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# Dynamic Programming Example Analysis of a Pump Station

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**Abstract**

For a pump station supplying water, this paper is analyzing the real situation of the station considering all the real existing factors related to set a mathematic model, in the goal of reducing the cost thru adjusting the water flow in different time sections. The dynamic programming model method with detailed analyses is induced illustrated by an example of optimizing the cost of a single unit in a single day with a fixed water-supply amount in the paper. More interesting, a special case of this model is illustrated to proof that non-stop pump working is more power saving than intermittent working given the unchangeable electricity price.



*Keyword: pump station, electricity price, time sections, mathematic model, dynamic programming, single unit*

## 1. Introduction

### *1.1 The general situation where we are*

In the irrigation work or the water conservancy work, we always use pumps. As the electricity cost varies in every time section with the flux and reflux etc., how to save the cost while assuring the water supply without increasing the other kinds of costs is the key question to solve in this paper. After investigation, it is found out that the water pump flow can be controlled in any time section. Let Q-the pump flow be the decision variable, now the problem becomes what we do to adjust Q in every time section to reduce the total electricity cost.

What's more, if the time section is big enough and we can warrant the water supply in that section. In this case, which is more electricity saving to run the pump in a non-stop way or intermittently to adjust the water flow in that time section is an interesting question to be solved. We need to give more detailed analyses about that. That's why we press forward a special case in the flowing part of this paper.

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*1.2 Why we use dynamic programming here*

Dynamic programming is a computational technique to solving dynamic optimization problems pioneered by Richard Bellman in the late 1950s. It was developed initially as the result of studying certain types of programming problems which arose in inventory theory. It is more powerful in deterministic and stochastic environments, especially in sequential decision of this kind, and it can be used in both continuous and discrete time. As we make different decisions in different time, we make sequential decisions, thus it can be taken as a multiple –stage deterministic problem. Here we can divide a whole 24h day into different time stages so that this optimization process of cost –save for the single pump in a single day can be done with this right fitting method.

## 2. Mathematic Model Setting

There are 24h a day, considering the peak-valley price, the range of water level etc, we divide a day into SN time sections. The objective function is the total electrical cost of the unit in one day. The decision variable is the water flow in every time section. The constraint condition is the fixed water-supply amount and the supply power or the limited supplying speed for the unit. According to the water project knowledge, we can get the model as:

$$\begin{cases} Min\, f_d = \sum_{i=1}^{SN} \dfrac{\rho \cdot g \cdot Q_i(\theta_i, n_i) \cdot H_i}{\eta_{z,i}(\theta_i, n_i) \cdot \eta_{mot} \cdot \eta_{\text{int}} \cdot \eta_f} \cdot \Delta T_i \cdot P_i & \text{(Objective Function)} \quad (1.1) \\ \sum_{i=1}^{SN} Q_i(\theta_i, n_i) \Delta T_i = W_e \quad ; \quad i=1，2，\ldots SN & \text{(Water Supply Constraint)} \quad (1.2) \\ Q_i \le a W_e \ (0<a \le 1) & \text{(Power Constraint)} \quad (1.3) \end{cases}$$

Where $f_d$ is the total electricity cost in one day (Let 10 thousand RMB as one unit), $\rho$ is the water density, g is the acceleration of the gravity, $H_i$ is the average pressure head, $\theta_i$ is the vane angle, $n_i$ is the rotation speed, $Q_i(\theta_i, n_i)$ means the water flow thru the unit in time section i, $\Delta T_i$ is the length of time section i, $P_i$ is the electricity price of time section i (RMB/KW), $W_e$ is the necessary daily water needs in one day, and $\eta_{z,i}(\theta_i, n_i)$, $\eta_{mot}$, $\eta_{\text{int}}$, $\eta_f$ are respectively plant efficiency, motor efficiency, transmission efficiency and conversion efficiency, $\eta_{z,i}$ is subject to water flow and pressure head.

## 3. Analyses of the Model

*3.1 How we get the dynamic programming model*

In equation 1.1, given the divided time sections, and electricity prices in every time section, therefore it is obvious that $\rho$, g, $\Delta T_i$, and $P_i$ are all known constants. According to the authority of water engineering, $\eta_{mot}$ keeps invariable as 94% for big engines when the load ratio is above 60%. The transmission efficiency $\eta_{\text{int}}$ of the direct motors can be considered as 1, $\eta_f$ of the higher power frequency convertor is around 96%, hence, $\eta_{mot}, \eta_{\text{int}}, \eta_f$ can all be taken as constants. And let's assume that $\theta_i$ is $0^0$ to get the equation of $\eta_{z,i}$ to make it simple to analyze.

Let $\frac{\rho \cdot g}{\eta_{mot} \cdot \eta_{\text{int}} \cdot \eta_f} \cdot P_i = b_i$ (2.1), where $b_i$ is a calculated constant.

Now, let's see how to do with $\frac{Q_i(\theta_i, n_i) \cdot H_i}{\eta_{z,i}(\theta_i, n_i)}$ (2.2)

Based on the previous research done by the other pump related professionals, by using the nonlinear programming method, in equation 2.2, we can get $H_i = \alpha_1 Q_i^2 + \alpha_2 Q_i + \alpha_3$ (2.2.1) where $\alpha_1, \alpha_2, \alpha_3$ are calculated constants. Similarly, given that $\theta_i = 0^0$, $\eta = \beta_1 Q_i^3 + \beta_2 Q_i^2 + \beta_3 Q_i + \beta_4$ (2.2.2), where $\beta_1, \beta_2, \beta_3, \beta_4$ can be calculated. Thru using these two results, consequently, we can have:

$$\begin{aligned}\frac{Q_i(\theta_i, n_i) \cdot H_i}{\eta_{z,i}(\theta_i, n_i)} &= \frac{\alpha_1 Q_i^2 + \alpha_2 Q_i + \alpha_3}{\beta_1 Q_i^3 + \beta_2 Q_i^2 + \beta_3 Q_i + \beta_4} \cdot Q_i \\ &= \frac{\gamma_1 Q_i^2 + \gamma_2 Q_i + \gamma_3}{\beta_1 Q_i^3 + \beta_2 Q_i^2 + \beta_3 Q_i + \beta_4} + \frac{\alpha_1}{\beta_1} \qquad (2.2.3),\end{aligned}$$

Where $\gamma_1, \gamma_2, \gamma_3$ are derived from $\alpha, \beta$ thru polynomial division. Thus, we can let

$$\frac{\gamma_1 Q_i^2 + \gamma_2 Q_i + \gamma_3}{\beta_1 Q_i^3 + \beta_2 Q_i^2 + \beta_3 Q_i + \beta_4} + \frac{\alpha_1}{\beta_1} = f(Q_i) \qquad (2.2.4)$$

Therefore, the equation 1.1, 1.2, 1.3 can be rewritten as :

$$\begin{cases} Min\, f_d = \sum_{i=1}^{SN} f(Q_i) \cdot \Delta T_i \cdot b_i \\ \sum_{i=1}^{SN} Q_i \cdot \Delta T_i = W_e; \quad i=1,\ 2,\ \ldots,\ SN \\ Q_i \le aW_e \ (0<\text{a} \le 1) \end{cases} \qquad (3.1)$$

Here 3.1 is an obvious dynamic programming model, it is efficient using dynamic optimization method to solve this problem. It can be solved thru the recursion algorithm. That can be shortly expressed as following.

$$F_d = opt[\sum_{i=1}^{i=k} f(Q_i) \cdot \Delta T_i \cdot b_i] + Min[\sum_{i=k+1}^{i=SN} f(Q_i) \cdot \Delta T_i \cdot b_i] \qquad (3.2)$$

In this equation, it means the optimization result of the first few stages can be inherited by the following steps, hence, the recursive algorithm can be implemented to solve the problem.

*3.2 One special but meaningful case*

When calculating the model, a special case is found very interesting. When we assume that the electricity price is always the same for all the time in a day, then $b_i$ will be always the same, let us say

$b$ according to equation 2.1. Also let us divide every time stage equally, so we have $\Delta T_i = \frac{24h}{SN}$. Based on 3.1, if the time section is big enough, the total water supply in that one stage can meet the whole water need, therefore, it can be deduced that $Q_i \le aW_e$ $(0<a \le 1)$ this constraint can be omitted. In this case, we can have the model as:

$$\begin{cases} Min\, f_d = \hat{b} \cdot \frac{24h}{SN} \cdot \sum_{i=1}^{SN} f(Q_i) \\ \sum_{i=1}^{SN} Q_i = \frac{SN}{24h} W_e; \quad i=1,\ 2,\ \ldots,\ SN \end{cases} \tag{4.1}$$

In 4.1, we have that $Q_i \ge 0$, and given the condition above, I can figure out, normally, we have the result that for any integer k between 0 and SN, when $Q_k = \frac{SN}{24h} W_e,\ 0<\mathrm{k} \le \mathrm{SN}$, and $\mathrm{Q}_i = 0$, $i \ne k$, we can have the minimized electricity cost. In another word, we just have the pump on in one time section while all the other off, in this condition, it is the most power saving solution. Non-stop way is the best solution to save the electricity in this case.

The following steps are about how to proof it.

1) In equation 2.2.3, according to the water engineering experiments, we can get the values of $\alpha, \beta, \gamma$ thru nonlinear programming. Normally, after calculation of the function, we have $f'(x+t) \le f'(x),\ \mathrm{x,t} \ge 0$, the increasing speed is descending.

2) Thus, we calculate the derivation of $f(Q_1) + f(Q_2) - f(Q_1 + Q_2)$, considering $Q_1$ as a variable and $Q_2$ as a constant, then, we have:

$$[f(Q_1) + f(Q_2) - f(Q_1+Q_2)]' = f'(Q_1) - f'(Q_1+Q_2) \ge 0, \quad \therefore f(Q_1) + f(Q_2) \ge f(Q_1+Q_2)$$

3) Using the recursion, we can speculate that

$$f(Q_1) + f(Q_2) + \cdots f(Q_{SN}) \ge f(Q_1 + Q_2 + \cdots + Q_{SN}) \tag{4.2}$$

4) Finally, we have the optimized and minimized value as:

$$\hat{b} \cdot \frac{24h}{SN} \cdot [f(0) \cdot (SN-1) + f(Q_k)]$$

$$= \hat{b} \cdot \frac{24h}{SN} \cdot [f(0) \cdot (SN-1) + \frac{SN}{24h} W_e] \tag{4.3}$$

Note that, as for equation 2.2.4, this rule only works, when function f(x) is constrained to $f'(x+t) \le f'(x),\ \mathrm{x,t} \ge 0$. From experiments, in most cases, with $\alpha$, $\beta$, $\gamma$ got from the nonlinear programming method, that 2.2.4 can meet this condition. It can only be tested by experience but hard to be logically proved.

## 4. Conclusion and Prospect

In the mathematic model, we can calculate to optimize the electric cost thru adjusting the water flow in every time section. In the special case, we can conclude that normally when a pump is non-stop working is more power efficient than intermittent working. Though the model and the general algorithm is given, there is still more work to optimize the recursion steps to save computational time and space. The blade angle can also be taken as the decision variable to optimize the energy cost. The dynamic programming method can be used to solve how to adjust the blade angle to optimize the efficiency. Questions like how to combine these decision factors together is still in front of us.

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