Journal of the Egyptian Mathematical Society (2014) 22, 6–10

Egyptian Mathematical Society
Journal of the Egyptian Mathematical Society
www.etms-eg.org
www.elsevier.com/locate/joems

ORIGINAL ARTICLE

On computation of real eigenvalues of matrices via the Adomian decomposition

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Received 23 March 2013; revised 14 May 2013; accepted 1 June 2013
Available online 24 July 2013

KEYWORDS
Eigenvalue;
Adomian decomposition;
Matrix computation;
Characteristic polynomial;
Adomian polynomials

Abstract The problem of matrix eigenvalues is encountered in various fields of engineering endeavor. In this paper, a new approach based on the Adomian decomposition method and the Faddeev-Leverrier's algorithm is presented for finding real eigenvalues of any desired real matrices. The method features accuracy and simplicity. In contrast to many previous techniques which merely afford one specific eigenvalue of a matrix, the method has the potential to provide all real eigenvalues. Also, the method does not require any initial guesses in its starting point unlike most of iterative techniques. For the sake of illustration, several numerical examples are included.

MATHEMATICS SUBJECT CLASSIFICATION: 15A18; 65H04; 32A70

1. Introduction

It goes without saying that many engineering disciplines are much indebted to matrix theory. Among most of matrix computations, computation of eigenvalues and eigenvectors is of predominant importance. Of aged algorithms to seek eigenvalues of symmetric matrices, mention can be made of an algorithm attributed to Jacobi which was long later revived by von Neumann in 1946. Givens was the one to employ the method of bisections for attaining eigenvalues of real symmetric matrices [1]. Later on, a rather simple alternative for obtaining eigenvalues of a matrix was provided by the Power Method; for background information see [2]. However, the approach suffers from the demerit of not providing all eigenvalues (i.e. it can only achieve those with algebraic multiplicity of one). The interested reader can find a lot on the topic in the fundamental book by Wilkinson [3]. Quite recently, Aishima et al. has come up with a new Wilkinson-like multishift QR algorithm to reassure that the quest for the eigenvalue problem is not over yet [4].

The Adomian decomposition method (ADM) is a powerful semi-analytical scheme to treat a wide span of functional equations including algebraic, differential, integral, and integro-differential both linear and nonlinear. Since its postulation

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Peer review under responsibility of Egyptian Mathematical Society.

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http://dx.doi.org/10.1016/j.joems.2013.06.004

by George Adomian in mid-80s [5], the ADM has been the center of attention owing to its remarkable efficiency. It is free of linearization, discretization or perturbation and converges to the exact analytical solutions in most cases rapidly. The literature abounds with real-world applications of the ADM [6–13]. We include a brief introduction on this method later.

It is the objective of this paper to apply the ADM in conjunction with the Faddeev-Leverrier's algorithm to afford real eigenvalues of a square matrix of an arbitrary size. The accuracy and rapid convergence of the proposed approach is well demonstrated in the given illustrative examples.

2. Statement of the problem

The quantity λ is said to be an eigenvalue of a $n \times n$ square matrix Λ if it satisfies the matrix equation below:

$$\Lambda x = \lambda x \tag{1}$$

where x is a nonzero column vector of dimension n.

Eq. (1) can be written in the form:

$$(\Lambda - \lambda I)x = 0 \tag{2}$$

where I is the identity matrix of order n.

For the Eq. (2) to hold true, the matrix $(\Lambda - \lambda I)$ has to be singular, that is:

$$\det(\Lambda - \lambda I) = 0 \tag{3}$$

The determinant on the left-hand side of Eq. (3) can be expanded to give:

$$a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \cdots + a_{n-1}\lambda + a_n = 0 \tag{4}$$

The previous equation is often referred to as the characteristic equation/polynomial of the matrix Λ in matrix algebra.

Among many rivals, Faddeev-Leverrier's scheme has maintained a good reputation in providing the characteristic polynomial of matrices [14,15]. Assuming that Λ is n-by-n, the method consists of the following step:

$$\begin{cases} \Lambda_1 = \Lambda \\ \Lambda_{i+1} = \Lambda(\Lambda_i + a_i I); \quad 1 \leqslant i \leqslant n \end{cases} \tag{5}$$

where,

$$\begin{cases} a_0 = 1 \\ a_i = -\frac{trace(\Lambda_i)}{i}; \quad 1 \leqslant i \leqslant n \end{cases} \tag{6}$$

In this regard, recursively all the $n + 1$ coefficients of the Eq. (4), or in other words the characteristic polynomial, are found conveniently by Eqs. (5), (6).

At a very first glance, the Eq. (4) may seem uncomplicated, however, upon a further scrutiny the opposite is revealed as the work by Abel and Galois suggests that no general formula for achieving zeroes of most polynomials with degree greater than four exists [16]. Moreover, a robust and efficient numerical algorithm to determine all the roots of a large-degree polynomial equation is hard to find [2].

In what follows, we make an effort to find λ values from Eq. (4) by means of the Adomian decomposition method.

3. How the ADM works

For the convenience of the reader, we present here a short review on the necessary background of the Adomian decomposition method.

Let us consider a general functional equation of type:

$$u - N(u) = f \tag{7}$$

where N is a nonlinear operator which maps a Hilbert space H into itself, f is a given function and u is an unknown function. The ADM decomposes u as an infinite series $u = \sum_{i=0}^{\infty} u_i$ and N as $N(u) = \sum_{i=0}^{\infty} A_i$, where A_i s are called the Adomian polynomials alternatively obtained by the traditional formula [17]:

$$A_i = A_i(u_0, u_1, \ldots, u_i) = \frac{1}{i!}\frac{d^i}{d\lambda^i} N\left(\sum_{k=0}^{\infty} u_k \lambda^k\right)\Bigg|_{\lambda=0} \tag{8}$$

By letting $u_0 = f$, the ADM constructs the following recurrence to generate other components of the solution, i.e. u_i s.

$$u_{i+1} = A_i; \quad i \geqslant 0 \tag{9}$$

The convergence and reliability of this method have been ascertained by prior works (e.g. [8]).

Elsewhere [18], Fatoorehchi and Abolghasemi have devised a completely different algorithm to generate the Adomian polynomials of any desired nonlinear operators. It is mainly based on string functions and symbolic programming. By setting the symbolic variable $NON = u_0 + u_1 + u_2 + \cdots + u_n$ and a large enough integer n, the following function in MATLAB can return the Adomian polynomial components of a nonlinear operator acting on NON.

Program 1. An alternative code for determination of the Adomian polynomials

```
function sol = AdomPoly(expression,nth)
Ch = char(expand(expression));
s = strread(Ch, '%s', 'delimiter', '+');
for i = 1:length(s)
t = strread(char(s(i)), '%s', 'delimiter', '*()expUlogsinh');
t = strrep(t,'^','*');
    if length(t) ~ = 2
p = str2num(char(t));
sumindex = sum(p)-p(1);
    else
sumindex = str2num(char(t));
    end
list(i) = sumindex;
end
A = '';
for j = 1:length(list)
    if nth = = list(j)
A = strcat(A,s(j),'+');
    end
end
N = length(char(A))-1;
F = strcat ('%',num2str(N),'c%n');
sol = sscanf(char(A),F);
```

4. The proposed method

In order to have the ADM give the sought-after eigenvalues, it is indispensable to write Eq. (4) in a fixed-point form, viz. $\lambda = g(\lambda)$. Provided that $a_{n-1} \neq 0$, the following equation can be expressed promptly as:

$$\lambda = -\frac{a_0}{a_{n-1}}\lambda^n - \frac{a_1}{a_{n-1}}\lambda^{n-1} - \cdots - \frac{a_{n-2}}{a_{n-1}}\lambda^2 - \frac{a_n}{a_{n-1}} \tag{10}$$

In keeping with the ADM, $\lambda = \sum_{i=0}^{\infty}\lambda_i$, $\lambda_0 = -a_n/a_{n-1}$ and the nonlinearities λ^n, λ^{n-1}, …, λ^2 shall be substituted by their corresponding Adomian polynomials, namely, A_n, ..., B_n, C_n. Consequently we get:

$$\lambda_{i+1} = -\frac{a_0}{a_{n-1}}A_i - \frac{a_1}{a_{n-1}}B_i - \cdots - \frac{a_{n-2}}{a_{n-1}}C_i; \quad i \geqslant 0 \tag{11}$$

Obviously, $\xi_1 = \lambda = \sum_{i=0}^{\infty}\lambda_i$ is an eigenvalue to the matrix Λ. It is worthwhile to mention that, in some cases, it may happen that the sequence produced by Eq. (11) diverges. To alleviate such a defect, one can add an auxiliary diagonal matrix αI to Λ to create a new matrix $\Theta = \Lambda + \alpha I$. Once α is suitably chosen, the ADM efficiently provides an eigenvalue for the matrix Θ. At this point with the help of the lemma described ahead, a correspondent eigenvalue for the matrix Λ can be immediately achieved. Such a situation is well illustrated within example 2.

Lemma 1. *Let Aand B be $n \times n$ matrices, I represent identity matrix in n dimensions, α denote a real number, and eig( ) stand for an operator returning an eigenvalue of its matrix argument. If $A = B + \alpha I$and $eig(A) = \lambda$, then it holds that $eig(B) = \lambda - \alpha$.*

Proof. From $eig(A) = \lambda$ it follows that

$$\det(A - \lambda I) = 0$$

Replacing matrix A with its equivalent gives

$$\det(B + \alpha I - \lambda I) = 0$$

or obviously

$$\det(B - (\lambda - \alpha)I) = 0$$

This asserts that the quantity $\lambda - \alpha$ is an eigenvlaue for the matrix B or in other words $eig(B) = \lambda - \alpha$. □

Once a first eigennvalue, say ξ_1, of a matrix is determined, we can proceed to find the others by this routine:

1. Extract out the root just been found out of Eq. (4) to yield a new equation:

$$\frac{a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \cdots + a_{n-1}\lambda + a_n}{\lambda - \xi_1} = 0 \tag{12}$$

2. Accordingly, build up a new fixed-point form equation, (new nonlinearities emerge, rational types to be specific).

$$\frac{b_0 + b_1(\lambda-\xi_1) + b_2(\lambda-\xi_1)^2 + \cdots + b_{n-1}(\lambda-\xi_1)^{n-1} + (\lambda-\xi_1)^n}{\lambda - \xi_1} = 0 \tag{13}$$

$$\frac{b_0}{\lambda-\xi_1} + b_1 + b_2(\lambda-\xi_1) + \cdots + b_{n-1}(\lambda-\xi_1)^{n-2} + (\lambda-\xi_1)^{n-1} = 0 \tag{14}$$

$$\frac{1}{b_2}\frac{b_0}{\lambda-\xi_1} + \lambda + \frac{b_1}{b_2} - \xi_1 + \cdots + \frac{b_{n-1}}{b_2}(\lambda-\xi_1)^{n-2} + \frac{1}{b_2}(\lambda-\xi_1)^{n-1} = 0 \tag{15}$$

$$\lambda = \xi_1 - \frac{b_1}{b_2} - \frac{1}{b_2}\frac{b_0}{\lambda-\xi_1} + \cdots - \frac{b_{n-1}}{b_2}(\lambda-\xi_1)^{n-2} - \frac{1}{b_2}(\lambda-\xi_1)^{n-1} \tag{16}$$

3. Follow back the ADM to yield a solution, a new eigenvalue, to this new equation. By invoking the mentioned procedure repeatedly, one can attain all real eigenvalues of a matrix; see example 4.

5. Numerical examples

To illustrate the proposed method and show its applicability, we present a number of numerical examples in this section. Certainly, one can adopt the procedure to treat any desired matrix.

Example 1. Assume that

$$\Lambda = \begin{bmatrix} 4 & 2 & 0 \\ 3 & 2 & 1 \\ 2 & 7 & 9 \end{bmatrix} \tag{17}$$

Following the described Faddeev-Leverrier algorithm, the characteristic equation to Λ reads

$$-6 - 49\lambda + 15\lambda^2 - \lambda^3 = 0 \tag{18}$$

According to what discussed, we set

$$\begin{cases} \lambda_0 = -\frac{6}{49} \\ \lambda_{i+1} = \frac{15}{49}A_i - \frac{1}{49}B_i; \quad i \geqslant 0 \end{cases} \tag{19}$$

with A_i and B_i being the Adomian polynomials replacing λ^2 and λ^3, orderly.

Thus, some first decomposition components are listed as

$$\begin{aligned} \lambda_0 &= -0.1224489796 \\ \lambda_1 &= 0.4627393036\text{e} \times 10^{-2} \\ \lambda_2 &= -0.3511578086 \times 10^{-3} \\ \lambda_3 &= 0.3336368140 \times 10^{-4} \\ \lambda_4 &= -0.3553111861 \times 10^{-5} \\ \lambda_5 &= 0.4056048425 \times 10^{-6} \\ \lambda_6 &= -0.4852021856 \times 10^{-7} \\ \lambda_7 &= 0.6003189951 \times 10^{-8} \\ \lambda_8 &= -0.7618918370 \times 10^{-9} \\ \lambda_9 &= 0.9863833101 \times 10^{-10} \end{aligned} \tag{20}$$

Approximately we have $\lambda = \sum_{i=0}^{\infty}\lambda_i = \sum_{i=0}^{9}\lambda_i = -0.1181425714$.

The accurate value of this eigenvalue obtained by the built-in eig() command in MATLAB equals to -0.118142571382007.

Example 2. Suppose that

$$\Lambda = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 9 & 5 \\ 6 & 1 & -2 \end{bmatrix} \tag{21}$$

By virtue of the Faddeev-Leverrier's algorithm, the characteristic equation promptly is yielded as

$$-66 + 38\lambda + 7\lambda^2 - \lambda^3 = 0 \tag{22}$$

Therefore, we will have

$$\begin{cases} \lambda_0 = \frac{66}{38} \\ \lambda_{i+1} = -\frac{7}{38}A_i + \frac{1}{38}B_i; \quad i \geqslant 0 \end{cases} \tag{23}$$

where A_i and B_i are the same polynomials as in example 1. With little computational effort, one can found that the sequence generated by Eq. (23) is nonconvergent. Consequently, we build the matrix Θ as $\Theta = \Lambda - I$ (note we choose $\alpha = -1$). So, the recurrence giving an eigenvalue of Θ reads

$$\begin{cases} \lambda_0 = \frac{22}{49} \\ \lambda_{i+1} = -\frac{4}{49} A_i + \frac{1}{49} B_i; \quad i \geqslant 0 \end{cases} \tag{24}$$

Similar to the procedure followed in example 1, an approximate eigenvalue of matrix Θ equals to $\lambda = \sum_{i=0}^{9} \lambda_i = 0.4352005837$. From the above-mentioned Lemma and its preceding discussion, we conclude that 1.4352005837 is an eigenvalue of the matrix Λ which is very close to what returned by the eig() function in MATLAB, that is 1.43520058370682.

Example 3. Define

$$\Lambda = \begin{bmatrix} 1 & -10 & 6 & 3 & 0 & 1 \\ 5 & 1 & 4 & -3 & 1 & 1 \\ 0 & 3 & 5 & 1 & 6 & 1 \\ 5 & 1 & 3 & 2 & 1 & 7 \\ -1 & -3 & 4 & 7 & 8 & 2 \\ 1 & 0 & 6 & 0 & 3 & 3 \end{bmatrix} \tag{25}$$

Similar to example 1, we obtain

$$\begin{aligned} &\lambda_0 = 0.5604312305 \\ &\lambda_1 = 0.6648917053 \times 10^{-1} \\ &\lambda_2 = 0.1476736549 \times 10^{-1} \\ &\lambda_3 = 0.3982022153 \times 10^{-2} \\ &\lambda_4 = 0.1183408891 \times 10^{-2} \\ &\lambda_5 = 0.3730426712 \times 10^{-3} \\ &\lambda_6 = 0.1223551136 \times 10^{-3} \\ &\lambda_7 = 0.4129699415 \times 10^{-4} \\ &\lambda_8 = 0.1424360086 \times 10^{-4} \\ &\lambda_9 = 0.4996808284 \times 10^{-5} \\ &\lambda_{10} = 0.1777066706 \times 10^{-5} \\ &\lambda_{11} = 0.6391513655 \times 10^{-6} \end{aligned} \tag{26}$$

Therefore, we reach to an eigenvalue $\lambda = \sum_{i=0}^{11} \lambda_i = 0.6474115490$ which is very close to the value 0.6474119158 obtained by eig() command in MATLAB.

Example 4. Suppose that we are after finding all eigenvalues of the following matrix

$$\Lambda = \begin{bmatrix} 0 & 1 \\ 3 & 5 \end{bmatrix} \tag{27}$$

It is conspicuous that Λ has two real eigenvalues: -0.541381265149110 and 5.54138126514911.

Clearly, the characteristic equation of Λ is:

$$-3 - 5\lambda + \lambda^2 = 0 \tag{28}$$

Thus,

$$\begin{cases} \lambda_0 = -\frac{3}{5} \\ \lambda_{i+1} = \frac{1}{5} A_i; \quad i \geqslant 0 \end{cases} \tag{29}$$

Following the same proposed routine, we get the first eigenvalue as $\xi_1 = \sum_{i=0}^{10} \lambda_i = -0.5413813106$.

For the other eigenvalue, we treat the following equation

$$\frac{\lambda^2 - 5\lambda - 3}{\lambda + 0.5413813106} = 0. \tag{30}$$

Equally,

$$\frac{(\lambda + 0.5413813106)^2 - 5(\lambda + 0.5413813106) - 1.0827626212\lambda - 0.5861871704}{\lambda + 0.5413813106} = 0 \tag{31}$$

or

$$\lambda = 4.4586186894 + \frac{1.0827626212\lambda + 0.5861871704}{\lambda + 0.5413813106} \tag{32}$$

According to the ADM principles we have

$$\begin{cases} \lambda_0 = 4.4586186894 \\ \lambda_{i+1} = C_i; \quad i \geqslant 0 \end{cases} \tag{33}$$

where C_i is the i-th component of the Adomian polynomial correspondent to the rational nonlinearity in Eq. (32). The scheme converges quickly to the other eigenvalue of Λ as listed below

$$\begin{aligned} &\lambda_0 = 4.458618689 \\ &\lambda_1 = 1.082762566 \\ &\lambda_2 = 0.1197682426 \times 10^{-7} \\ &\lambda_3 = -0.2593611261 \times 10^{-8} \\ &\lambda_4 = 0.5616529794 \times 10^{-9} \\ &\lambda_5 = -0.1216273456 \times 10^{-9} \\ &\lambda_6 = 0.2633870198 \times 10^{-10} \\ &\lambda_7 = -0.5703710650 \times 10^{-11} \\ &\lambda_8 = 0.1235152497 \times 10^{-11} \\ &\lambda_9 = -0.2674752818 \times 10^{-12} \end{aligned} \tag{34}$$

As a result, $\xi_2 = \sum_{i=0}^{9} \lambda_i = 5.541381265$.

Example 5. Given

$$\Lambda = \begin{bmatrix} 1.5968 & 0.4067 & 1.6821 & 1.4823 & 0.6130 & 1.1581 & 1.5300 & 1.1833 & 0.2666 & 1.0769 & 1.4653 & 1.4803 \\ 0.8807 & 0.3468 & 0.2109 & 0.5649 & 1.5044 & 1.2754 & 1.6692 & 1.6797 & 0.5653 & 1.6124 & 1.4401 & 1.8586 \\ 0.6203 & 1.4092 & 1.2080 & 0.3199 & 1.9308 & 0.7978 & 0.0071 & 1.2795 & 1.3947 & 1.6545 & 0.1160 & 0.0624 \\ 0.0938 & 1.1709 & 1.3741 & 1.3825 & 1.0859 & 0.1856 & 1.2819 & 0.1112 & 0.3984 & 0.4782 & 0.1618 & 0.8526 \\ 0.9762 & 1.6188 & 0.7041 & 0.4304 & 0.2122 & 1.2874 & 1.0752 & 0.2664 & 0.6232 & 1.5480 & 1.6760 & 1.6336 \\ 0.3047 & 1.6140 & 1.3658 & 1.6641 & 0.7625 & 0.2963 & 1.4066 & 0.3558 & 0.2621 & 0.8672 & 1.5098 & 0.4322 \\ 1.8240 & 1.6339 & 1.8158 & 0.4552 & 0.6035 & 1.8558 & 1.6444 & 1.8738 & 1.0063 & 1.0032 & 1.1697 & 0.8651 \\ 0.8179 & 0.1329 & 1.2771 & 0.1165 & 1.3292 & 0.9613 & 0.8639 & 0.5311 & 0.5369 & 0.8005 & 1.4301 & 1.4941 \\ 0.2433 & 0.3539 & 0.0535 & 0.3553 & 1.6574 & 1.4749 & 0.8957 & 0.4362 & 1.2782 & 0.7119 & 0.4300 & 1.0833 \\ 0.7743 & 0.9288 & 1.7279 & 1.6630 & 1.6252 & 1.3282 & 0.8261 & 1.6116 & 0.3586 & 0.5905 & 1.3632 & 1.1777 \\ 1.1773 & 1.6594 & 0.0833 & 1.2124 & 0.3794 & 0.3002 & 1.1942 & 0.3272 & 1.0876 & 0.6965 & 1.4851 & 1.8546 \\ 0.6501 & 1.6867 & 0.6138 & 0.5528 & 0.3832 & 1.4644 & 0.7726 & 0.8012 & 0.0982 & 1.7683 & 0.3814 & 1.6697 \end{bmatrix}. \tag{35}$$

The characteristic equation pertaining to the matrix Λ can be determined by virtue of Eq. (3) as,

$$\lambda^{12} - 12.2416\lambda^{11} + 5.3606\lambda^{10} - 17.5256\lambda^{9} + 8.8741\lambda^{8} - 98.5986\lambda^{7} + 94.8460\lambda^{6} - 142.1029\lambda^{5} + 183.8437\lambda^{4} - 11.3085\lambda^{3} - 213.8051\lambda^{2} + 383.2418\lambda + 55.6948 = 0 \quad (36)$$

Converting Eq. (35) into a fixed-point form gives

$$\lambda = -0.1453254918 + 0.5578855770\lambda^{2} + 0.0295074446\lambda^{3} - 0.4797067092\lambda^{4} + 0.3707917655\lambda^{5} - 0.2474833309\lambda^{6} + 0.2572750669\lambda^{7} - 0.0231554630\lambda^{8} + 0.0457299193\lambda^{9} - 0.0139875269\lambda^{10} + 0.0319422343\lambda^{11} - 0.0026093185\lambda^{12} \quad (37)$$

Following the principles of the ADM, one can construct a solution to Eq. (36) as

$$\begin{cases} \lambda_0 = -0.1453254918 \\ \lambda_{i+1} = 0.5578855770J_i + 0.0295074446L_i - 0.4797067092M_i \\ +0.3707917655N_i - 0.2474833309O_i + 0.2572750669P_i \\ -0.0231554630Q_i + 0.0457299193R_i - 0.0139875269S_i \\ +0.0319422343T_i - 0.0026093185U_i; \quad i \geqslant 0 \end{cases} \quad (38)$$

where $J, L, \ldots, U$ are the Adomian polynomials decomposing the nonlinearities $\lambda^2, \lambda^3, \ldots, \lambda^{12}$, respectively.

Therefore, by Eq. (37), one easily obtains

$$\lambda_0 = -0.1453254919 \quad \lambda_3 = 0.3313773364 \times 10^{-3}$$
$$\lambda_6 = -0.3544496172 \times 10^{-5}$$
$$\lambda_1 = 0.1145101001 \times 10^{-1} \quad \lambda_4 = 0.6918827032 \times 10^{-4}$$
$$\lambda_2 = -0.1757165434 \times 10^{-2} \quad \lambda_5 = 0.1534869885 \times 10^{-4} \quad (39)$$

and can approximate an eigenvalue of Λ as $\lambda = \sum_{i=0}^{6}\lambda_i = -0.1353576540$ which is so close to the result returned by the eig() command in MATLAB, -0.1353569759.

6. Conclusion

Based on the Adomian decomposition method combined with the Faddeev-Leverrier's algorithm, a novel method is proposed to handle the eigenvalue problem of real matrices. The scheme is simple and computationally robust. Unlike many previous methods which offer only one eigenvalue of a matrix, the current method is shown to be capable of providing all real eigenvalues. The illustrative examples given in the paper, ascertained the accuracy and efficiency of the method.

Acknowledgment

We, H.F. and H.A., are grateful to the reviewers and the editor for their helpful comments and suggestions which indeed improved the quality of this paper.

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