is investigated. Different solution behavior with multiple solution branches has been found and compared with the existing literature.

1. It is seen that with the elastic parameter increases correspond to the curve for local skin friction decreases gradually.
2. It is observed that for decreasing values of elastic parameter We both the branches of skin friction coefficient present same sort of increasing behavior.
3. We get the critical values of λ i.e. $\lambda_c \approx -5.06, -6.03$ and -6.63 where curves of skin friction being divided into two branches (that is lower and upper branch).
4. It is observed that in both the cases lower and upper branch give same increasing behavior for higher values of M.
5. It is observed throughout trend of the graphs of skin friction coefficient against each parameter show increasing behavior.
6. When we increase the non-Newtonian fluid parameter We correspond to resist the motion of the fluid.

Acknowledgment

The 3rd author (Rizwan Ul haq) is thankful to Higher Education Commission Pakistan for research funding through International Research Support Initiative Program (IRSIP).

References

[1] Hiemenz K. Die Grenzschicht an einem in den gleichförmigen Flüessigkeitsstrom eingetauchten geraden Kreiszylinder. Ding Polym J 1911;321.
[2] Sakiadis BC. Boundary layer behavior on continuous solid flat surfaces. J AICHE 1961;7:26–8.
[3] Crane L. Flow past a stretching plate. Zeit Für Angew Math und Phy 1970;21:645–7.
[4] Nazar R, Amin N, Filip D, Pop I. Unsteady boundary layer flow in the region of the stagnation-point on a stretching sheet. Int J Eng Sci 2004;42:1241–53.
[5] Sadeghy K, Hajibeygib H, Taghavia SM. Stagnation-point flowof upper-convected Maxwell fluids. Int J Non-Linear Mech 2006;41:1242–7.
[6] Kumari) M, Nath G. Steady mixed convection stagnation-point flow of upper convected Maxwell fluids with magnetic field. Int J Non-Linear Mech 2009;44:1048–55.
[7] Ishak A, Nazar R, Pop I. Heat transfer over a stretching surface with variable heat flux in micropolar fluids. Phys Lett A 2008;372(5):559–61.
[8] Nadeem S, Hussain A. HAM solutions for boundary layer flow in the region of the stagnation point towards a stretching sheet. Commun Nonlin Sci Numer Sim 2010;15:475–81.
[9] Nadeem S, Haq RU, Lee C. MHD flow of a Casson fluid over an exponentially shrinking sheet. Sci Iranica 2012;19(6):1550–3.
[10] Nadeem S, Haq RU, Akbar NS, Khan ZH. MHD three-dimensional Casson fluid flow past a porous linearly stretching sheet. Alexandria Eng J 2013;52(4):577–82.
[11] Noor NFM. Analysis for MHD flow of a Maxwell fluid past a vertical stretching sheet in the presence of thermophoresis and chemical reaction. World Acad Sci Eng Technol 2012;64:1019–23.
[12] Akbar Ns, Nadeem S, Haq RU, Khan ZH. Numerical solutions of Magnetohydrodynamic boundary layer flow of tangent hyperbolic fluid towards a stretching sheet. Indian J Phys 2013;87(11):1121–4.
[13] Nadeem S, Haq RU. Effect of thermal radiation for megnetohydrodynamic boundary layer flow of a nanofluid past a stretching sheet with convective boundary conditions. J Comput Theor Nanosci 2014;11(1):32–40.
[14] Mahapatra TR, Nandy. Stability of dual solutions in stagnation-point flow and heat transfer over a porous shrinking sheet with thermal radiation. Meccanica 2013;48:23–32.

[15] Akbar NS, Nadeem S. Combined effects of heat and chemical reactions on the peristaltic flow of Carreau fluid model in a diverging tube. Int J Numer Methods Fluid 2011;67:1818–32.
[16] Lok YY, Amin N, Pop I. Non-orthogonal stagnation point towards a stretching sheet. Int J Non-Linear Mech 2006;41(4):622–7.
[17] Wang CY. Stagnation flow towards a shrinking sheet. Int J Non-Linear Mech 2008;43:377–82.
[18] Khan ZH, Khan WA, Qasim M, Shah IA. MHD stagnation point ferrofluid flow and heat transfer toward a stretching sheet, Nanotechnology. IEEE Trans 2014(13):35–40.
[19] Akbar NS, Nadeem S, Haq RU, Khan ZH. Radiation effects on MHD stagnation point flow of nano fluid towards a stretching surface with convective boundary condition. Chin J Aeronaut 2013;26(6):1389–97.
[20] Bhattacharyya K. Dual solutions in unsteady stagnation-point flow over a shrinking sheet. Chin Phys Lett 2011;28:084702.

Noreen. Sher. Akbar. DBS&H, CEME, National University of Sciences and Technology, Islamabad, Pakistan

Rizwan Ul Haq. Department of Mathematics, Quaid-i-Azam University 45320 , Islamabad 44000 Pakistan

Sohail Nadeem. Department of Mathematics, Quaid-i-Azam University 45320 , Islamabad 44000 Pakistan

Shi-wei Ye was born in Zhejiang, China, on April 3, 1987. He received the B.S. degree in mathematics from Nankai University, Tianjin, China, in 2007. He is studying the third year of the P.H.D degree in Computational Mathematics from Peking University. His current research interest is the modeling and simulation of soft matter.