



# Anomalous $CP$ -violation in $B_s-\bar{B}_s$ mixing due to a light spin-one particle

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## ABSTRACT

The recent measurement of the like-sign dimuon charge asymmetry in semileptonic  $b$ -hadron decays by the D0 Collaboration is about three sigmas away from the standard-model prediction, hinting at the presence of  $CP$ -violating new physics in the mixing of  $B_s$  mesons. We consider the possibility that this anomalous result arises from the contribution of a light spin-1 particle. Taking into account various experimental constraints, we find that the effect of such a particle with mass below the  $b$ -quark mass can yield a prediction consistent with the anomalous D0 measurement within its one-sigma range.

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## 1. Introduction

The D0 Collaboration has recently reported a new measurement of the like-sign dimuon charge asymmetry in semileptonic  $b$ -hadron decays,  $A_{sl}^b = [-9.57 \pm 2.51 \text{ (stat)} \pm 1.46 \text{ (sys)}] \times 10^{-3}$  [1]. It disagrees with the standard model (SM) prediction  $A_{sl}^{b,SM} = (-2.3^{+0.5}_{-0.6}) \times 10^{-4}$  [2,3] by 3.2 standard deviations, thereby providing evidence for anomalous  $CP$ -violation in the mixing of neutral  $B$ -mesons. This observable is related to the charge asymmetry  $a_{sl}^s$  for “wrong-charge” semileptonic  $B_s$  decay induced by oscillations. The above values of  $A_{sl}^b$  thus translate into [1,3]

$$a_{sl}^{s,exp} = -(14.6 \pm 7.5) \times 10^{-3}, \quad (1)$$

$$a_{sl}^{s,SM} = (2.1 \pm 0.6) \times 10^{-5}. \quad (2)$$

Although not yet conclusive, this sizable discrepancy between experiment and theory suggests that new physics beyond the SM may be responsible for it. Consequently, it has attracted a great deal of attention in the literature [4–6].

In addition to  $a_{sl}^s$ , the observables of interest in this case are the mass and width differences  $\Delta M_s$  and  $\Delta \Gamma_s$ , respectively, between the heavy and light mass-eigenstates in the  $B_s-\bar{B}_s$  system. Their experimental values are [7]

$$\begin{aligned} \Delta M_s^{exp} &= 17.77 \pm 0.12 \text{ ps}^{-1}, \\ \Delta \Gamma_s^{exp} &= 0.062^{+0.034}_{-0.037} \text{ ps}^{-1}. \end{aligned} \quad (3)$$

These three observables are related to the off-diagonal elements  $M_s^{12}$  and  $\Gamma_s^{12}$  of the mass and decay matrices, respectively, which characterize  $B_s-\bar{B}_s$  mixing. The relationship is described by [8]

$$(\Delta M_s)^2 - \frac{1}{4}(\Delta \Gamma_s)^2 = 4|M_s^{12}|^2 - |\Gamma_s^{12}|^2, \quad (4)$$

$$\Delta M_s \Delta \Gamma_s = 4|M_s^{12}| |\Gamma_s^{12}| \cos \phi_s, \quad \phi_s = \arg(-M_s^{12}/\Gamma_s^{12}), \quad (5)$$

$$a_{sl}^s = \frac{4|M_s^{12}| |\Gamma_s^{12}| \sin \phi_s}{4|M_s^{12}|^2 + |\Gamma_s^{12}|^2} \quad (6)$$

in the notation of Ref. [1]. The SM predicts [3,6]

$$\begin{aligned} 2M_s^{12,SM} &= 20.1(1 \pm 0.40)e^{-0.035i} \text{ ps}^{-1}, \\ 2|\Gamma_s^{12,SM}| &= 0.096 \pm 0.039 \text{ ps}^{-1}, \\ \phi_s^{SM} &= (4.2 \pm 1.4) \times 10^{-3} = 0.24^\circ \pm 0.08^\circ. \end{aligned} \quad (7)$$

Since  $\Delta \Gamma_s \ll \Delta M_s$  and [8]  $|\Gamma_s^{12}| \ll |M_s^{12}|$ , the commonly used expressions are

$$\Delta M_s \simeq 2|M_s^{12}|, \quad \Delta \Gamma_s \simeq 2|\Gamma_s^{12}| \cos \phi_s, \quad (8)$$

leading to

$$a_{sl}^s \simeq \frac{|\Gamma_s^{12}| \sin \phi_s}{|M_s^{12}|} \simeq \frac{2|\Gamma_s^{12}| \sin \phi_s}{\Delta M_s}. \quad (9)$$

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The preceding equation for  $a_{s1}^s$  implies that any new physics which is to provide a successful explanation for the anomalous value of  $a_{s1}^s$  reported by D0 needs to affect both  $M_s^{12}$  and  $\Gamma_s^{12}$ . However, as Eqs. (3) and (8) indicate, the magnitude of  $M_s^{12}$  is strongly constrained by the experimental data, and so the possible room for new physics lies mostly in  $\Gamma_s^{12}$  and the relative phase  $\phi_s$  between  $M_s^{12}$  and  $\Gamma_s^{12}$  [5]. A related observation is that the smallness of the SM prediction  $\phi_s^{\text{SM}}$  suggests that any new-physics effects which can significantly enhance  $\phi_s$  as well as  $|\Gamma_s^{12}|$  with respect to their SM values are likely to account for the unexpectedly large value of  $a_{s1}^{s,\text{exp}}$ .

Here we consider the possibility that this  $a_{s1}^s$  anomaly arises from the contribution of a new particle of spin one and mass under the  $b$ -quark mass. Nonstandard spin-1 particles with masses of a few GeV or less have been explored to some extent in various other contexts beyond the SM in the literature. Their existence is in general still allowed by presently available data and also desirable, as they may offer possible explanations for some of the recent experimental anomalies and unexpected observations. For instance, a spin-1 boson having mass of a few GeV and couplings to both quarks and leptons has been proposed to explain the measured value of the muon  $g-2$  and the NuTeV anomaly simultaneously [9]. As another example,  $\mathcal{O}(\text{MeV})$  spin-1 bosons which can interact with dark matter as well as leptons may be responsible for the observed 511-keV emission from the Galactic bulge and are potentially detectable by future neutrino telescopes [10]. If its mass is of  $\mathcal{O}(\text{GeV})$ , such a particle may be associated with the unexpected excess of positrons recently observed in cosmic rays, possibly caused by dark-matter annihilation [11]. In the context of hyperon decay, a spin-1 boson with mass around 0.2 GeV, flavor-changing couplings to quarks, and a dominant decay mode into  $\mu^+\mu^-$  can explain the three anomalous events of  $\Sigma^+ \rightarrow p\mu^+\mu^-$  reported by the HyperCP experiment several years ago [12]. Although in these few examples the spin-1 particles tend to have very small couplings to SM particles, it is possible to test their presence in future high-precision experiments [10–13]. It is therefore also interesting to explore a light spin-1 boson as an explanation for the  $a_{s1}^s$  anomaly.

In this Letter we adopt a model-independent approach, assuming only that the spin-1 particle, which we shall refer to as  $X$ , is lighter than the  $b$  quark, carries no color or electric charge, and has some simple form of flavor-changing interactions with quarks. As we will elaborate, it is possible for  $X$  with mass below the  $b$ -quark mass and couplings satisfying current experimental constraints to yield a value of  $a_{s1}^s$  which is within the one-sigma range of the new D0 data.

## 2. Interactions and amplitudes

With  $X$  being colorless and electrically neutral, we can express the Lagrangian describing its effective flavor-changing couplings to  $b$  and  $s$  quarks as

$$\begin{aligned} \mathcal{L}_{bsX} &= -\bar{s}\gamma_\mu(g_V - g_A\gamma_5)bX^\mu + \text{H.c.} \\ &= -\bar{s}\gamma_\mu(g_L P_L + g_R P_R)bX^\mu + \text{H.c.}, \end{aligned} \quad (10)$$

where  $g_V$  and  $g_A$  parametrize the vector and axial-vector couplings, respectively,  $g_{L,R} = g_V \pm g_A$ , and  $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ . Generally, the constants  $g_{V,A}$  can be complex. In principle,  $X$  can have additional interactions, flavor-conserving and/or flavor-violating, with other fermions which are parametrized by more coupling constants. We assume that these additional parameters already satisfy other experimental constraints to which they are subject, but with which we do not deal in this study. Hence we will not consider

much further phenomenological implications of such a particle, beyond those directly related to the D0 anomalous finding. In the following, we derive the contributions of  $\mathcal{L}_{bsX}$  to the amplitudes for several processes involving the  $B_s$  meson.

For the mixing-matrix elements  $M_s^{12}$  and  $\Gamma_s^{12}$ , including the  $X$  contributions we have

$$M_s^{12} = M_s^{12,\text{SM}} + M_s^{12,X}, \quad \Gamma_s^{12} = \Gamma_s^{12,\text{SM}} + \Gamma_s^{12,X}. \quad (11)$$

To determine  $M_s^{12,X}$ , we apply the general relation  $2m_{B_s}M_s^{12} = \langle B_s^0 | \mathcal{H}_{b\bar{s} \rightarrow \bar{b}s} | \bar{B}_s^0 \rangle$  [15] to the effective Hamiltonian  $\mathcal{H}_{b\bar{s} \rightarrow \bar{b}s}^X$  derived from the amplitude for the tree-level transition  $b\bar{s} \rightarrow \bar{b}s$  mediated by  $X$  in the  $s$  and  $t$  channels induced by  $\mathcal{L}_{bsX}$ . Thus

$$\begin{aligned} \mathcal{H}_{b\bar{s} \rightarrow \bar{b}s}^X &= \frac{\bar{s}\gamma^\mu(g_L P_L + g_R P_R)b\bar{s}\gamma_\mu(g_L P_L + g_R P_R)b}{2(m_X^2 - m_{B_s}^2)} \\ &+ \frac{\{\bar{s}[(g_L m_s - g_R m_b)P_L + (g_R m_s - g_L m_b)P_R]b\}^2}{2(m_X^2 - m_{B_s}^2)m_X^2}, \end{aligned} \quad (12)$$

where we have used in the denominators the approximation  $p_X^2 \simeq m_{B_s}^2 \sim m_b^2$  appropriate for the  $B_s$  rest-frame and included an overall factor of  $1/2$  to account for the products of two identical operators. This Hamiltonian was earlier obtained in a different context in Ref. [14]. In evaluating its matrix element at energy scales  $\mu \sim m_b$ , one needs to include the effect of QCD running from high energy scales which mixes different operators. The resulting contribution of  $X$  is

$$\begin{aligned} M_s^{12,X} &= \frac{f_{B_s}^2 m_{B_s}}{3(m_X^2 - m_{B_s}^2)} \left[ (g_V^2 + g_A^2) P_1^{\text{VLL}} \right. \\ &+ \frac{g_V^2 (m_b - m_s)^2 + g_A^2 (m_b + m_s)^2}{m_X^2} P_1^{\text{SLL}} \\ &+ (g_V^2 - g_A^2) P_1^{\text{LR}} \\ &\left. + \frac{g_V^2 (m_b - m_s)^2 - g_A^2 (m_b + m_s)^2}{m_X^2} P_2^{\text{LR}} \right], \end{aligned} \quad (13)$$

where

$$\begin{aligned} P_1^{\text{VLL}} &= \eta_1^{\text{VLL}} B_1^{\text{VLL}}, \\ P_1^{\text{SLL}} &= -\frac{5}{8} \eta_1^{\text{SLL}} R_{B_s} B_1^{\text{SLL}}, \quad \text{and} \\ P_j^{\text{LR}} &= -\frac{1}{2} \eta_{1j}^{\text{LR}} R_{B_s} B_1^{\text{LR}} + \frac{3}{4} \eta_{2j}^{\text{LR}} R_{B_s} B_2^{\text{LR}}, \end{aligned}$$

$j = 1, 2$  [16], with the  $\eta$ 's denoting QCD-correction factors, the  $B$ 's being bag parameters defined by the matrix elements

$$\begin{aligned} \langle B_s^0 | \bar{s}\gamma^\mu P_L b \bar{s}\gamma_\mu P_L b | \bar{B}_s^0 \rangle &= \langle B_s^0 | \bar{s}\gamma^\mu P_R b \bar{s}\gamma_\mu P_R b | \bar{B}_s^0 \rangle \\ &= \frac{2}{3} f_{B_s}^2 m_{B_s}^2 B_1^{\text{VLL}}, \end{aligned}$$

$$\langle B_s^0 | \bar{s} P_L b \bar{s} P_L b | \bar{B}_s^0 \rangle = \langle B_s^0 | \bar{s} P_R b \bar{s} P_R b | \bar{B}_s^0 \rangle = -\frac{5}{12} f_{B_s}^2 m_{B_s}^2 R_{B_s} B_1^{\text{SLL}},$$

$$\langle B_s^0 | \bar{s}\gamma^\mu P_L b \bar{s}\gamma_\mu P_R b | \bar{B}_s^0 \rangle = -\frac{1}{3} f_{B_s}^2 m_{B_s}^2 R_{B_s} B_1^{\text{LR}}, \quad \text{and}$$

$$\langle B_s^0 | \bar{s} P_L b \bar{s} P_R b | \bar{B}_s^0 \rangle = \frac{1}{2} f_{B_s}^2 m_{B_s}^2 R_{B_s} B_2^{\text{LR}}, \quad \text{and}$$

$$R_{B_s} = m_{B_s}^2 / (m_b + m_s)^2.$$

As for  $\Gamma_s^{12}$ , it is in general affected by any physical state  $f$  into which both  $B_s$  and  $\bar{B}_s$  can decay. Mathematically,  $\Gamma_s^{12}$  is given by [8]

$$\Gamma_s^{12} = \sum_f' (\mathcal{M}(B_s \rightarrow f))^* \mathcal{M}(\bar{B}_s \rightarrow f), \quad (14)$$

the prime indicating that final-state kinematical factors and integrations are to be properly incorporated. In the SM, this is dominated by the CKM-favored  $b \rightarrow c\bar{c}s$  tree-level processes [3]. In contrast, with the  $X$  mass  $m_X < m_b$ , the dominant processes contributing to  $\Gamma_s^{12,X}$  arise from decays induced by  $b(\bar{b}) \rightarrow s(\bar{s})X$ , such as  $\bar{B}_s(B_s) \rightarrow \eta X$ ,  $\bar{B}_s(B_s) \rightarrow \eta' X$ , and  $\bar{B}_s(B_s) \rightarrow \phi X$ . It follows that  $\Gamma_s^{12,X}$  can be written as

$$\Gamma_s^{12,X} = \sum_{f_X}' (\mathcal{M}(B_s \rightarrow f_X))^* \mathcal{M}(\bar{B}_s \rightarrow f_X), \quad (15)$$

where  $f_X = \eta X, \eta' X, \phi X, \dots$  for kinematically allowed  $B_s \rightarrow f_X$ . Now, apart from the presence of squares of the coupling constants,  $g_{V,A}^2$ , instead of their absolute values, this sum is the same in form as the sum of rates  $\sum_{f_X} \Gamma(B_s \rightarrow f_X)$ , which is approximately equivalent to the rate  $\Gamma(b \rightarrow sX)$  of the inclusive decay  $b \rightarrow sX$  for  $m_X < m_b - m_s$ . Accordingly, one can rewrite  $\Gamma_s^{12,X}$  using the formula for  $\Gamma(b \rightarrow sX)$  derived from  $\mathcal{L}_{bsX}$  above, with  $|g_{V,A}|^2$  replaced with  $g_{V,A}^2$ . Thus

$$\Gamma_s^{12,X} \simeq \frac{|\vec{p}_X|}{8\pi m_b^2 m_X^2} \{g_V^2 [(m_b + m_s)^2 + 2m_X^2] [(m_b - m_s)^2 - m_X^2] + g_A^2 [(m_b - m_s)^2 + 2m_X^2] [(m_b + m_s)^2 - m_X^2]\}, \quad (16)$$

where  $\vec{p}_X$  is the 3-momentum of  $X$  in the rest frame of  $b$ .

As it turns out, however, for  $m_X \gtrsim 3$  GeV we find that  $\Gamma_s^{12,X}$  evaluated using Eq. (16) is numerically less than that using Eq. (15) with the sum being over  $f_X = \eta X, \eta' X$ , and  $\phi X$  alone. This is an indication that the approximation in Eq. (16) is no longer good for these larger values of  $m_X$ , as soft QCD effects are no longer negligible in relating  $b \rightarrow sX$  to the corresponding  $\bar{B}_s$  process. To get around this problem, for  $m_X \gtrsim 3$  GeV we take  $\Gamma_s^{12,X}$  to be that given by Eq. (15) with the sum being over  $f_X = \eta X, \eta' X, \phi X$  and neglect the effects of states  $f_X$  involving mesons heavier than the  $\phi$  due to the smaller phase space of those states. Hence, to evaluate  $\Gamma_s^{12,X}$  in this case requires the  $\bar{B}_s \rightarrow (\eta, \eta', \phi)$  matrix elements of the  $b \rightarrow s$  operators in  $\mathcal{L}_{bsX}$ , which we expect take into account, at least partly, the soft QCD effects not included in Eq. (16). The matrix element relevant to  $\bar{B}_s \rightarrow PX$ , with  $P = \eta$  or  $\eta'$ , is

$$\varepsilon_X^{*\mu} \langle P(p_P) | \bar{s} \gamma_\mu b | \bar{B}_s(p_{B_s}) \rangle = 2\varepsilon_X^* \cdot p_P F_1^{B_s P}, \quad (17)$$

where  $k = p_{B_s} - p_P = p_X$ , the form-factor  $F_1^{B_s P}$  depends on  $k^2 = m_X^2$ , and we have used the fact that the  $X$  polarization  $\varepsilon_X$  and momentum  $p_X$  satisfy the relation  $\varepsilon_X^* \cdot p_X = 0$ . For  $\bar{B}_s \rightarrow \phi X$  we need

$$\varepsilon_X^{*\mu} \langle \phi(p_\phi) | \bar{s} \gamma_\mu b | \bar{B}_s(p_{B_s}) \rangle = \frac{2V^{B_s \phi}}{m_{B_s} + m_\phi} \epsilon_{\mu\nu\sigma\tau} \varepsilon_X^{*\mu} \varepsilon_\phi^{*\nu} p_{B_s}^\sigma p_\phi^\tau, \quad (18)$$

$$\varepsilon_X^{*\mu} \langle \phi(p_\phi) | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}_s(p_{B_s}) \rangle = iA_1^{B_s \phi} (m_{B_s} + m_\phi) \varepsilon_X^* \cdot \varepsilon_\phi^* - \frac{2iA_2^{B_s \phi} \varepsilon_\phi^* \cdot k}{m_{B_s} + m_\phi} \varepsilon_X^* \cdot p_\phi, \quad (19)$$

where  $k = p_{B_s} - p_\phi = p_X$ , and the form-factors  $V^{B_s \phi}$  and  $A_{1,2}^{B_s \phi}$  are all functions of  $k^2 = m_X^2$ . The amplitudes for  $\bar{B}_s \rightarrow PX$  and  $\bar{B}_s \rightarrow \phi X$  are then

$$\mathcal{M}(\bar{B}_s \rightarrow PX) = 2g_V F_1^{B_s P} \varepsilon_X^* \cdot p_P, \quad (20)$$

$$\mathcal{M}(\bar{B}_s \rightarrow \phi X) = -ig_A \left[ A_1^{B_s \phi} (m_{B_s} + m_\phi) \varepsilon_\phi^* \cdot \varepsilon_X^* - \frac{2A_2^{B_s \phi} (\varepsilon_\phi^* \cdot p_X) (\varepsilon_X^* \cdot p_\phi)}{m_{B_s} + m_\phi} \right] + \frac{2g_V V^{B_s \phi}}{m_{B_s} + m_\phi} \epsilon_{\mu\nu\sigma\tau} \varepsilon_\phi^{*\mu} \varepsilon_X^{*\nu} p_\phi^\sigma p_X^\tau. \quad (21)$$

It follows that for  $m_X \gtrsim 3$  GeV we have

$$\Gamma_s^{12,X} \simeq \Gamma_s^{12,X}(\eta X) + \Gamma_s^{12,X}(\eta' X) + \Gamma_s^{12,X}(\phi X), \quad (22)$$

$$\Gamma_s^{12,X}(PX) = \frac{g_V^2 |\vec{p}_P|^3}{2\pi m_X^2} (F_1^{B_s P})^2,$$

$$\Gamma_s^{12,X}(\phi X) = \frac{|\vec{p}_\phi|}{8\pi m_{B_s}^2} (H_0^2 + H_+^2 + H_-^2) \quad (23)$$

in the  $B_s$  rest-frame, where [17]  $H_0 = -ax - b(x^2 - 1)$  and  $H_\pm = a \pm c\sqrt{x^2 - 1}$ , with

$$\begin{aligned} a &= g_A A_1^{B_s \phi} (m_{B_s} + m_\phi), & b &= -\frac{2g_A A_2^{B_s \phi} m_\phi m_X}{m_{B_s} + m_\phi}, \\ c &= -\frac{2g_V V^{B_s \phi} m_\phi m_X}{m_{B_s} + m_\phi}, & x &= \frac{m_{B_s}^2 - m_\phi^2 - m_X^2}{2m_\phi m_X}, \\ |\vec{p}_M| &= \frac{1}{2m_{B_s}} [(m_{B_s}^2 + m_M^2 - m_X^2)^2 - 4m_{B_s}^2 m_M^2]^{1/2}, \end{aligned} \quad (24)$$

and  $F_1^{B_s P}$ ,  $A_{1,2}^{B_s \phi}$ ,  $V^{B_s \phi}$  all evaluated at  $k^2 = m_X^2$ . For numerical work in the next section, we employ  $F_1^{B_s \eta}(k^2) = -F_1^{B_d K}(k^2) \sin \varphi$ , and  $F_1^{B_s \eta'}(k^2) = F_1^{B_d K}(k^2) \cos \varphi$  [18], with  $\varphi = 39.3^\circ$  [19] and the  $B_d \rightarrow K$  form-factor  $F_1^{B_d K}(k^2)$  from Ref. [20], as well as  $A_{1,2}^{B_s \phi}(k^2)$  and  $V^{B_s \phi}(k^2)$  from Ref. [21].

### 3. Numerical analysis

We start with the constraints imposed by  $\Delta M_s^{\text{exp}}$  in Eq. (3). In this case, it is appropriate to use the approximate formula  $\Delta M_s \simeq 2|M_s^{12}|$ , from Eq. (8), with  $M_s^{12}$  given in Eq. (11) and the  $X$  contribution in Eq. (13). For numerical inputs, we adopt the SM numbers in Eq. (7),  $f_{B_s} = 240$  MeV,  $m_b(m_b) = 4.20$  GeV,  $m_s(m_b) = 80$  MeV [3,6],  $P_1^{\text{VLL}} = 0.84$ ,  $P_1^{\text{SL}} = -1.47$ ,  $P_1^{\text{LR}} = -1.62$ ,  $P_2^{\text{LR}} = 2.46$  [16], and meson masses from Ref. [7]. In Fig. 1 we show the ranges of  $\text{Re } g_V$  and  $\text{Im } g_V$  satisfying  $\Delta M_s^{\text{exp}} = 2|M_s^{12}|$  for  $m_X = 2$  and 4 GeV, respectively, where for simplicity we have set the coupling  $g_A$  to zero. The contours in each of the plots correspond to variations of the SM contribution  $M_s^{12,\text{SM}}$ , which has an error of 40%, as quoted in Eq. (7). Evidently, both  $\text{Re } g_V$  and  $\text{Im } g_V$  can be as large as a few times  $10^{-5}$ . Assuming  $g_V = 0$  instead, we get allowed regions for  $\text{Re } g_A$  and  $\text{Im } g_A$  which are roughly almost three times smaller, with the vertical and horizontal axes interchanged. These restrictions from  $\Delta M_s^{\text{exp}} = 2|M_s^{12}|$  turn out to be weaker than the ones we consider below using other  $B_s$  observables.

Before proceeding, it is of interest also at this point to see how  $g_{V,A}$  may compare to the analogous flavor-changing couplings  $\bar{g}_{V,A}$  of  $X$  to a pair of  $d$  and  $s$  quarks, subject to constraints from kaon-mixing data. For definiteness, we take  $m_X = 2$  GeV, which is one of the values considered in our numerical examples. In this case, the pertinent observables are the mass difference

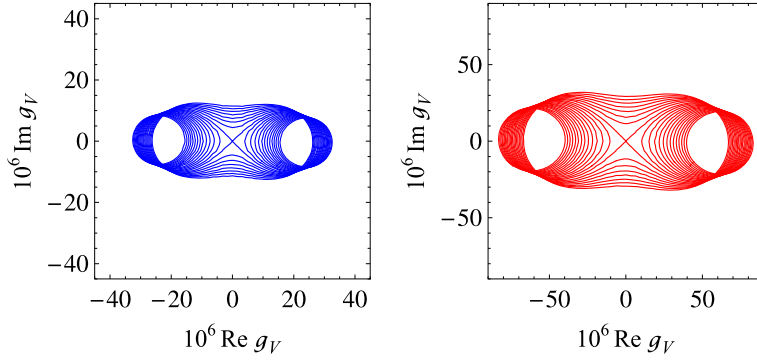


Fig. 1. Regions of  $\text{Re } g_V$  and  $\text{Im } g_V$  allowed by  $\Delta M_S^{\text{exp}} = 2|M_S^{12}|$  constraint for  $m_X = 2$  GeV (left plot) and  $m_X = 4$  GeV (right plot) under the assumption  $g_A = 0$ .

$\Delta M_K$  between  $K_L$  and  $K_S$  and the  $CP$ -violation parameter  $\epsilon_K$ , which are related to the mass matrix element  $M_K^{12} = M_K^{12,\text{SM}} + M_K^{12,X}$  by  $\Delta M_K = 2 \text{Re } M_K^{12} + \Delta M_K^{\text{LD}}$  and  $\epsilon_K = \text{Im } M_K^{12} / (\sqrt{2} \Delta M_K^{\text{exp}})$ , where  $M_K^{12,\text{SM}}$  ( $M_K^{12,X}$ ) parameterizes the short-distance SM ( $X$ ) contribution and  $\Delta M_K^{\text{LD}}$  contains long-distance effects [15]. The SM can accommodate the measured value  $\Delta M_K^{\text{exp}} = (3.483 \pm 0.006) \times 10^{-12}$  MeV [7], although the calculation of  $\Delta M_K^{\text{LD}}$  suffers from significant uncertainties [15], whereas the SM prediction  $|\epsilon_K|_{\text{SM}} = (2.01^{+0.59}_{-0.66}) \times 10^{-3}$  [22] agrees well with the data,  $|\epsilon_K|_{\text{exp}} = (2.228 \pm 0.011) \times 10^{-3}$  [7]. Accordingly, it is reasonable to require the  $X$  contributions to satisfy  $2 \text{Re } M_K^{12,X} < 3.4 \times 10^{-12}$  MeV and  $|\text{Im } M_K^{12,X}| / (\sqrt{2} \Delta M_K^{\text{exp}}) < 0.7 \times 10^{-3}$ . The expression for  $M_K^{12,X}$  can be obtained from Eq. (13) after making the appropriate replacements, namely with the new numbers  $f_K = 160$  MeV,  $m_K = 498$  MeV,  $m_s(\mu) = 115$  MeV,  $m_d/m_s \simeq 0$ ,  $P_1^{\text{VLL}} = 0.48$ ,  $P_1^{\text{SLL}} = -18.1$ ,  $P_1^{\text{LR}} = -36.1$ , and  $P_2^{\text{LR}} = 59.3$  at the scale  $\mu = 2$  GeV [16]. With  $m_X = 2$  GeV and the above requirements on the  $X$  contributions, assuming  $\bar{g}_A = 0$  then leads one to  $(\text{Im } \bar{g}_V)^2 - (\text{Re } \bar{g}_V)^2 \lesssim 4 \times 10^{-14}$  and  $|(\text{Re } \bar{g}_V)(\text{Im } \bar{g}_V)| \lesssim 4 \times 10^{-17}$ , implying that  $|\bar{g}_V| \lesssim 2 \times 10^{-7}$ . Similarly, setting  $\bar{g}_V = 0$  instead, one extracts  $|\bar{g}_A| \lesssim 2 \times 10^{-7}$ . These bounds on  $\bar{g}_{V,A}$  from kaon data are much stronger than those on  $g_{V,A}$  derived from  $\Delta M_S^{\text{exp}}$  in the previous paragraph. However, as we will see in the following, the corresponding values of  $g_{V,A}$  that can reproduce the  $D_0$  measurement are much smaller and have bounds roughly similar to these  $\bar{g}_{V,A}$  numbers. Thus, although in our model-independent approach  $\bar{g}_{V,A}$  are not necessarily related to  $g_{V,A}$ , this exercise serves to illustrate that in models where the two sets of couplings are expected to be comparable in size it is possible to satisfy both kaon-mixing and  $B_s$  data.

To explore the ranges of  $g_{V,A}$  allowed by the other  $B_s$  quantities,  $\Delta \Gamma_s^{\text{exp}}$  and  $a_{\text{sl}}^{s,\text{exp}}$ , besides  $\Delta M_S^{\text{exp}}$ , their values being quoted in Eqs. (1) and (3), we employ the exact relations written in Eqs. (5) and (6), although one would arrive at similar results with the approximate formulas in Eqs. (8) and (9). The relevant SM numbers are listed in Eq. (7). For the  $X$  contributions, we have  $M_S^{12,X}$  in Eq. (13), whereas  $\Gamma_s^{12,X}$  is from Eq. (16) if  $m_X < 3$  GeV and from Eq. (22) otherwise. To simplify our analysis, we again assume only one of  $g_{V,A}$  to be contributing at a time, setting the other one to zero.

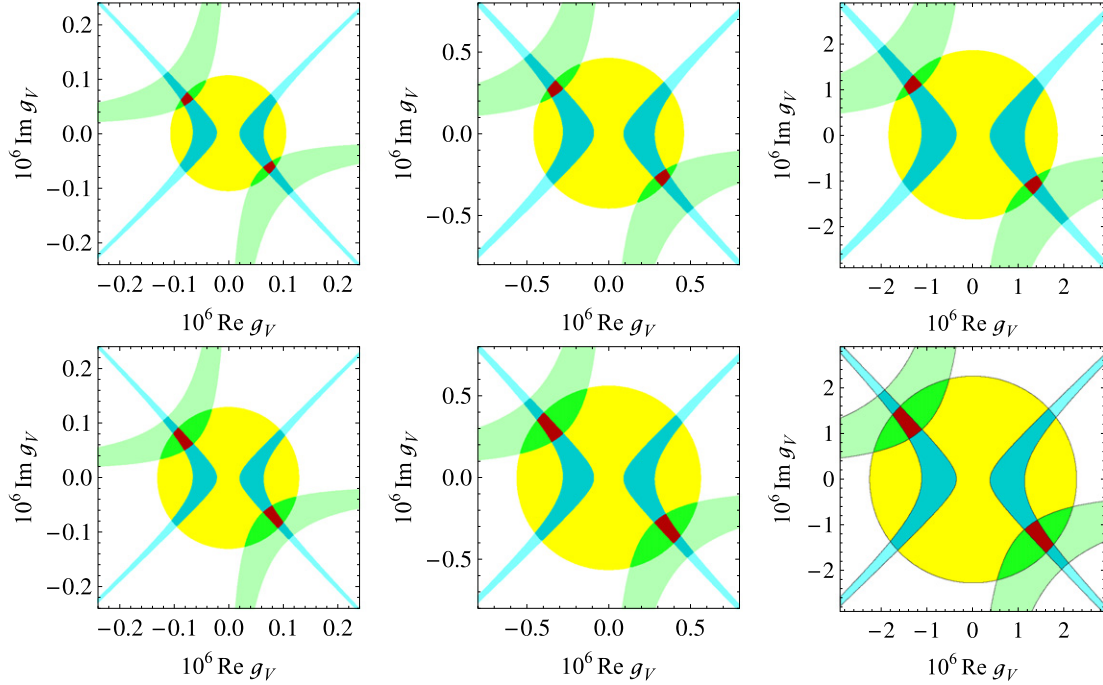
We also need to take into account the inclusive decay  $b \rightarrow sX$  because it provides constraints on  $g_{V,A}$  via its contribution,  $\Gamma(b \rightarrow sX)$ , to the  $B_s$  total-width  $\Gamma_{B_s}$ . Though the experimental value of  $\Gamma_{B_s}$  is fairly well determined,  $\Gamma_{B_s}^{\text{exp}} = 0.70 \pm 0.02 \text{ ps}^{-1}$  [7], its theoretical prediction in the SM involves significant uncertainties, mainly due to  $\Gamma_{B_s}^{\text{SM}}$  being proportional to  $m_b^5$  at leading order in the  $1/m_b$  expansion [23]. With  $m_b$  having its PDG value [7],

the error of  $\Gamma_{B_s}^{\text{SM}}$  from  $m_b^5$  alone would be of order 20%. There are additional uncertainties from the  $f_{B_s}^2$  dependence of  $\Gamma_{B_s}^{\text{SM}}$ , as  $f_{B_s} = 240 \pm 40$  MeV [6], but they occur at subleading order in the  $1/m_b$  expansion [24]. Conservatively, we then require  $\Gamma(b \rightarrow sX)$  to be smaller than  $0.15 \Gamma_{B_s} \simeq 0.1 \text{ ps}^{-1}$ , but we will also assume, alternatively, the somewhat bigger upper-bound of  $0.15 \text{ ps}^{-1}$ . We will comment on the implications of  $\Gamma(b \rightarrow sX)$  bounds stricter than  $\Gamma(b \rightarrow sX) < 0.1 \text{ ps}^{-1}$  as well.

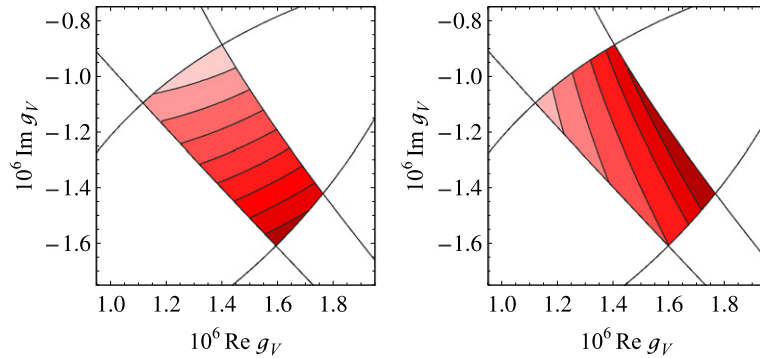
We remark that the same  $\Gamma(b \rightarrow sX)$  also contributes to the total widths  $\Gamma_{B_d}$  and  $\Gamma_{B_u}$  of the  $B_d$  and  $B_u^+$  mesons, respectively, the SM calculations of which involve sizable uncertainties similar to that of  $\Gamma_{B_s}^{\text{SM}}$ . Since the SM predicts the width ratios  $\Gamma_{B_d}/\Gamma_{B_s}$  and  $\Gamma_{B_u}/\Gamma_{B_s}$  to be only a few percent away from unity [24], it follows that the  $\Gamma(b \rightarrow sX)$  contributions to  $\Gamma_{B_s, B_d, B_u}$  respect the experimental numbers  $\Gamma_{B_d}/\Gamma_{B_s} = 1.05 \pm 0.06$  and  $\Gamma_{B_u}/\Gamma_{B_s} = 1.071 \pm 0.009$  [7].

In Fig. 2, we display the allowed values of  $\text{Re } g_V$  and  $\text{Im } g_V$ , assuming  $g_A = 0$ , subject to the requirements from the one-sigma ranges of  $\Delta M_S^{\text{exp}}$ ,  $\Delta \Gamma_s^{\text{exp}}$ , and  $a_{\text{sl}}^{s,\text{exp}}$  applied in Eqs. (5) and (6), plus the restrictions on  $\Gamma(b \rightarrow sX)$ . For the reasons described in the preceding section, in drawing this figure we have employed the expression for  $\Gamma(b \rightarrow sX)$  from  $\Gamma_s^{12,X}$  in Eq. (16) if  $m_X < 3$  GeV and Eq. (22) otherwise, with  $g_V^2$  and  $g_A^2$  replaced by  $|g_V|^2$  and  $|g_A|^2$ , respectively. Choosing  $m_X = 0.5, 2, 4$  GeV for illustration, we have imposed  $\Gamma(b \rightarrow sX) < 0.1 \text{ ps}^{-1}$  in the upper plots and  $\Gamma(b \rightarrow sX) < 0.15 \text{ ps}^{-1}$  in the lower ones. On each plot, the (blue) regions satisfying the  $\Delta M_S^{\text{exp}} \Delta \Gamma_s^{\text{exp}}$  constraint lie on all the four quadrants and are narrower than the (green) regions satisfying  $a_{\text{sl}}^{s,\text{exp}}$ , which lie on only the second and fourth quadrants. The circular (yellow) regions represent the  $\Gamma(b \rightarrow sX)$  bounds. Clearly, there is parameter space of  $X$  (in dark red) that can cover part of the one-sigma range of  $a_{\text{sl}}^{s,\text{exp}}$  and is simultaneously allowed by the other two constraints. In each  $m_X$  case, the overlap area allowed by all the constraints is significantly larger with the less restrictive bound  $\Gamma(b \rightarrow sX) < 0.15 \text{ ps}^{-1}$ . Evidently, the size of each of the areas corresponding to the different constraints is sensitive to the value of  $m_X$  and increases as the latter grows. The overlap region satisfying all the constraints also increases in size with  $m_X$ .

To illustrate in more detail the impact of  $X$  on the values of  $|\Gamma_s^{12}|$  and  $\sin \phi_s$  corresponding to the parameter space allowed by all the constraints, we display the graphs in Fig. 3 in the case of  $m_X = 4$  GeV and  $\Gamma(b \rightarrow sX) < 0.15 \text{ ps}^{-1}$ . These (red) shaded regions are none other than the (dark red) overlap region in the fourth quadrant of the lower-right plot in Fig. 2, but one could alternatively use the overlap region in the second quadrant. The left plot in Fig. 3 indicates that the size of  $|\Gamma_s^{12}|$  can be enhanced to 3.1 times the central value of  $|\Gamma_s^{12,\text{SM}}|$ . Furthermore, from the right plot, the magnitude of  $\sin \phi_s$  can be increased to almost 1, which is roughly a few hundred times larger than its SM value.



**Fig. 2.** Regions of  $\text{Re } g_V$  and  $\text{Im } g_V$  allowed by  $a_{\text{sl}}^{s,\text{exp}}$  constraint (green),  $\Delta M_s^{\text{exp}} \Delta \Gamma_s^{\text{exp}}$  constraint (blue),  $\Gamma(b \rightarrow sX) < 0.1 \text{ ps}^{-1}$  (yellow), and all of them (dark red) for  $m_X = 0.5 \text{ GeV}$  (upper left plot),  $2 \text{ GeV}$  (upper middle plot), and  $4 \text{ GeV}$  (upper right plot), under the assumption  $g_A = 0$ . The lower plots are the same as the upper ones, except that  $\Gamma(b \rightarrow sX) < 0.15 \text{ ps}^{-1}$ . (For interpretation of the references to color, the reader is referred to the web version of this Letter.)



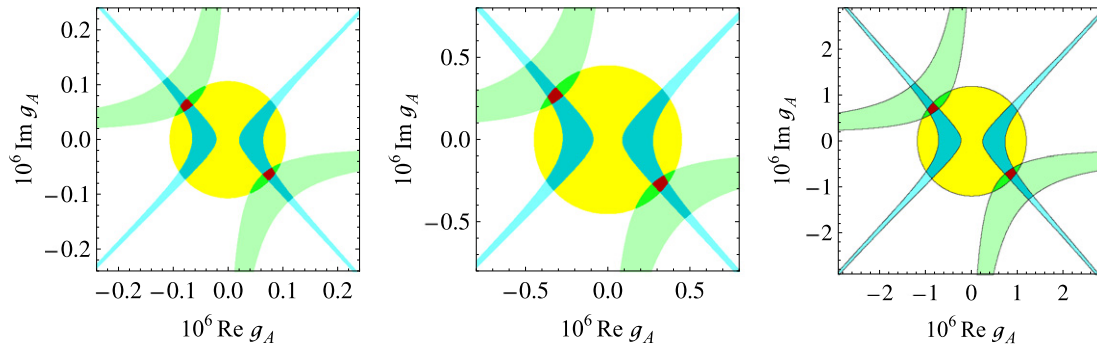
**Fig. 3.** Values of  $|\Gamma_s^{12}|$  (left plot) and  $\sin \phi_s$  (right plot) for  $m_X = 4 \text{ GeV}$  and the  $(\text{Re } g_V, \text{Im } g_V)$  overlap region in the fourth quadrant of the lower-right plot in Fig. 2 allowed by all the constraints, with  $\Gamma(b \rightarrow sX) < 0.15 \text{ ps}^{-1}$ . In the left plot, from darkest to lightest, the differently shaded (red colored) areas correspond to  $|\Gamma_s^{12}|/\Gamma_s^{12,\text{SM}} > 3.1, 2.9, 2.7, \dots, 1.5$ , respectively, with each region including the area of the next darker region and  $|\Gamma_s^{12,\text{SM}}|$  being its central value. Similarly, in the right plot, from darkest to lightest  $\sin \phi_s < -0.99, -0.98, -0.96, -0.93, -0.89, -0.85$ . (For interpretation of the references to color, the reader is referred to the web version of this Letter.)

Combining them leads to  $-0.016 \lesssim a_{\text{sl}}^s \lesssim -0.007$ . Thus the enhancement of  $|\Gamma_s^{12}| \sin \phi_s$  generated by the  $X$  contribution can be sufficiently sizable to yield a prediction for  $a_{\text{sl}}^s$  which can reach most of the one-sigma range of the anomalous  $a_{\text{sl}}^{s,\text{exp}}$ , including its central value. For lower values of  $m_X$ , the situations are similar, as can be inferred from the lower plots in Fig. 2, although the allowed  $(\text{Re } g_V, \text{Im } g_V)$  areas are smaller. With the more restrictive bound  $\Gamma(b \rightarrow sX) < 0.1 \text{ ps}^{-1}$ , part of the one-sigma range of  $a_{\text{sl}}^{s,\text{exp}}$  can still be reproduced, as the upper plots in Fig. 2 imply, but its central value is no longer reachable.

If we require a bound stricter than  $\Gamma(b \rightarrow sX) < 0.1 \text{ ps}^{-1}$ , then the  $X$  contribution may not be able to lead to any of the one-sigma values of  $a_{\text{sl}}^{s,\text{exp}}$ . However, in that case there can still be  $(\text{Re } g_V, \text{Im } g_V)$  regions allowed by all the constraints if one considers instead 90%-C.L. ranges of  $a_{\text{sl}}^{s,\text{exp}}$ ,  $\Delta M_s^{\text{exp}}$ , and  $\Delta \Gamma_s^{\text{exp}}$ . More definite statements about this would have to await more precise data on  $a_{\text{sl}}^s$  from future experiments.

In the case that  $g_V = 0$ , we show in Fig. 4 the values of  $\text{Re } g_A$  and  $\text{Im } g_A$  allowed by the constraints from  $a_{\text{sl}}^{s,\text{exp}}$  and  $\Delta M_s^{\text{exp}} \Delta \Gamma_s^{\text{exp}}$  at the one-sigma level. We have chosen  $m_X = 0.5, 2, 4 \text{ GeV}$  as before, but imposed only  $\Gamma(b \rightarrow sX) < 0.1 \text{ ps}^{-1}$ . The effects of  $X$  here can be seen to be qualitatively similar to those in the  $g_V \neq 0$  and  $g_A = 0$  case.

Finally, a few comments on distinguishing the scenario that we have proposed to reproduce the D0 result from the other proposals in the literature seem to be in order. Since the main feature in our proposal is the presence of a new light spin-1 boson with flavor-changing couplings to  $b$  and  $s$  quarks, if the D0 finding is confirmed by other experiments, the results of our model-independent study can serve to help motivate experimentalists to look for the particle in various  $b \rightarrow s$  transitions, such as by scrutinizing the dilepton-mass distributions in  $\bar{B}_s \rightarrow \eta^{(\prime)} \ell^+ \ell^-$ ,  $\phi \ell^+ \ell^-$ , and  $\bar{B}_d \rightarrow \bar{K}^{(*)0} \ell^+ \ell^-$  in case it has sufficient branching ratios into  $\ell^+ \ell^-$ . Since its couplings tend to be very small, the searches for



**Fig. 4.** Regions  $\text{Re } g_A$  and  $\text{Im } g_A$  allowed by  $a_{\text{sl}}^{s,\text{exp}}$  constraint (green),  $\Delta M_s^{\text{exp}} \Delta \Gamma_s^{\text{exp}}$  constraint (blue),  $\Gamma(b \rightarrow sX) < 0.1 \text{ ps}^{-1}$  (yellow), and all of them (dark red) for  $m_X = 0.5 \text{ GeV}$  (left plot),  $2 \text{ GeV}$  (middle plot), and  $4 \text{ GeV}$  (right plot), under the assumption  $g_V = 0$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

the particle would require a high degree of precision, which could hopefully be realized at LHCb or future  $B$  factories. If a new spin-1 particle is discovered in a measurement of some  $b \rightarrow s$  transition, to proceed and examine if the particle is the one that can reproduce the D0 anomaly, it would be necessary to invoke model dependence, as different models containing such a particle would likely have different values of the additional flavor-conserving and flavor-violating couplings which the particle might have to various fermions, subject to other experimental data. The adoption of model specifics would also be unavoidable in order to distinguish this scenario from other new-physics scenarios which could account for the D0 anomaly without a nonstandard light spin-1 boson, especially if it turned out to be experimentally elusive. If a new light spin-1 particle were to be detected first outside the  $B$  sector, it would again be necessary to have a model to make connections to the  $B$  sector.

#### 4. Conclusions

We have investigated the possibility that the anomalous like-sign dimuon charge asymmetry in semileptonic  $b$ -hadron decays recently measured by the D0 Collaboration arises from the contribution of a light spin-1 particle,  $X$ , to the mixing of  $B_s$  mesons. Taking a model-independent approach, we have assumed only that  $X$  is lighter than the  $b$  quark, carries no color or electric charge, and has vector and axial-vector  $bsX$  couplings. Thus, in contrast to a heavy  $Z'$  particle,  $X$  can be produced as a physical particle in  $B_s$  decay, and so it affects not only the mass matrix element  $M_s^{12}$ , but also the decay matrix element  $\Gamma_s^{12}$ . We have found that the  $X$  contribution can enhance the magnitude of  $\Gamma_s^{12}$  as well as the relative  $CP$ -violating phase  $\phi_s$  between  $M_s^{12}$  and  $\Gamma_s^{12}$  by a significant amount. More precisely, taking into account experimental constraints from a number of  $B_s$  observables, namely  $\Delta M_s$ ,  $\Delta \Gamma_s$ , and  $\Gamma_{B_s}$ , we have shown that the effect of  $X$  can increase  $|\Gamma_s^{12}|$  to become a few times greater than its SM prediction and enlarge the size of  $\sin \phi_s$  by a factor of a few hundred. As a consequence, the  $X$  contribution can lead to a prediction for  $a_{\text{sl}}^s$  which is consistent with its anomalous value as measured by D0 within one standard-deviation and possibly even reaches its central value. We have therefore demonstrated that a light spin-1 particle can offer a viable explanation for the D0 anomaly. Whether or not future  $B_s$  experiments confirm the new D0 finding, the coming data will likely be useful for further probing new-physics scenarios involving light spin-1 particles.

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